ELECTRIC CHARGE & DIPOLE MOMENTS

- MASS & INTRINSIC ANGULAR MOMENTUM ARE UNIQUE ATTRIBUTES OF PARTICLES → CLASSIFICATION
- WHAT ABOUT ELECTRIC CHARGE?
- IT'S QUANTIZED – BUT OVERALL CHARGE MAY DEPEND ON INTERAL STRUCTURE → ATOMS

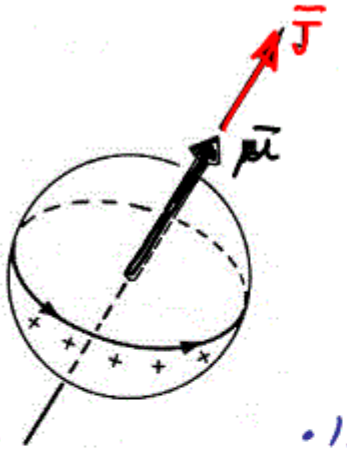
PROTON → CHARGE $+e$ → COMPOSITE

POSITRON → CHARGE $+e$ → POINTLIKE

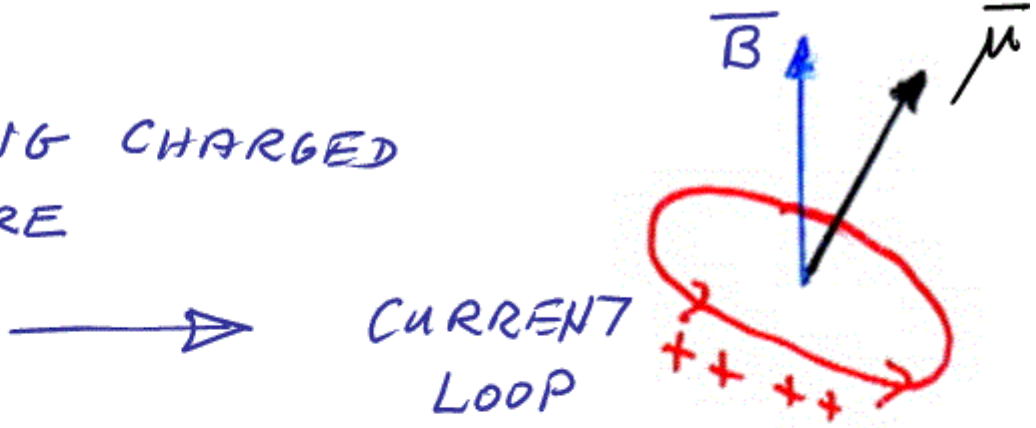
- MAGNETIC DIPOLE MOMENTS CAN ALLOW US TO CLASSIFY PARTICLES AS:

COMPOSITE ⟷ POINTLIKE

SEMICLASSICAL MAGNETIC MOMENTS



SPINNING CHARGED SPHERE



CURRENT LOOP

• INTERACTION ENERGY BETWEEN $\bar{\mu}$ AND APPLIED $\bar{B}$ FIELD

$$E_{MAG} = -\bar{\mu} \cdot \bar{B}$$

DO WORK

$$\mu = \frac{1}{c} \text{ CURRENT} \times \text{AREA}$$

SEE ELEMENTARY E&M

• PARTICLE CHARGE q, VELOCITY v IN CIRCULAR ORBIT — $\frac{v}{2\pi r}$ IS ORBITAL FREQUENCY

• CURRENT AROUND ORBIT $qv/2\pi r$

$$\mu = \text{CURRENT} \times \text{AREA}$$

$$= \frac{1}{c} \frac{qv}{2\pi r} \cdot \pi r^2$$

$$= \frac{1}{c} q \frac{vr}{2}$$

$$\bar{L} = \bar{r} \times \bar{p} \rightarrow m(\bar{r} \times \bar{v})$$

$$\bar{\mu} = \frac{q}{2mc} \bar{L}$$

CHARGE

MASS

$\bar{L}$ ← ANGULAR MOMENTUM

CLASSICALLY $\bar{\mu} = \frac{q}{2mc}\bar{L} \rightarrow \frac{\bar{\mu}}{\bar{L}} = q/2mc$

SAME RESULT IN QUANTUM MECHANICS FOR ORBITAL ANGULAR MOMENTUM CORRESPONDENCE PRINCIPLE

$$\bar{\mu} = g\frac{e}{2mc}\bar{J}$$

g IS MEASURE OF HOW $\bar{\mu}$ DEVIATES FROM $\frac{e}{2mc}J$

SHOULD BE 1 FOR CLASSICAL SYSTEM.

$$[J] \rightarrow [\hbar] \rightarrow \frac{J}{\hbar} \quad \text{DIMENSIONLESS}$$

$$\bar{\mu} = g\mu_0\frac{\bar{J}}{\hbar}$$

$$\mu_0 = \frac{e\hbar}{2mc}$$

$\mu_0 = e\hbar/2m_c \longrightarrow$ MAGNETON

$$\mu_B = \frac{e\hbar}{2m_e c} = 0.5788 \times 10^{-14} \text{ MeV/Gauss}$$

BOHR MAGNETON

m_e ELECTRON MASS

$$\mu_N = \frac{e\hbar}{2m_p c} = 3.1525 \times 10^{-18} \text{ MeV/G}$$

$$\mu_N \approx 2 \times 10^{-3} \mu_B$$

EVEN THOUGH e^+ AND PROTON HAVE IDENTICAL CHARGES, $\vec{\mu}$ ARE VERY DIFFERENT

DUE TO DIFFERENT MASSES

- ORBITAL ANGULAR MOMENTUM SEEMS TO WORK SAME CLASSICALLY & IN QUANTUM MECHANICS

- WHAT ABOUT INTRINSIC ANGULAR MOMENTUM WHICH HAS NO CLASSICAL ANALOG ?

$$\bar{\mu} = g \frac{e}{2mc} \bar{J}$$

- CLASSICAL POINT PARTICLE OR ORBITAL ANGULAR MOMENTUM $g = 1$

- FOR AN ELECTRON, IN THE DIRAC EQUATION, AND EXPERIMENTALLY $g = 2$

$$\mu_e = 2 \mu_B$$

ANOMALOUS MAGNETIC MOMENT

↑ CLASSICAL POINT

MAGNETIC MOMENT & STRUCTURE

- ELECTRON (OR μ, τ) IS A POINT DIRAC PARTICLE

  ← NO INTERNAL STRUCTURE

$$\mu_e = 2\,\mu_B$$

- IF A PROTON WERE A POINT DIRAC PARTICLE

$$\mu_p = 2\,\mu_N$$

- ACTUALLY $\mu_p = 2.79\,\mu_N$ ← STRUCTURE

- ELECTRICALLY NEUTRAL, NEUTRON $\mu_n = -1.91\,\mu_N$

  ELECTRICALLY CHARGED STRUCTURE