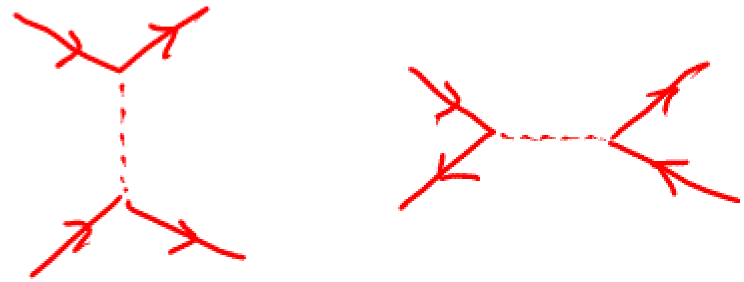
DECAYS MEDIATED BY FORCES

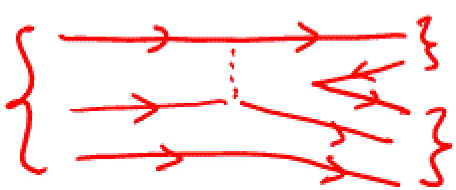
- GAUGE BOSONS

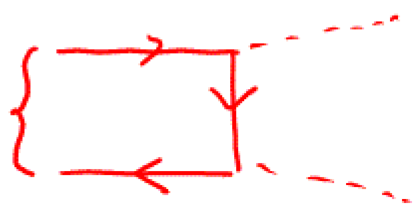
  γ^μ, $W^\pm$ Z^0, g

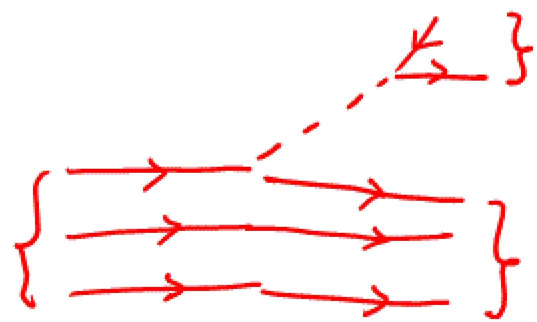
  MEDIATE INTERACTIONS



- THE GAUGE BOSONS CAN ALSO MEDIATE DECAYS

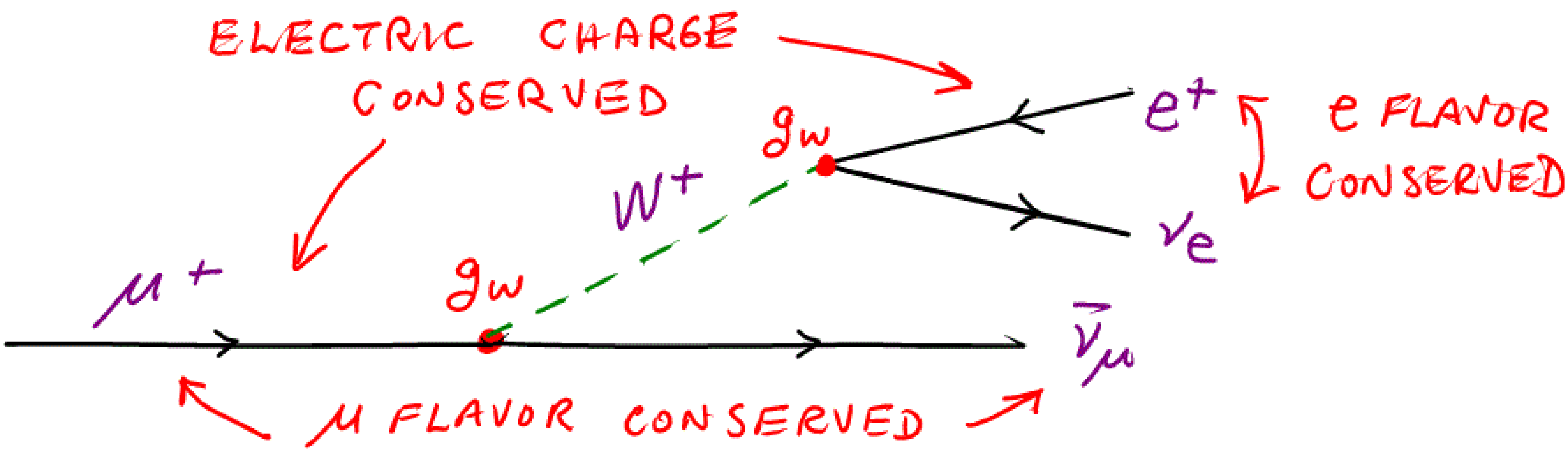






WEAK DECAYS OF LEPTONS

WEAK FORCE → "LONG" TIME TO INDUCE DECAY

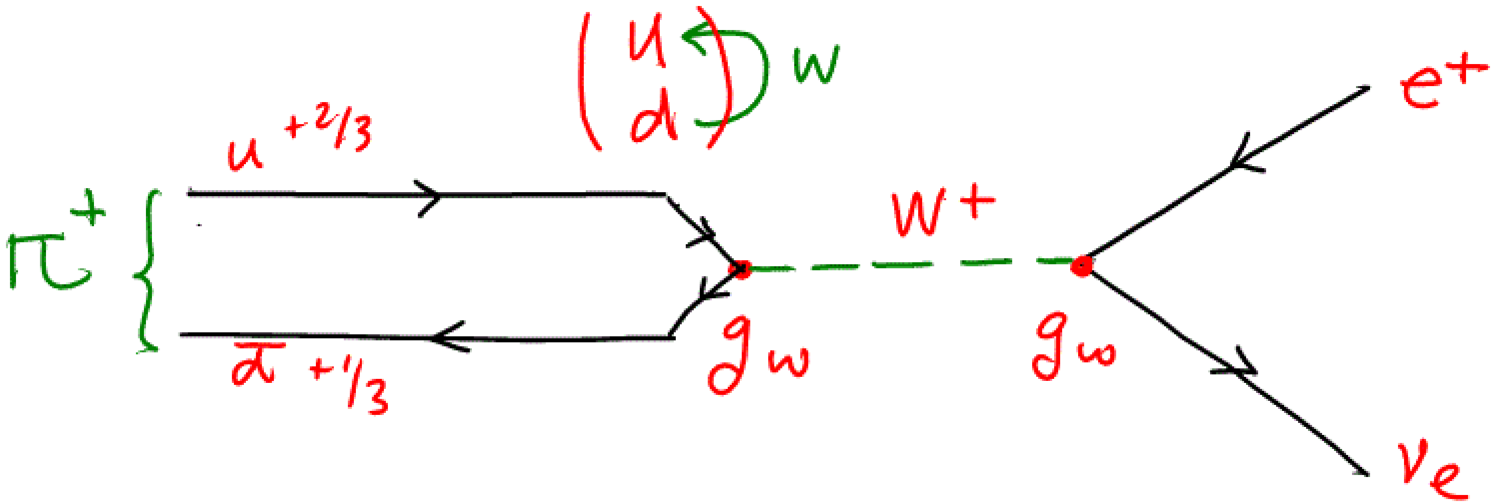


$$\tau(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \sim 2.2 \times 10^{-6}\,s$$

- τ - LEPTON DECAY IS EXACTLY SAME
- g_W SAME ; $\tau_\tau = 2.9 \times 10^{-13}\,s$ $\quad m_\tau >> m_\mu$ PHASE SPACE

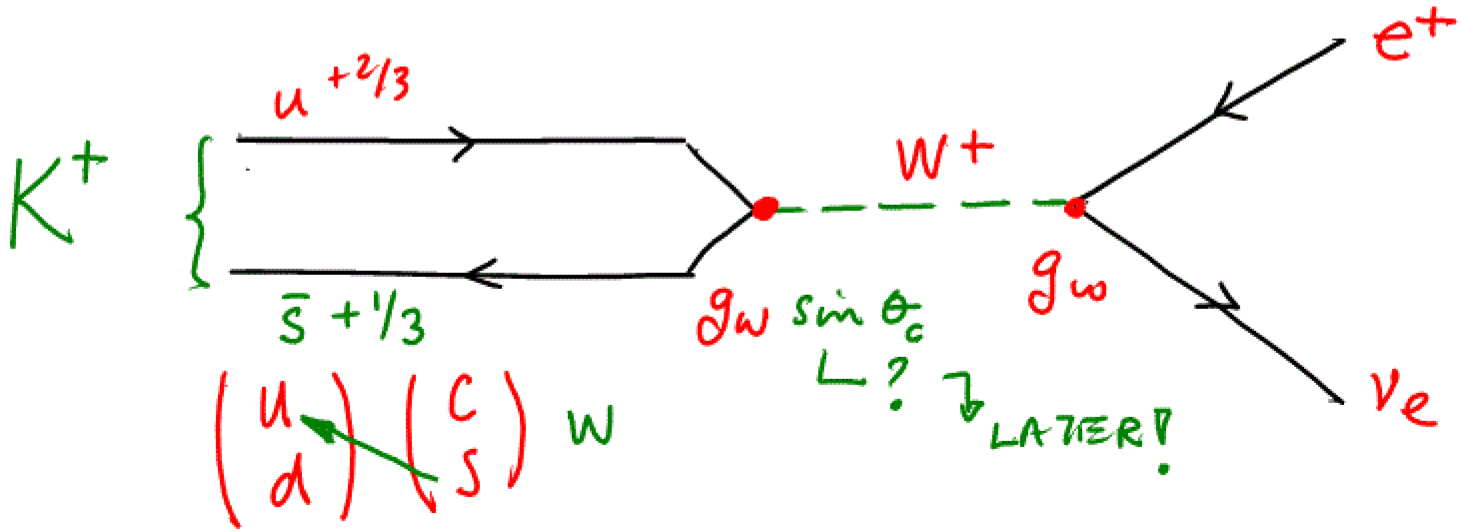
WEAK DECAYS OF MESONS & BARYONS

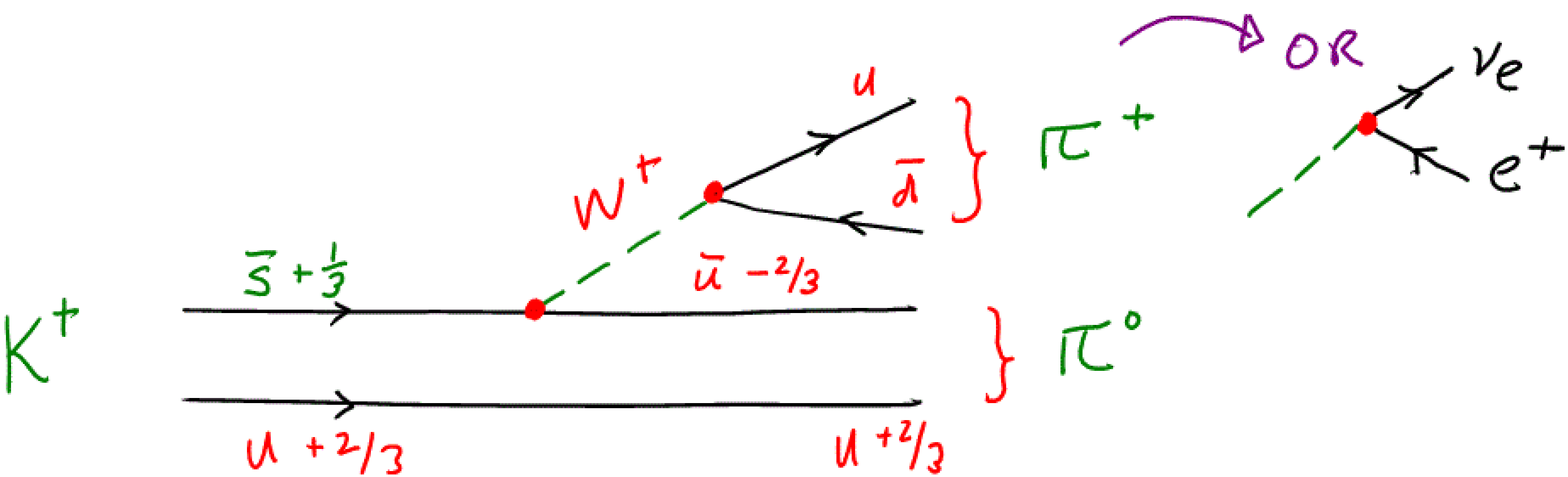
- ACTUALLY QUARK TRANSITIONS



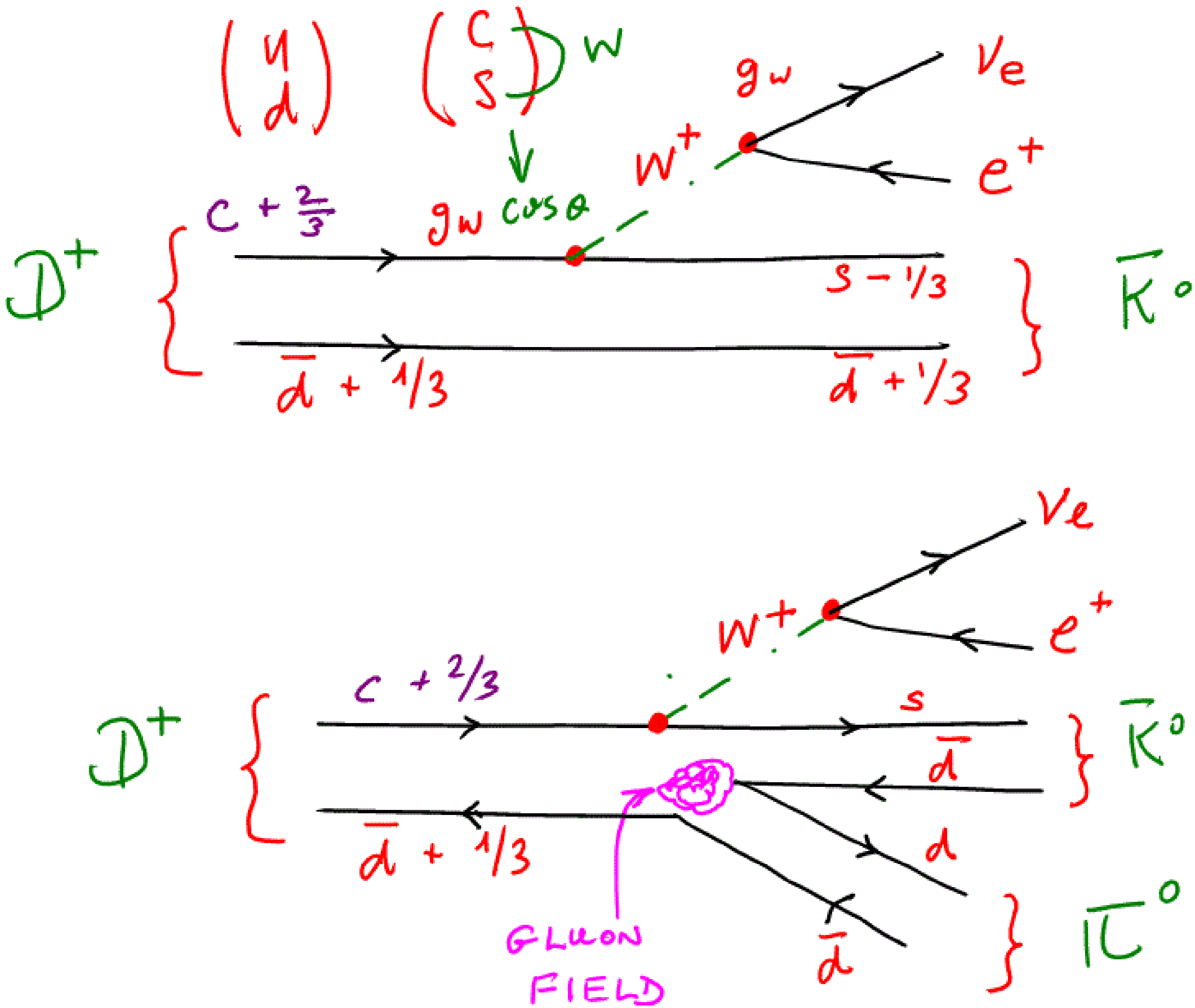
- ELECTRIC CHARGE CONSERVED AT BOTH VERTICES
- QUARK & LEPTONS HAVE SAME WEAK COUPLING CONSTANT g_w
- LEPTON FLAVOUR CONSERVED $e^+ \nu_e$
- QUARK FLAVOUR NOT CONSERVED.

KAON DECAY HAS AN ADDED TWIST

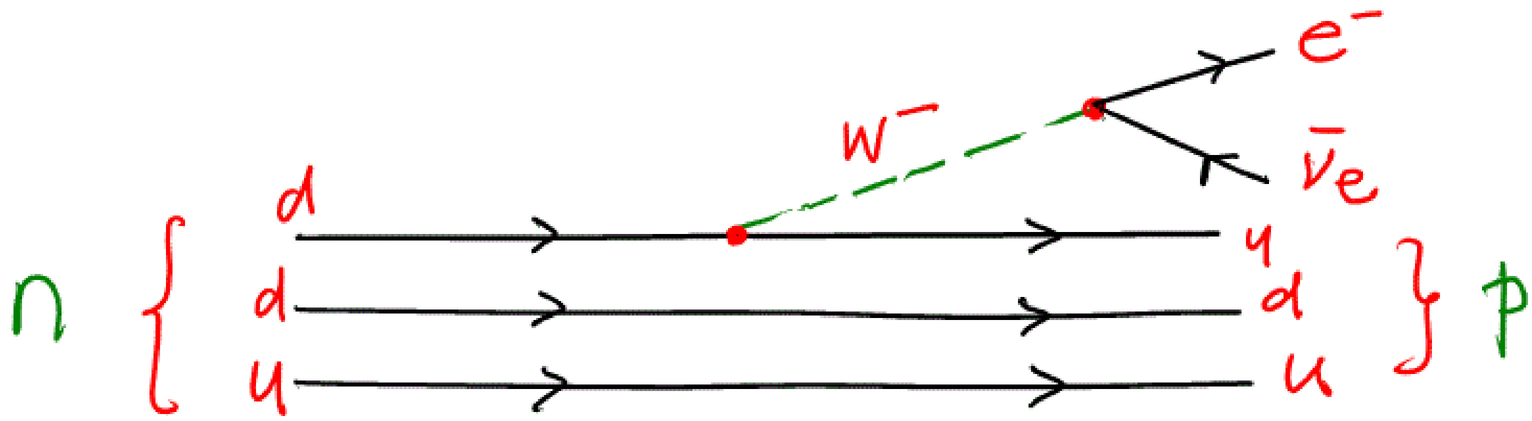


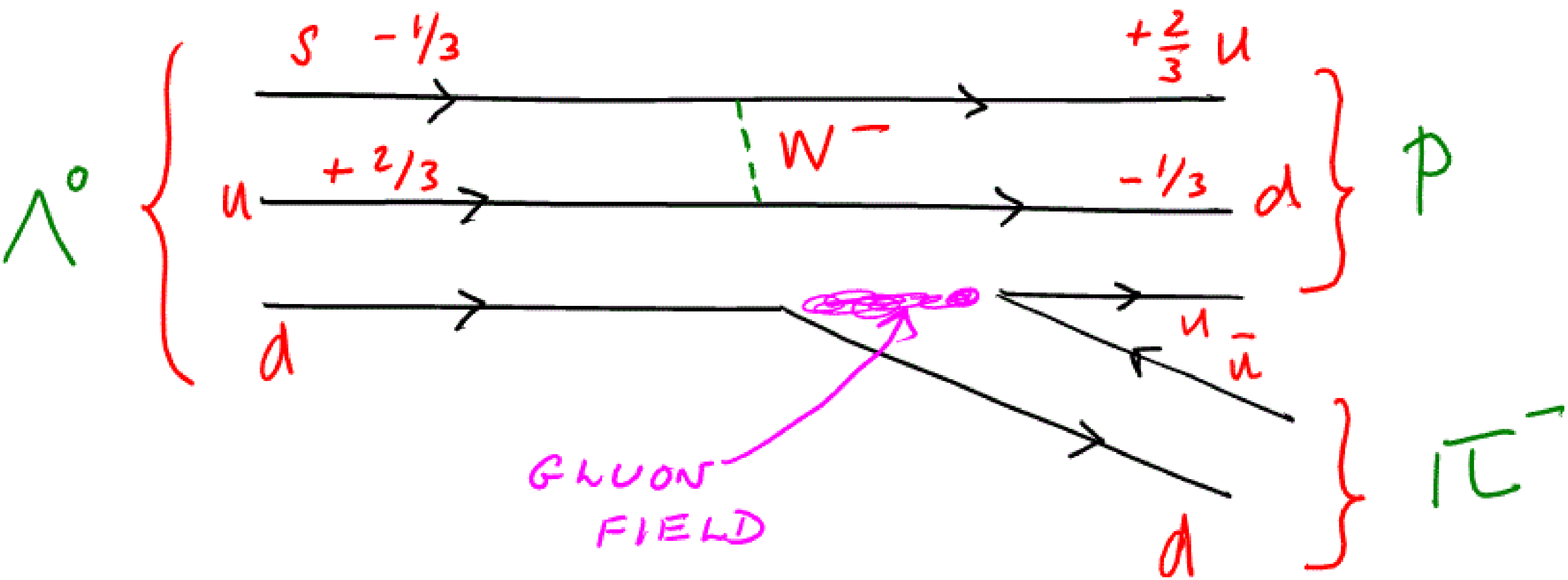


NEXT QUARK GENERATION

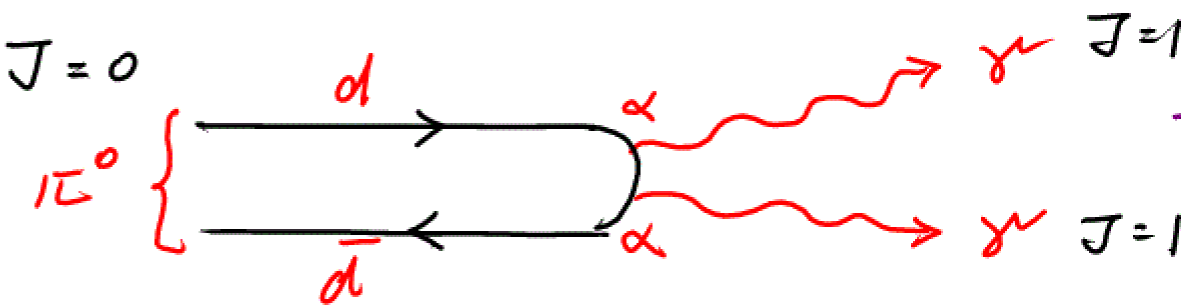


WEAK DECAYS OF BARYONS



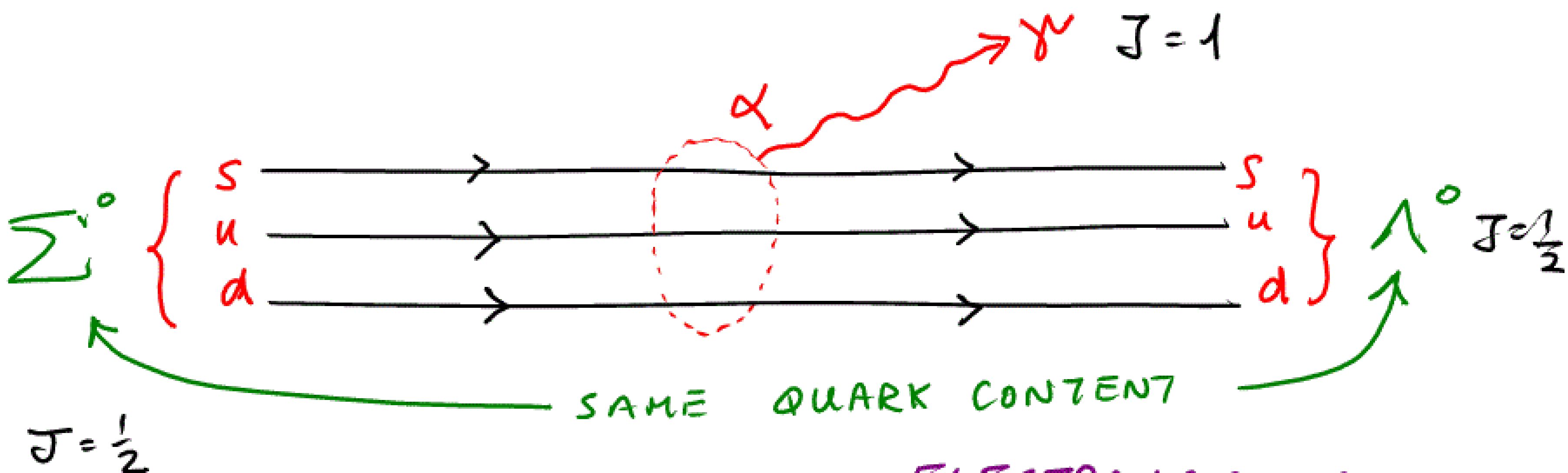


ELECTROMAGNETIC DECAYS



$\tau_{\pi^0} \sim 9 \times 10^{-17}$ s

- ELECTRIC CHARGE CONSERVED
- QUARK FLAVOR CONSERVED



$\tau_{\Sigma^0} \sim 7 \times 10^{-20}$ s

ELECTROMAGNETIC DECAYS MUCH FASTER THAN WEAK.

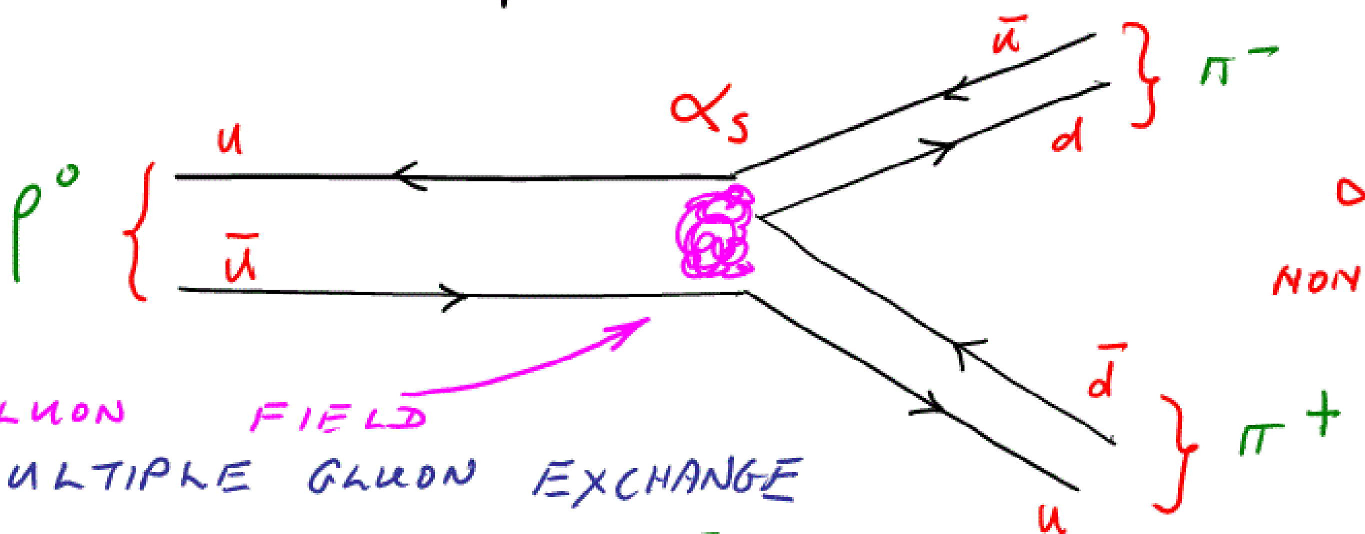
STRONG = COLOR = HADRONIC DECAYS

- LOOK BACK AT INVARIANT MASS OF ρ^0

  $\tau \sim 4 \times 10^{-24}\,s$ ← VERY FAST.

- THESE DIAGRAMS ARE MNEMONICS NOT FEYNMAN DIAGS.

  eg $|\rho\rangle \approx |u\bar{u}\rangle + |d\bar{d}\rangle$ SUPERPOSITION



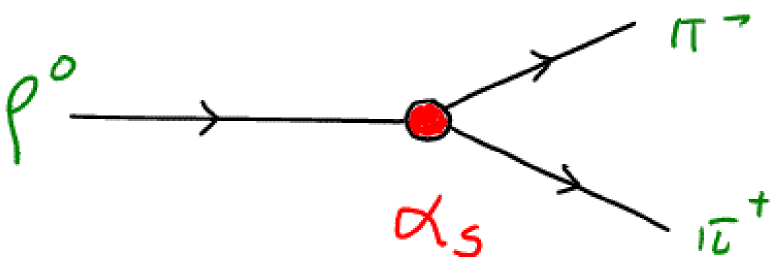
GLUON FIELD

MULTIPLE GLUON EXCHANGE

$\alpha_s \equiv \alpha_s(q^2)$

NON PERTURBATIVE

$\alpha_s(q^2) \sim \alpha_s(m_\pi)^2 \geq 1$



- IN BOTH INTERACTION & DECAYS

— STRONGEST FORCE WHICH CONSERVES RELEVANT QUANTUM NUMBERS DOMINATES

" THAT WHICH IS NOT FORBIDDEN IS COMPULSORY "

- DECAYS

$n \to p e^- \bar{\nu}_e$	WEAK
$\pi^0 \to \gamma\gamma$	EM
$\rho^0 \to \pi^+\pi^-$	STRONG

- INTERACTIONS

$\sigma(\nu_e n \to p e^-)$	WEAK
$\sigma(e^+e^- \to e^+e^-)$	EM
$\sigma(\pi p \to \pi p \pi \pi)$	STRONG

DECAYS IN (NONRELATIVISTIC) QUANTUM MECHANICS

- UNSTABLE STATE DECAYS $N(t) = N(0)\, e^{-t/\tau}$

- WANT TO FIND TRANSITION AMPLITUDE

$$\langle \text{FINAL} | \mathcal{H} | \text{INITIAL} \rangle$$

- PARTICLE IN REST FRAME

DEFINITE ENERGY

$\Downarrow$

MASS

$$\psi(t) = \psi(0)\, e^{-iEt/\hbar}$$

$\hbar \to 1$

- PROBABILITY OF FINDING PARTICLE TO HAVE ENERGY E DOES NOT DEPEND ON TIME

$$|\psi(t)|^2 = |\psi(0)|^2$$

← STABLE

← NO DECAY

TO DESCRIBE AN UNSTABLE STATE

HAVE TO MAKE:

$$|\psi(t)|^2 \rightarrow \text{FUNCTION OF TIME}$$

SO WRITE:

$$E = E_0 - \frac{1}{2} i \Gamma$$

ADD A SMALL IMAGINARY PART TO ENERGY

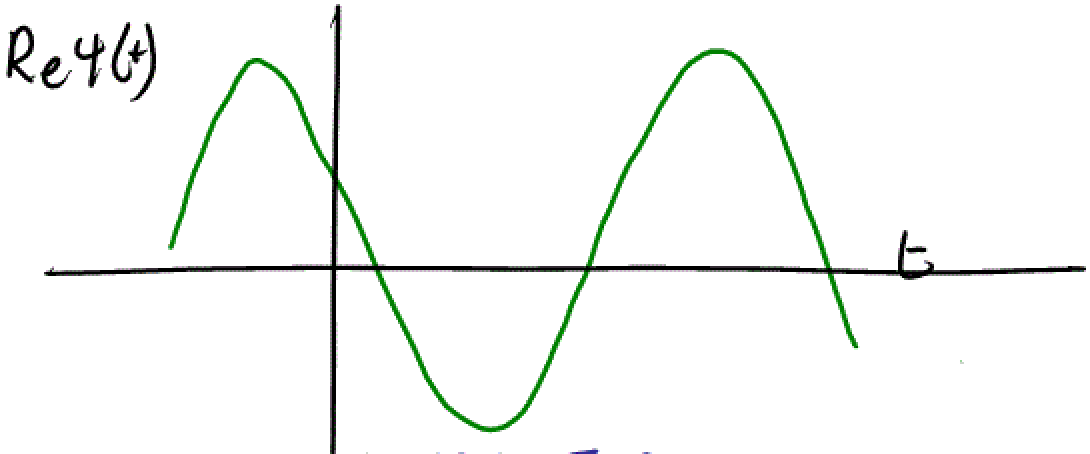
THEN

$$|\psi(t)|^2 = |\psi(0)|^2 \exp[-\Gamma t]$$

. COMPARE TO $N(t) = N(0) \exp[-\lambda t]$

$$\lambda = \frac{1}{\tau} = \frac{\Gamma}{\hbar}$$

WAVE FUNCTION OF WELL DEFINED ENERGY (MASS)

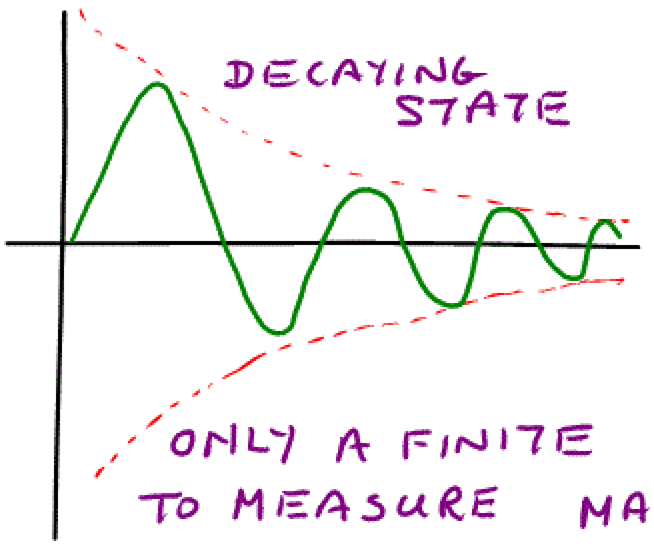


$$\psi(0) = e^{-iEt}$$

TAKE AS LONG AS YOU LIKE TO MEASURE MASS

$$\Delta E = \frac{\hbar}{\Delta t} \to 0$$

• FOR AN UNSTABLE PARTICLE ENERGY NOT EXACTLY DEFINED



ONLY A FINITE TIME TO MEASURE MASS

$$\Delta E = \frac{\hbar}{\Delta t} > 0$$

• PROBABILITY OF FINDING PARTICLE DROPS OFF IN TIME

$$\psi(t) = \psi(0)\exp(-iEt)$$

$$E \to E_0 - \frac{i\Gamma}{2}$$

$$\psi(t) = \psi(0)\exp(-iE_0 t)\exp\left(-\frac{\Gamma t}{2}\right)$$

$\psi(t) \rightarrow$ PROBABILITY OF FINDING PARTICLE WITH A DEFINITE ENERGY E_0 AS A FUNCTION OF TIME

- FOR DECAYS WANT PROBABILITY OF FINDING A PARTICLE WITH VARIABLE ENERGY E_0

$$\text{INITIAL STATE} \left\{ \begin{array}{ccc} E_0 & \rightarrow & E \\ m_0 & \rightarrow & m \end{array} \right\} \text{FINAL STATE}$$

$$\psi(t) \xrightarrow{\text{FOURIER}} \psi(E)$$

$$\psi(t) = \psi(0)\, e^{-i\omega}$$

E

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt\, f(t) \exp(+i\omega t)$$

NEED FOURIER TRANSFORM OF

$$\psi(t) = \psi(0)\exp\left(-iE_0 t\right)\exp\left(-\frac{\Gamma}{2}t\right)$$

$$\Downarrow$$

$$g(\omega) = \frac{\psi(0)}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} dt \exp\left[-iE_0 t\right]\exp\left[-\frac{\Gamma}{2}t\right]\exp\left[i\omega t\right]$$

$$g(\omega) = \frac{\psi(0)}{\sqrt{2\pi}}\,\frac{i\hbar}{\left(\hbar\omega - E_0\right) + i\Gamma/2}$$

THIS IS 1ST ORDER PERTURBATION

FREE PARTICLE

$$\mathcal{H}_0 \rightarrow \mathcal{H}_0 + \mathcal{E}$$

$$E_0 \rightarrow E_0 - \frac{i\Gamma}{2}$$

HAMILTONIAN OF FORCE CAUSING PARTICLE DECAY

$$g(\omega) = \frac{\psi(0)}{\sqrt{2\pi}} \frac{i\hbar}{(\hbar\omega - E_0) + i\Gamma/2}$$

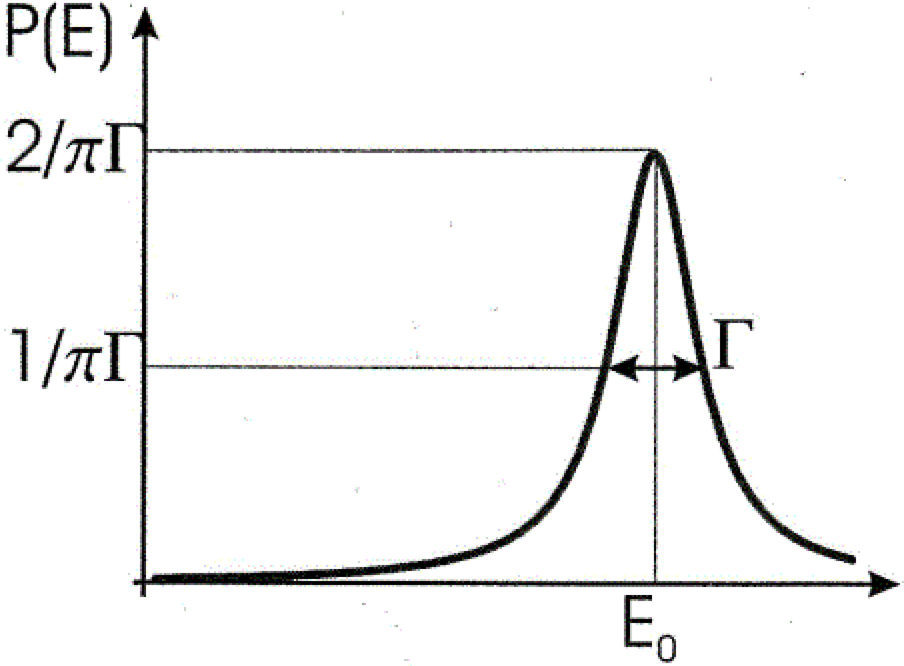
$$E = \hbar\omega$$

$g(\omega) \propto$ PROBABILITY AMPLITUDE FOR FINDING A FREQUNCY COMPONENT ω IN FOURIER EXPANSION OF $\psi(t)$

- PROBABILITY AMPLITUDE OF

$$|E_0\rangle \rightarrow |E\rangle$$

$$|m_0\rangle \rightarrow |m\rangle$$



PROBABILITY DENSITY OF FINDING A STATE WITH ENERGY E

$$P(E) \propto g^*(\omega)\, g(\omega)$$

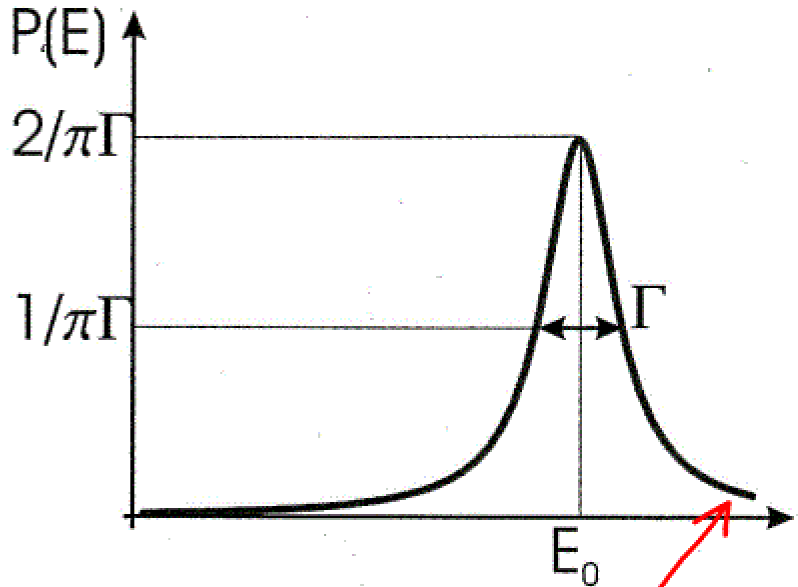
$$P(E) = \frac{k\hbar^2}{2\pi} \frac{|\psi(0)|^2}{(E-E_0)^2 + \frac{\Gamma^2}{4}}$$

- NORMALIZATION

$$\int_{-\infty}^{+\infty} P(E)\, dE = 1 \rightarrow k = \frac{\Gamma}{\hbar^2 |\psi(0)|^2}$$

$$P(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-E_0)^2 + (\Gamma/2)^2}$$

BREIT - WIGNER DECAY FORMULA



$$P(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-E_0)^2 - \left(\frac{\Gamma}{2}\right)^2}$$

NEVER GOES TO ZERO

- PERTURBATION TO FREE PARTICLE HAMILTONIAN

  — FREE PARTICLE STATE DECAYS

  — STATE NO LONGER HAS WELL-DEFINED ENERGY / MASS

  — HOW ILL-DEFINED DEPENDS ON HOW SHORT LIVED STATE IS

BREIT-WIGNER $\tau \Gamma = \hbar$ cf $\Delta t \cdot \Delta E \sim \hbar$

$$P(E) = \frac{\Gamma}{2\pi} \frac{1}{(E-E_0)^2 + \left(\Gamma/2\right)^2} \qquad \Gamma = \frac{\hbar}{\tau}, \frac{1}{\tau}$$

SAY AN INITIAL STATE CAN DECAY MANY WAYS!

$$Z^0 \to e^+e^-, \mu^+\mu^-, u\bar{u}, c\bar{c}, b\bar{b} \cdots\cdots$$

$$\Gamma = \sum_f \Gamma_f$$

BRANCHING RATIO
(BRANCHING FRACTION)

$$B_f = \frac{\Gamma_f}{\Gamma}$$

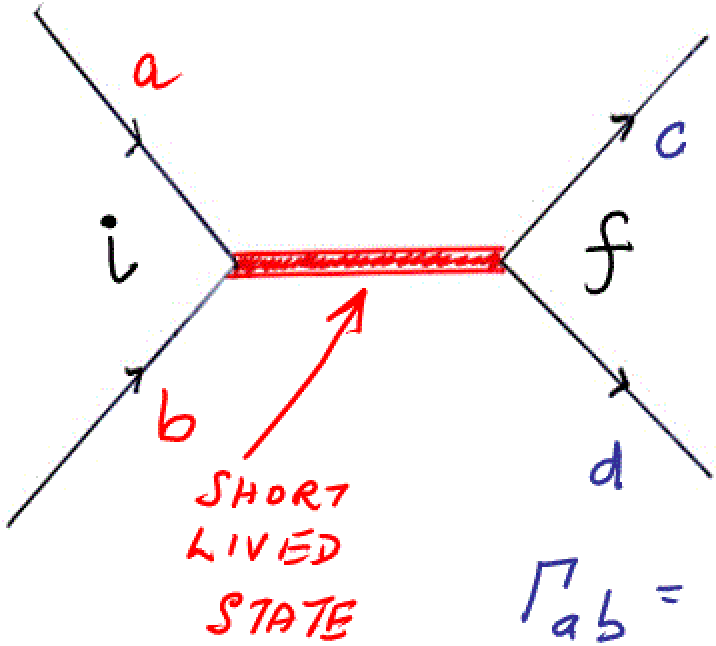
FOR ANY GIVEN FINAL STATE

$$P_f(E) = \frac{1}{2\pi} \frac{\Gamma_f}{(E-E_0)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

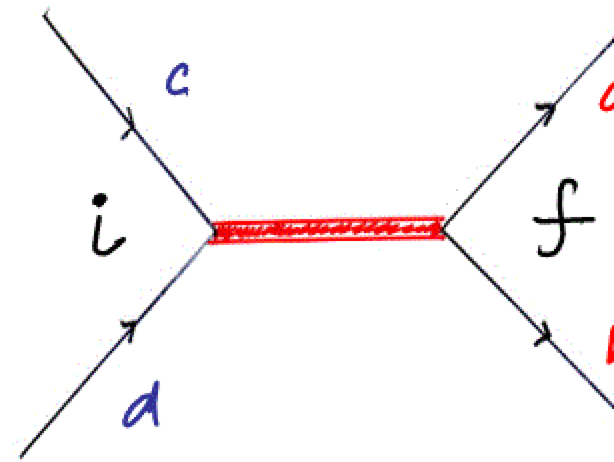
PARTIAL WIDTH

TOTAL WIDTH

RESONANT SCATTERING

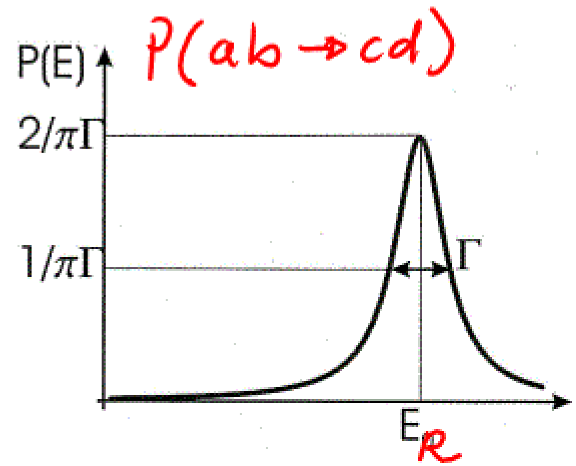


$$\equiv$$



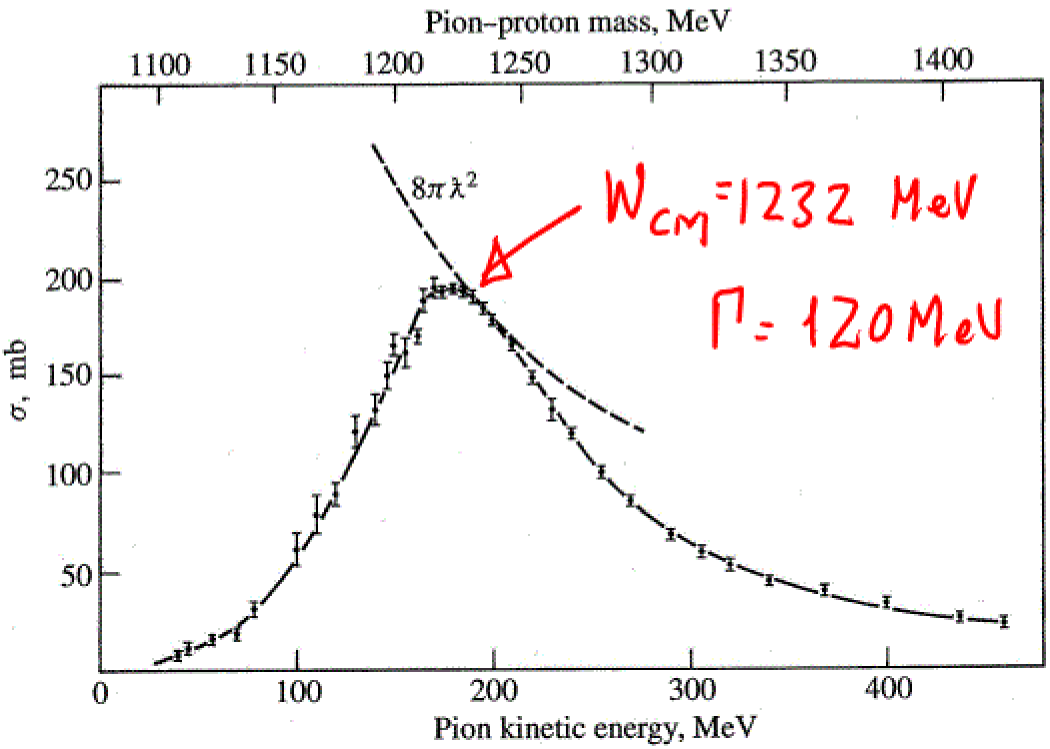
$$\Gamma_{ab} = \Gamma_i \quad ; \quad \Gamma_{cd} = \Gamma_f$$

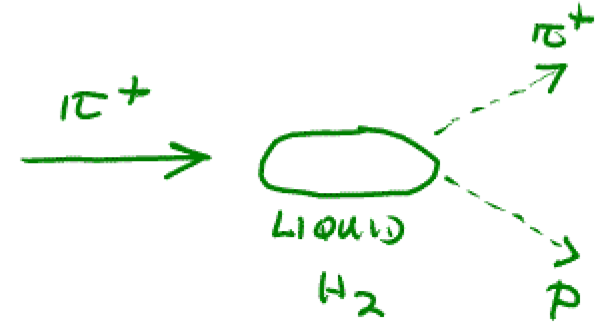
$$P(ab \to cd) \propto \frac{\Gamma_i \, \Gamma_f}{(E-E_R)^2 + \Gamma^2/4}$$



LOW ENERGY RESONANT SCATTERING $\pi^+ p \to \pi^+ p$

$P(E)$





FIRST EVIDENCE FOR STRUCTURE INSIDE THE PROTON

$$\pi p \to \Delta^{++}(1232) \to \pi^+ p$$

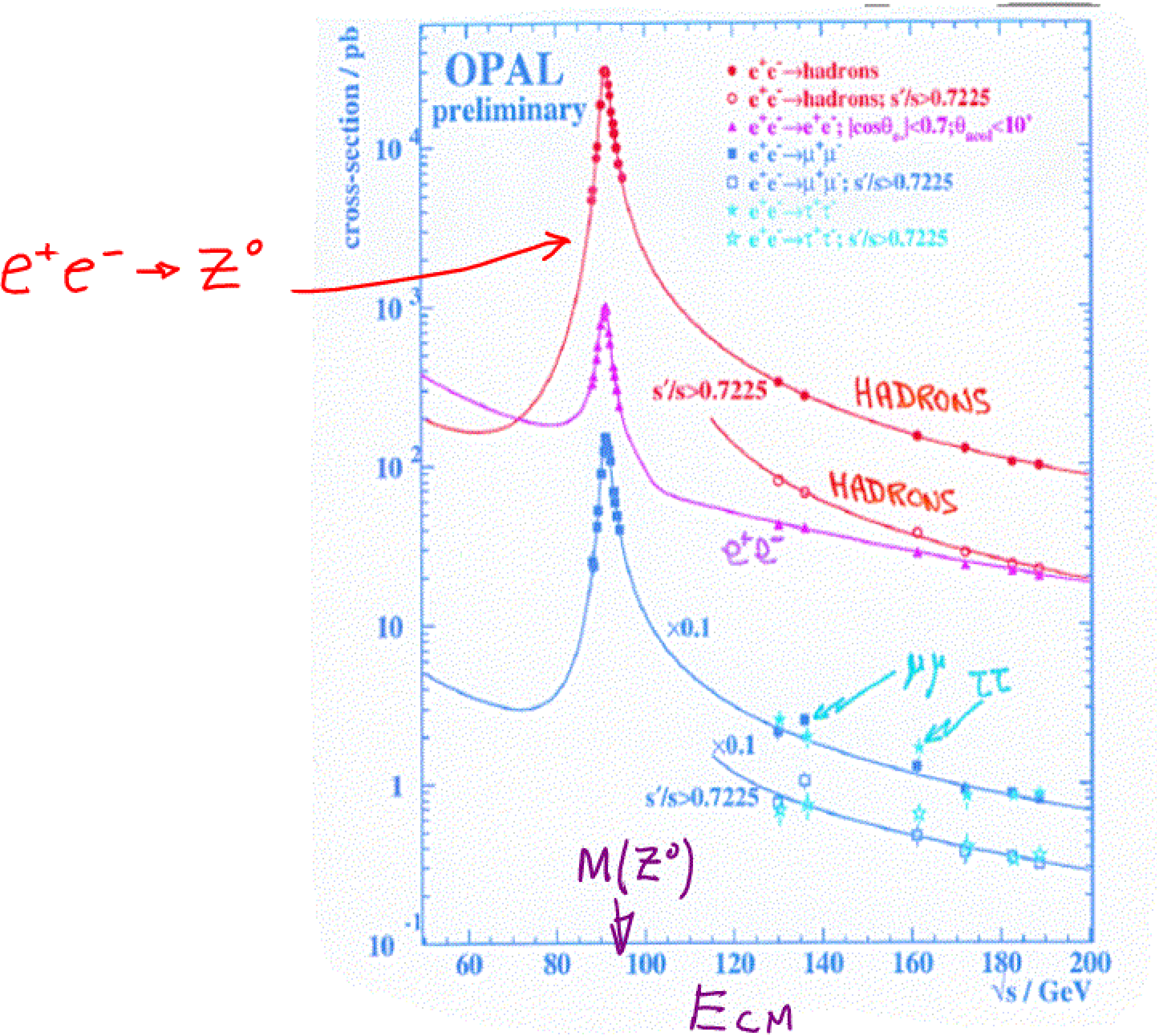
SHORT LIVED STATE, $\tau \sim 10^{-23}$ s

$\Delta^{++} \to uuu$

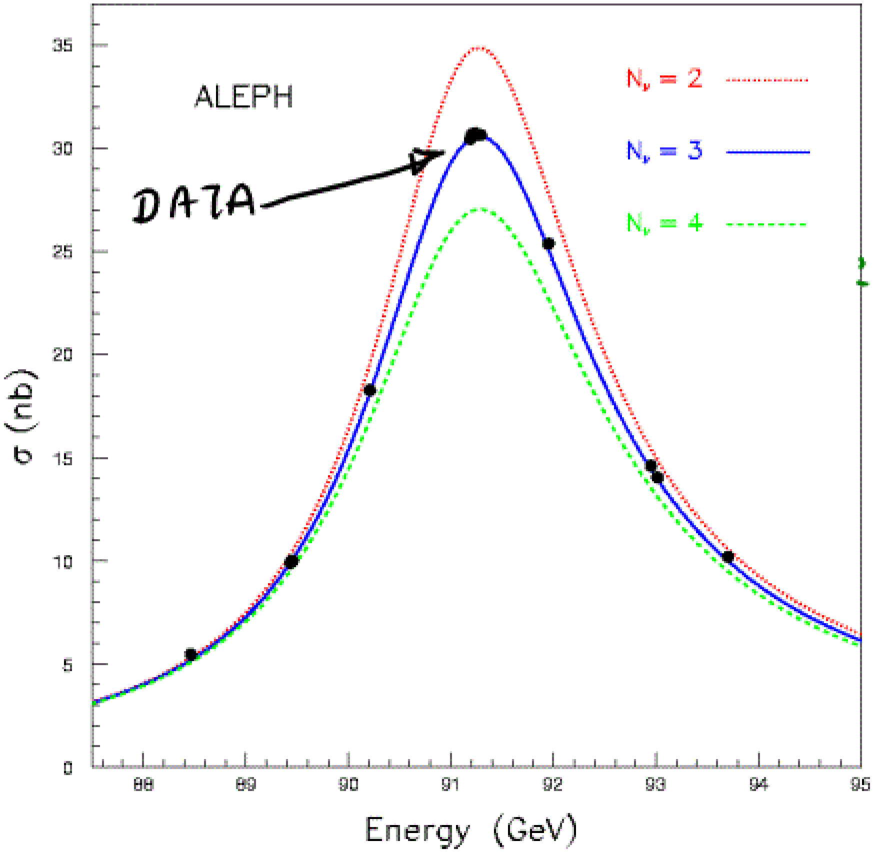
$Q = +2$

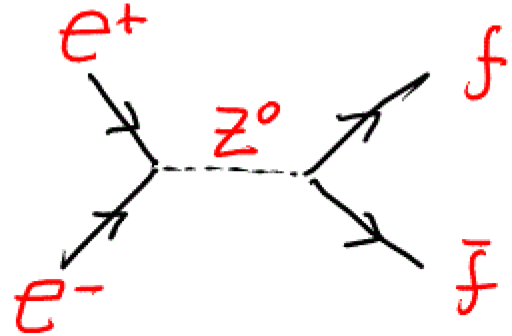
$J = \frac{3}{2}$

HIGH ENERGY RESONANT SCATTERING $e^+e^- \to Z^0 \to f\bar{f}$



HOW MANY NEUTRINO GENERATIONS?





ANYTHING

$\Gamma = 2.5\ \text{GeV}$

$e^+e^- \to$ ANYTHING

eg $e^+e^- \to \nu_e \bar{\nu}_e$

$\nu_\mu \bar{\nu}_\mu$

$\nu_\tau \bar{\nu}_\tau$

$\nu_N \bar{\nu}_N$???

$$\Gamma_{TOTAL} = \sum \Gamma_i = \underbrace{\sum \Gamma_i}_{\text{VISIBLE}} + \underbrace{\sum \Gamma_j}_{\text{INVISIBLE}}$$

NUMBER OF NEUTRINO GENERATIONS

COMMENT ON MULTIPLE DECAY PATHS

REMEMBER $dN(t) = -\Gamma N(t)\, dt$ $\quad \Gamma = 1/\tau$

$$N(t) = N(0)\, e^{-\Gamma t}$$

IF TWO DECAY PATHS — PARTIAL WIDTHS $\Gamma_1 \quad \Gamma_2$

$$\frac{dN}{dt} = -\Gamma_1 N - \Gamma_2 N$$

$$N(t) = N(0) \exp\left[\left(-\Gamma_1 - \Gamma_2\right) t\right]$$

BRANCHING FRACTIONS $f_1 = \Gamma_1/\Gamma \;,\; f_2 = \Gamma_2/\Gamma$

PARENT DECAYS WITH TRANSITION RATE

$$\Gamma = \Gamma_1 + \Gamma_2$$