

REPRIS ON GAUGE THEORIES

QED IS GLOBALLY GAUGE INVARIANT

$$G \mathcal{L}(\psi_e) \rightarrow \mathcal{L}(\psi_e^*)$$

SAME
LAGRANGIAN

SYMMETRY GROUP
IS $U(1)$

IF APPLY LOCAL PHASE TRANSFORMATION
DEPENDING ON $\bar{\alpha}, t$

$$G(\bar{\alpha}, t) \mathcal{L}(\psi_e) \rightarrow \mathcal{L}^*(\psi_e^*)$$

$$\mathcal{L}(\psi_e) \neq \mathcal{L}^*(\psi_e^*)$$

UNLESS WE INTRODUCE NEW FIELD (PHOTON)

$$G(\bar{\alpha}, t) \mathcal{L}(\psi_e, A) \rightarrow \mathcal{L}(\psi_e^*, A^*)$$

SAME LAGRANGIAN
INVARIANT.

ψ DESCRIBES PROPAGATION OF ELECTRON

$\mathcal{L}(\psi_1, \psi_2)$ DESCRIBES TWO ELECTRONS

GAUGE INVARIANCE $\rightarrow \psi, \mathcal{L}$ INVARIANT UNDER
ARBITRARY PHASE REDEFINITION AT \vec{x}, t POINTS

$$G(\vec{x}, t) \mathcal{L}(\psi_1, \psi_2, A) \rightarrow \mathcal{L}(\psi_1^*, \psi_2^*, A^*)$$

← INVARIANT →

THIS REQUIRES A GAUGE BOSON WHICH COMMUNICATES
THE LOCAL PHASE CONVENTION BETWEEN POINTS
IN SPACE-TIME



WE USUALLY CALL THIS
THE ELECTROMAGNETIC FORCE
REMEMBER? A FORCE
CHANGES THE PHASE OF A
WAVE FUNCTION LECTURE #3

HIERARCHY OF GAUGE FORCES

LOOKED AT TOY MODEL WITH SYMMETRY

$$\psi \rightarrow e^{i\alpha(\bar{x})} \psi$$

AND $\bar{\psi} \rightarrow \bar{\psi} - i\bar{A}(\bar{x})$ AND $\bar{A}(\bar{x}) \rightarrow \bar{A}(\bar{x}) + \bar{\nabla}\alpha(\bar{x})$

REAL QED \rightarrow DIRAC EQUATION AND

$$\psi \rightarrow e^{i\alpha(\bar{x}, t)} \psi$$

AND

$$\partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu - ieA_\mu$$

$$A_\mu \rightarrow A_\mu + \frac{1}{e} \partial_\mu \alpha(\bar{x}, t)$$

KLEIN-GORDON

SCALAR

$$(\partial^\mu \partial_\mu + m^2) \phi = 0$$

"SCALAR" PARTICLE
ONLY E, \vec{p}

SPACE-TIME
ENERGY-MOMENTUM

DIRAC

NOT A
SCALAR

$$(i \gamma^\mu \partial_\mu - m) \psi = 0$$

NO LONGER
2ND ORDER

E, \vec{p} }
SOME THING
EXTRA

'SPIN'

γ^μ →
DIRAC
 γ
MATRICES

$$D_\mu = \partial_\mu - i e \underbrace{A_\mu}_{\text{PHOTON}}$$

FREE DIRAC LAGRANGIAN

$$\mathcal{L} = i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m \bar{\Psi} \Psi$$

$$\partial_\mu \rightarrow \partial_\mu + i q A_\mu$$

$$\Psi \rightarrow \exp(i q \Lambda(x)) \Psi$$

$$A_\mu \rightarrow A_\mu(x) - \partial_\mu \Lambda(x)$$

$$\mathcal{L} = i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m \bar{\Psi} \Psi - q A_\mu \bar{\Psi} \gamma^\mu \Psi$$

$$\mathcal{L} = \mathcal{L}_{\text{FREE}} - j'^\mu A_\mu$$

WEAK INTERACTIONS

IF IGNORE ELECTRIC CHARGE SHOULD BE ABLE TO ARBITRARILY DEFINE ν_l AND e ($= e, \mu, \tau$) AT ANY POINT IN SPACE TIME

$$\begin{pmatrix} l' \\ \nu_l' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} l \\ \nu_l \end{pmatrix}$$

$$\psi \rightarrow e^{ig\bar{a} \cdot \bar{\tau}} \psi$$

ROTATION IN WEAK ISOSPACE

$$\tau_+ = \begin{pmatrix} 0 & \sqrt{2} \\ 0 & 0 \end{pmatrix}$$

$$\tau_- = \begin{pmatrix} 0 & 0 \\ \sqrt{2} & 0 \end{pmatrix}$$

$$\tau_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

GENERATORS OF SU(2)

SU(2) GAUGE SYMMETRY

$$2 \times 2 = 4 - 1 = 3 \text{ GAUGE BOSONS}$$

\uparrow ONLY 3 INDEPENDENT 2×2 MATRICES
 $W^\pm Z^0$

COLOUR INTERACTION

SHOULD BE ABLE TO CHOOSE ARBITRARY DEFINITION OF RGB AT ANY POINT IN SPACE TIME

$$\begin{pmatrix} R' \\ G' \\ B' \end{pmatrix} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

$$\psi \rightarrow e^{i\vec{\alpha} \cdot \vec{T}} \psi \quad \leftarrow \begin{matrix} 3 \times 3 \text{ TRACELESS} \\ \text{MATRICES} \end{matrix}$$

\vec{T} ARE GENERATORS OF SU(3)

\rightarrow SU(3) GAUGE SYMMETRY

$$T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$3 \times 3 = 9 - 1 = 8$ GAUGE BOSONS

\uparrow 8 BI-COLORED GLUONS

ONLY 8 INDEPENDENT 3×3 TRACELESS MATRICES

CONSERVED QUANTUM NUMBERS

COLOUR FORCE

8 BI-COLORED GLUONS

ELECTRO MAGNETIC

1 NEUTRAL PHOTON

WEAK FORCE

$W^+ W^- Z^0$

⏟

CARRY WEAK
CHARGE

QUARK FLAVOR

LEPTON FLAVOR

ISOSPIN

ELECTRIC CHARGE

BARYON NUMBER

QUARK FLAVOR

LEPTON FLAVOR

3-COMP OF ISOSPIN

ELECTRIC CHARGE

BARYONS NUMBER

LEPTON FLAVOR

ELECTRIC CHARGE

BARYON NUMBER

VIOLATES P

CP

NON-ABELIAN GAUGE FORCES

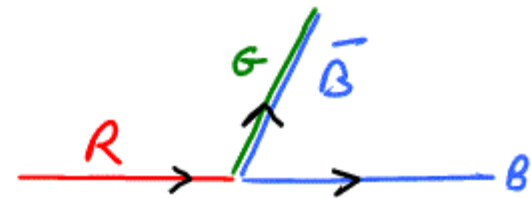
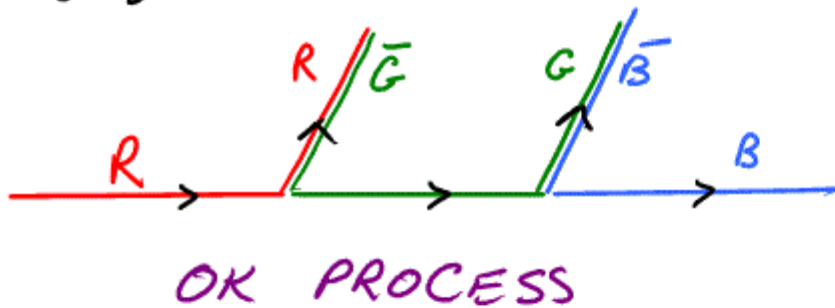
ELECTROMAGNETISM $e^{i\alpha(\vec{x})}$ \leftarrow GENERATORS COMMUTE

COLOR FORCE $e^{i\vec{a}\cdot\vec{T}}$ \leftarrow 3x3 MATRICES — DO NOT COMMUTE

$U(1) \rightarrow$ ABELIAN $\rightarrow \gamma^\mu$ HAS NO CHARGE

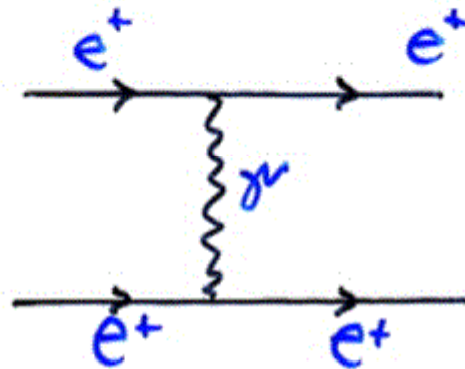
$$e \xrightarrow{\gamma_1} \xrightarrow{\gamma_2} e = e \xrightarrow{\gamma_2} \xrightarrow{\gamma_1} e$$

$SU(3) \rightarrow$ NON ABELIAN \rightarrow GLUONS CARRY COLOR

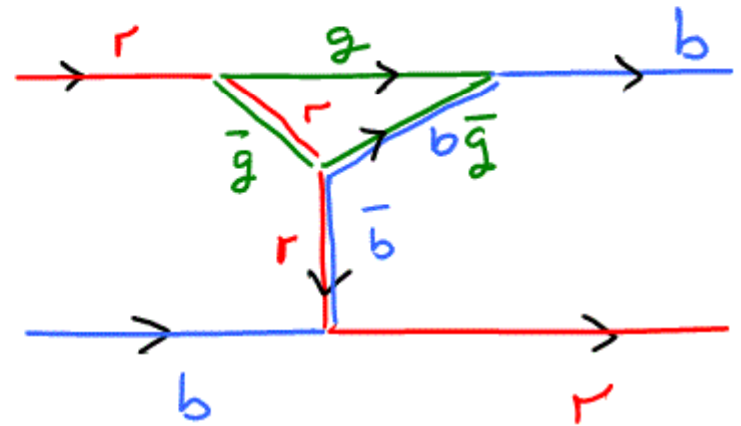
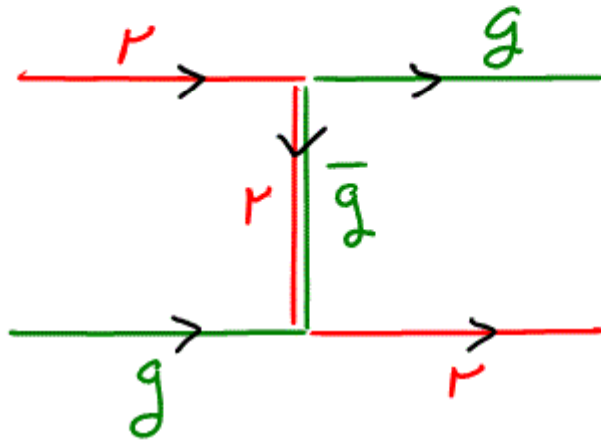


IMPLICATIONS OF GAUGE BOSON CARRYING CHARGE

QED \rightarrow



QCD \rightarrow



SELF COUPLING OF GLUONS
 \rightarrow MUCH GREATER COMPLEXITY

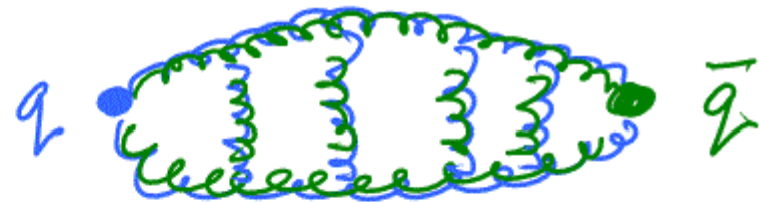
NO FREE QUARKS - CONFINEMENT

SELF COUPLING OF GLUONS MAKES IT IMPOSSIBLE TO SEPARATE QUARKS

QED

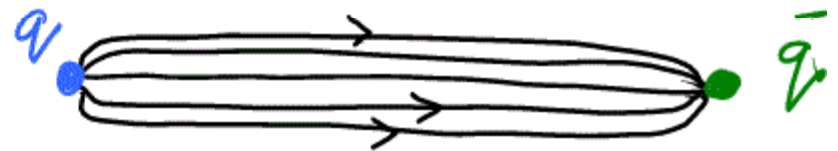


QCD

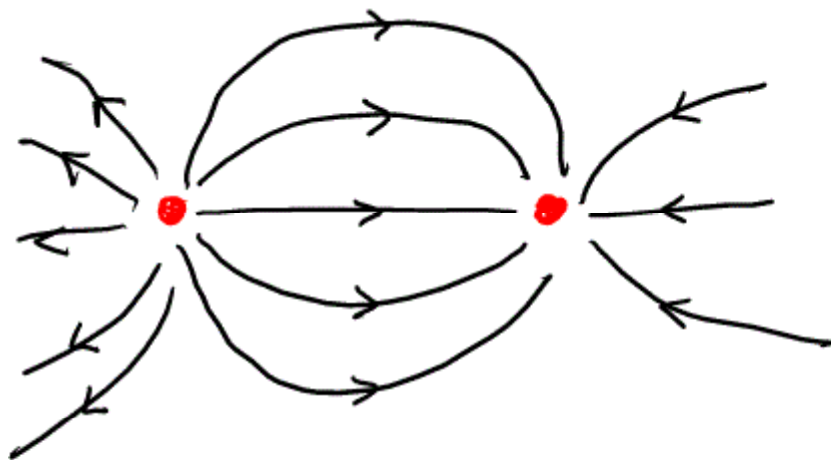


GLUONS INTERACT

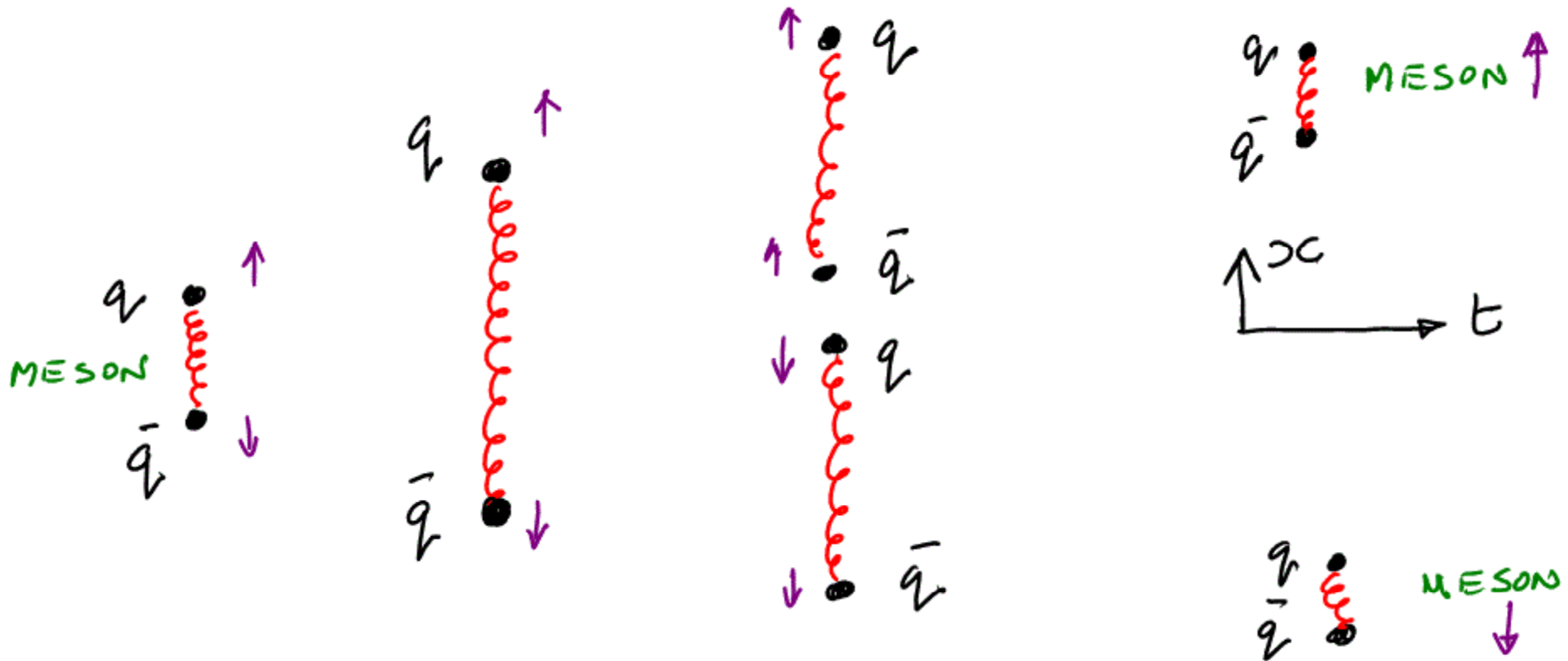
↓
FLUX TUBE



BEHAVES LIKE A SPRING JOINING QUARKS



FIELD LINES

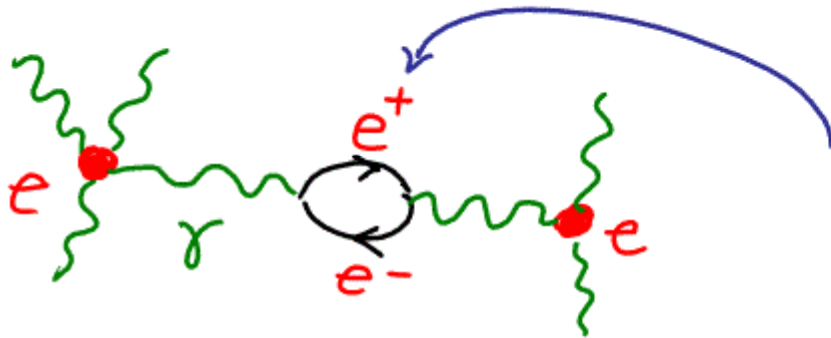


$q\bar{q}$ PAIR FORMS
 WHEN ENERGY DENSITY
 IN GLUON "SPRING"
 $> 2m_q$

\rightarrow MESON \rightarrow MESON + MESONS
 QUARKS CONFINED
 INSIDE HADRONS

BEHAVIOUR AS DISTANCE SCALE CHANGES

QED



ABELIAN GROUP $U(1)$
 γ DOES NOT CARRY CHARGE

VIRTUAL e^+e^- PAIRS
SCREEN CHARGE
EFFECTIVE COUPLING
STRENGTH INCREASES AT
SHORT DISTANCE

QCD

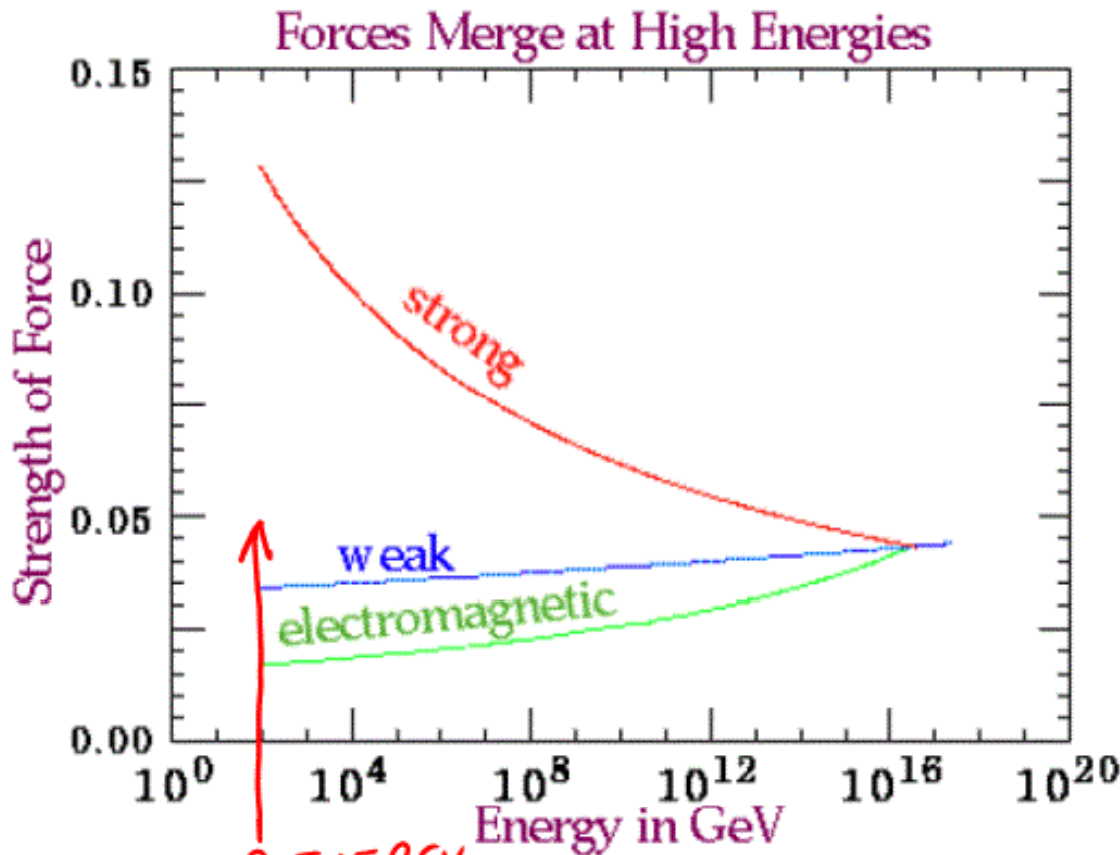


NON-ABELIAN $SU(3)$ GLUONS
CARRY CHARGE

AT SMALL DISTANCES SEE
LESS COLOUR CHARGE AS
GLUONS "CARRY IT AWAY"
EFFECTIVE COUPLING STRENGTH
DECREASE AT SMALL DISTANCE

WHY IS THE STRONG FORCE STRONG?

QCD IS "ASYMPTOTICALLY FREE"



OUR ENERGY SCALE

$$\alpha_s(q^2) = \frac{1}{B \ln q^2/\Lambda}$$

SCALE OF STRONG FORCE

NOTE THAT STRENGTH OF ALL FORCES VARY WITH ENERGY SCALE

GAUGE BOSONS → 1 3 8
 γ $W^\pm Z^0$ GLUONS

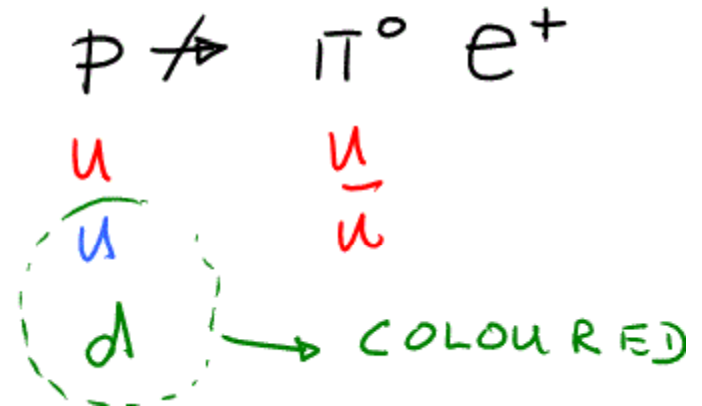
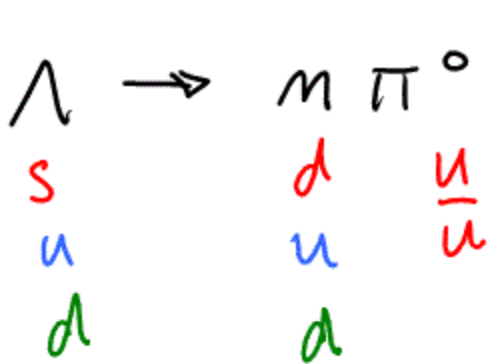
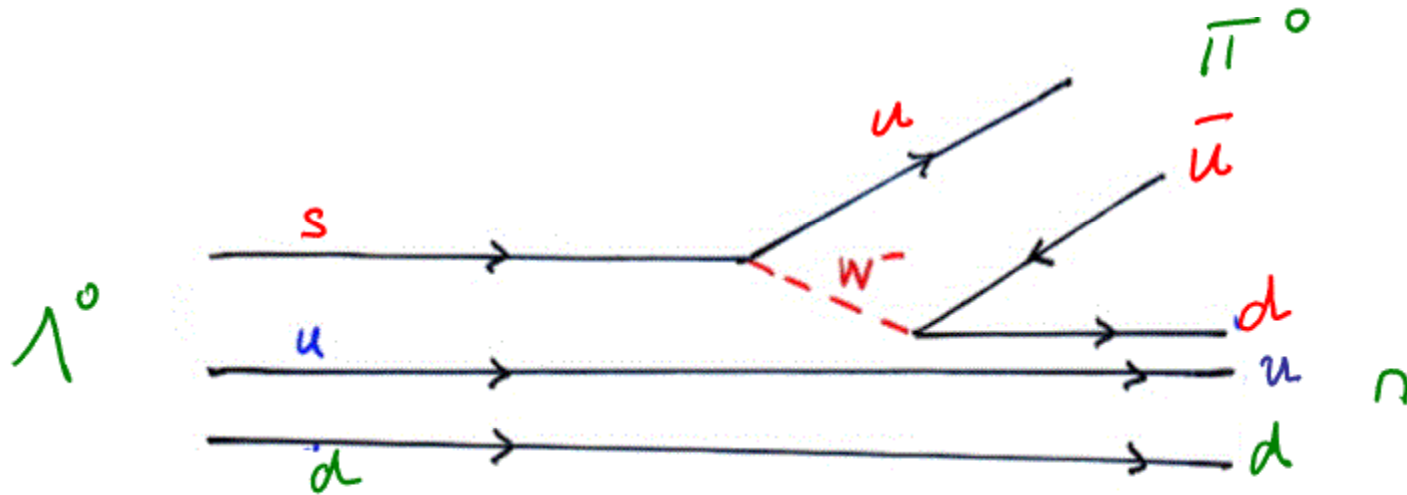
GAUGE SYMMETRY GROUP → $U(1) \times SU(2) \times SU(3)$

BARYON NUMBER CONSERVATION

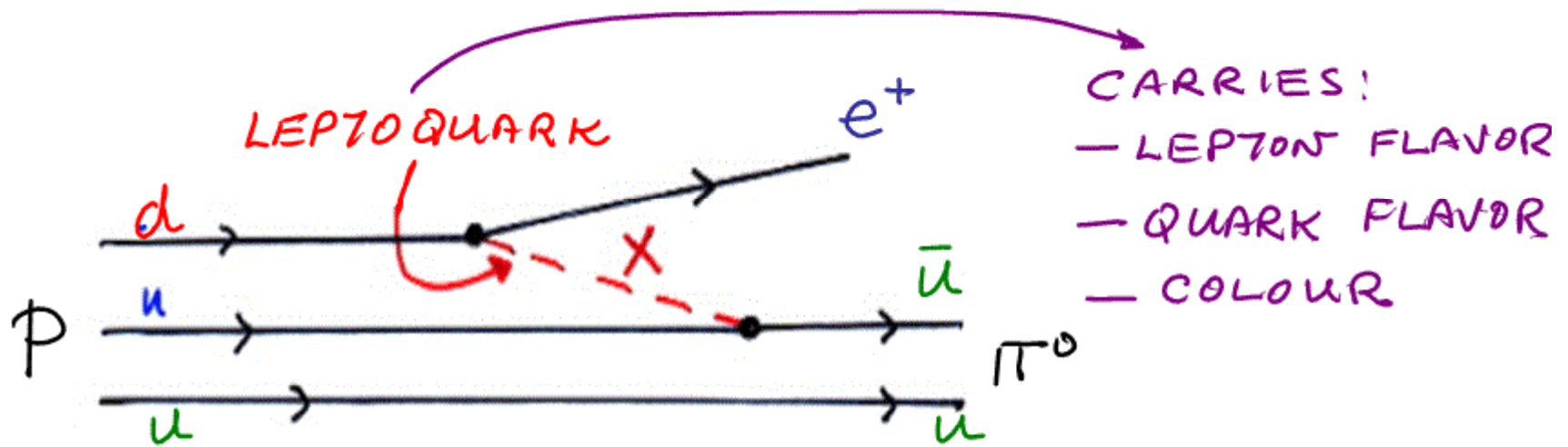
BARYON NUMBER CONSERVATION IS DUE TO!

→ NO FORCE CHANGES LEPTONS → QUARKS

→ ALL HADRONS "COLOURLESS"



FORCES BEYOND EM, WEAK, COLOUR



CAN IMAGINE A FIFTH FORCE

NEW GAUGE GROUP $SU(5)$? (NO)

NOT DETECTED YET \rightarrow GAUGE BOSON
VERY MASSIVE

COULD INDUCE PROTONS DECAY

\rightarrow BARYON NUMBER VIOLATION

CHARGE CONSERVATION $\tau(n \rightarrow p \nu_e \bar{\nu}_e) > 10^{18} \text{ yr}$

LEPTON CONSERVATION $\tau(\tau^b G_e \rightarrow \tau^b S_e e^- e^-) > 10^{26} \text{ yr}$

0ν 

BARYON CONSERVATION $\tau(p \rightarrow e^+ \pi^0) > 10^{33} \text{ yr}$