

CABIBBO - KOBAYASHI - MASKAWA MATRIX

FOR 3 GENERATIONS OF QUARK - GENERALIZE THE CABIBBO FLAVOUR MIXING MATRIX.

$$\begin{array}{l} \text{WEAK} \\ \text{EIGENSTATES} \end{array} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \begin{array}{l} \text{COLOUR} \\ \text{EIGENSTATES} \end{array}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$V_{ab} \rightarrow$ STRENGTH OF TRANSITION $a \rightarrow b$

$$V_{ud} \sim \cos \theta_c$$

$$V_{us} \sim \sin \theta_c$$

MOST GENERAL 3×3 MATRIX

$$\begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix}$$

9 COMPLEX ELEMENTS

\rightarrow 18 REAL NUMBERS

THIS MATRIX MUST BE UNITARY - PRESERVE PROBABILITY

$$V_{\alpha\beta}^+ V_{\beta\gamma} = \delta_{\alpha\gamma} \rightarrow \left(\begin{array}{c} \end{array} \right)^+ \left(\begin{array}{c} \end{array} \right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

eg $V_{11}^+ V_{11} + V_{12}^+ V_{21} + V_{13}^+ V_{31} = 1$

$$V_{11}^+ V_{12} + V_{12}^+ V_{22} + V_{13}^+ V_{32} = 0$$

OF PARAMETERS

FOR $2 \times 2 \rightarrow$ 4 CONSTRAINT EQUATIONS $8 \rightarrow 4$

FOR $3 \times 3 \rightarrow$ 9 CONSTRAINT EQUATIONS $18 - 9$

GENERALLY $N \times N \rightarrow 2N^2$ REAL PARAMETERS

UNITARITY REDUCES $2N^2 \rightarrow N^2$ REAL
PARAMETERS

RECALL CABIBBO MATRIX - WRITE AS

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} e^{i\gamma} & e^{i\beta} \\ e^{i\delta} & e^{i\alpha} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

EACH ELEMENT
DEPENDS ON ONLY ONE PARAMETER

THESE ARE
ACTUALLY
WAVE FUNCTIONS
COMPLEX NUMBERS

CAN ABSORB A PHASE INTO EACH QUARK WAVE FN.

$d \rightarrow d e^{i\theta_1}$, $d' \rightarrow d' e^{i\theta_2}$ CHANGES NOTHING $[\psi\psi^*]$

ABSORB ALL 4 PHASES? \rightarrow NO, SINCE $V \equiv V e^{i\phi}$
 \therefore ONE DEGREE OF FREEDOM (ONE PARAMETER)

CALL IT " θ_c "

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$N \times N$ MATRIX $\rightarrow 2N^2$ REAL PARAMETERS

\downarrow UNITARITY

N^2 REAL PARAMETERS

\downarrow ABSORB PHASES
INTO QUARK WAVE
FUNCTIONS

NUMBER OF
GENERATIONS

$(2N-1)$

ABSORBED

NOW HAVE $N^2 - (2N-1) = (N-1)^2$

REAL PARAMETERS IN THE MATRIX

$N=2 \Rightarrow$ ONE REAL PARAMETER

2×2 IS A VERY SPECIAL CASE

LET'S LOOK AT 3×3

CONSIDER CASE WHEN HAVE REAL $N \times N$ MATRIX

REAL UNITARY MATRIX \rightarrow ORTHOGONAL MATRIX

$$A \tilde{A} = I$$

eg. $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$a^2 + b^2 = 1$$

REMOVES ONE PARAMETER

$$c^2 + d^2 = 1$$

"

$$ac = -db$$

"

$$2 \times 2 \Rightarrow 4 - 3 = 1$$

GENERALLY $\frac{N}{2} (N-1)$

INDEPENDANT REAL
PARAMETERS

PUTTING THE WHOLE ARGUMENT TOGETHER

- $N \times N$ COMPLEX UNITARY MATRIX

$(N-1)^2$ INDEPENDENT PARAMETERS

- OF THESE $\frac{N}{2}(N-1)$ ARE REAL

- SO THERE ARE

$$(N-1)^2 - \frac{N}{2}(N-1) = \frac{(N-1)(N-2)}{2}$$

REMAINING COMPLEX NUMBERS OR PHASES

• $N \times N$ GENERAL MATRIX \leftarrow UNITARY

COMPLEX
ELEMENTS

$$\frac{N(N-1)}{2} \text{ REAL PARAMETERS}$$

$$\frac{(N-1)(N-2)}{2} \text{ PHASES}$$

• 2 GENERATIONS

1 REAL PARAMETER θ_c
0 PHASES

• 3 GENERATIONS

3 PARAMETERS $\theta_1, \theta_2, \theta_3$

1 PHASE δ

THIS IS THE INTERESTING THING

CKM - MATRIX

$$\begin{pmatrix}
 C_{12} C_{13} & S_{12} C_{13} & S_{13} e^{-i\delta} \\
 -C_{23} S_{12} - C_{12} S_{23} S_{13} e^{i\delta} & C_{12} C_{23} - S_{12} S_{23} S_{13} e^{i\delta} & C_{13} S_{23} \\
 S_{12} S_{23} - C_{12} C_{23} S_{13} e^{i\delta} & -C_{12} S_{23} - C_{23} S_{12} S_{13} e^{i\delta} & C_{13} C_{23}
 \end{pmatrix}$$

eg $C_{12} = \cos \theta_{12}$, $S_{23} \equiv \sin \theta_{23}$ $\theta_{ij} \rightarrow$ SMALL

EXPERIMENTALLY $V \approx$

$$\begin{pmatrix}
 1 & S_{12} & S_{13} e^{i\delta} \\
 -S_{12} & 1 & S_{23} \\
 -S_{13} e^{i\delta} & -S_{23} & 1
 \end{pmatrix}$$

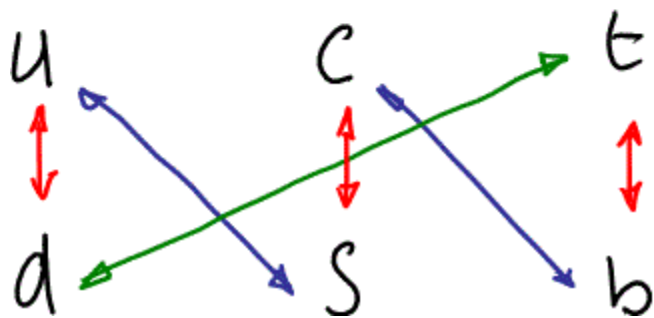
$\theta_{12} =$ CABIBBO ANGLE

RED \Rightarrow BLUE \Rightarrow GREEN

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

EXPERIMENT \rightarrow

$$\begin{pmatrix} 0.975 & 0.22 & 0.003 \\ 0.22 & 0.974 & 0.04 \\ 0.01 & -0.04 & 0.999 \end{pmatrix}$$



THE CLOSER QUARKS ARE
IN THE GENERATION
PATTERN \rightarrow THE MORE
PROBABLE ARE TRANSITIONS

SAW THAT TIME REVERSAL T IS

$$\psi \xrightarrow{T} \psi^*(-t)$$

IN STANDARD MODEL, EACH QUARK WAVE FUNCTION

$$\psi = e^{i\omega t} e^{i\delta} \rightarrow \text{FROM CKM}$$

$$= e^{i(\omega t + \delta)}$$

$$\downarrow T$$
$$= e^{-i[\omega(-t) + \delta]}$$

$$= e^{i(\omega t - \delta)}$$

cf

NOT TIME REVERSAL INVARIANT \rightarrow ~~CPT~~ \rightarrow ~~CP~~~~T~~ \rightarrow CPT.

IN THE LARGE SCALE UNIVERSE

MATTER \neq ANTIMATTER

IN ELEMENTARY PARTICLES

$K^0 \rightarrow \frac{2\pi}{3\pi} \rightarrow \cancel{CP} \rightarrow$ MATTER \neq ANTIMATTER

NEED 3 GENERATIONS FOR \cancel{CP}

NEED 3 GENERATIONS FOR MATTER \neq ANTIMATTER

\cancel{CP} IN QUARK SECTOR NOT ENOUGH TO
EXPLAIN LARGE SCALE MATTER - ANTIMATTER
ASYMMETRY

BUT WE HAVE $m_\nu \neq 0$ & 3 GENERATIONS OF

LEPTONS \rightarrow HOW MUCH $\cancel{CP} \rightarrow$ WE'LL
SEE.

WHY IS $m_\nu \neq 0$ SO IMPORTANT?

REMEMBER $\begin{pmatrix} u \\ d \end{pmatrix}$ ARE MASS EIGENSTATES

WE CAN LABEL THEM $\begin{pmatrix} m_u \\ m_d \end{pmatrix}$

FOR LEPTONS "CABIBBO" MIXING WOULD BE

$$\begin{pmatrix} m_e \\ m_{\nu_e} \cos \theta + m_{\nu_\mu} \sin \theta \end{pmatrix} \quad \begin{pmatrix} m_\mu \\ m_{\nu_e} \sin \theta - m_{\nu_\mu} \cos \theta \end{pmatrix}$$

FOR $m_\nu = 0 \rightarrow \begin{pmatrix} m_e \\ 0 \end{pmatrix} \quad \begin{pmatrix} m_\mu \\ 0 \end{pmatrix}$

\rightarrow NO MIXING \rightarrow NO CP IN LEPTON SECTOR

WHAT IS MASS "FOR" ?

UMM ----> HOLDS ME ON THE EARTH?

NO!

HOLDS GALAXIES TOGETHER?

NO!

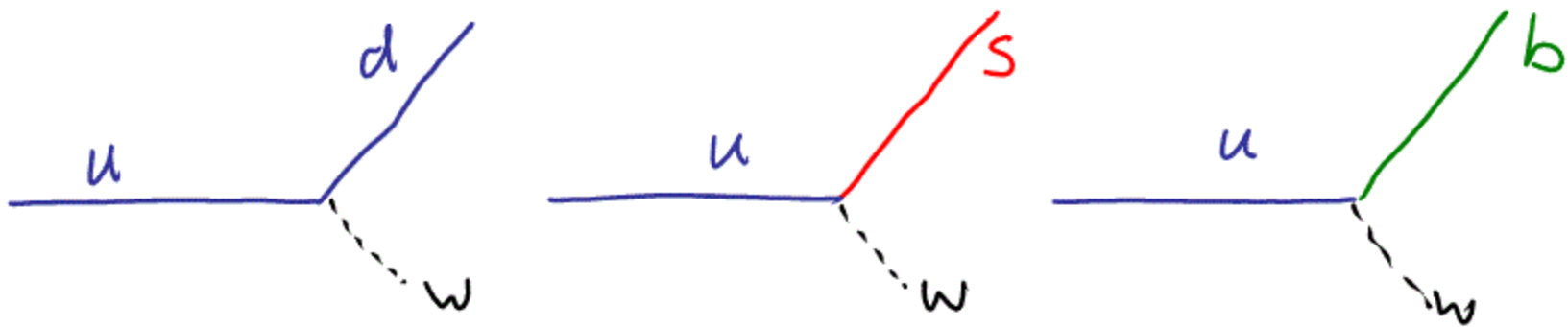
GOVERNS EXPANSION OF UNIVERSE?

WELL -----

LABELS THE 3 GENERATIONS

↓

~~CP~~



HOW DO I DISTINGUISH d s b ?

- SAME ELECTRIC CHARGE
- SAME COLOUR CHARGE
- SAME WEAK CHARGE
- ud cs tb COUPLINGS SAME
- DIFFERENT ub , us , ud COUPLINGS
- DIFFERENT MASS
- IF $m_d = m_s = m_b \rightarrow \pi^+(u\bar{d}) = K^+(u\bar{s}) = B^+(u\bar{b})$

$B^+ \rightarrow K^+ \pi^-$ INDISTINGUISHABLE
 $\pi^+ \pi^-$

NO MASS \rightarrow 3 GENERATIONS \rightarrow DON'T
INDISTINGUISHABLE EXIST

NO 3 GENERATIONS \rightarrow NO \cancel{CP}

NO \cancel{CP} \rightarrow NO MATTER \neq ANTIMATTER

MATTER \equiv MATTER \rightarrow ONLY γ 'S IN THE
UNIVERSE