UNITS IN PARTICLE PHYSICS

SI UNITS $\rightarrow$ MACROSCOPIC OBJECTS

$m_e = 9.1 \times 10^{-31}\,kg$ $\rightarrow$ USE UNITS APPROPRIATE TO SCALE OF PHYSICS

NATURAL UNITS $\rightarrow$ BASED ON FUNDAMENTAL CONSTANTS OF RELATIVITY & QUANTUM MECHANICS

$[kg, m, s] \longrightarrow [\hbar, c, GeV]$

$$\hbar = 1.055 \times 10^{-34}\ J.s$$

$$c = 2.998 \times 10^{8}\ ms^{-1}$$

$$GeV = 10^{9}\ eV = 1.602 \times 10^{-10}\ J$$

ROUGHLY MASS OF PROTON

	SI	$[\hbar, c, \mathrm{GeV}]$	$\hbar = c = 1$
ENERGY	$\mathrm{kg\, m^2\, s^{-2}}$	GeV	GeV
MOMENTUM	$\mathrm{kg\, m^2\, s^{-1}}$	GeV/c	GeV
MASS	kg	GeV/c^2	GeV
TIME	s	$(\mathrm{GeV}/\hbar)^{-1}$	$(\mathrm{GeV})^{-1}$
LENGTH	m	$(\mathrm{GeV}/\hbar c)^{-1}$	$(\mathrm{GeV})^{-1}$
AREA	$\mathrm{m^2}$	$(\mathrm{GeV}/\hbar c)^{-2}$	$(\mathrm{GeV})^{-2}$

EXAMPLE – LENGTH

$$\frac{\mathrm{GeV}}{\hbar c} = [J]\,\frac{1}{[J \cdot S]} \cdot \frac{[S]}{[m]}$$

$$= \frac{1}{[m]}$$

USING THIS SYSTEM

$$E^2 = p^2c^2 + m^2c^2 \longrightarrow E^2 = p^2 + m^2$$

AT END OF CALCULATION PUT FACTORS OF c, $\hbar$ BACK IN USING DIMENSIONAL ANALYSIS

$$E^2 = p^2 + m^2$$

$$E^2\,[\mathrm{GeV}]^2 = p^2 \underbrace{\left[\frac{\mathrm{GeV}}{c}\right]}_{\text{ABSORBED INTO DIMENSION OF } p} + m^2 \left[\frac{\mathrm{GeV}}{c^2}\right]$$

TO CANCEL THE FACTORS OF c, NEED TO WRITE

$$E^2\,[\mathrm{GeV}]^2 = p^2 \left[\frac{\mathrm{GeV}}{c}\right]^2 c^2 + m^2 \left[\frac{\mathrm{GeV}}{c^2}\right]^2 c^4$$

EXAMPLE: RMS RADIUS OF PROTON

$$\langle r^2 \rangle^{\frac{1}{2}} = 4.1 \ GeV^{-1}$$

$m = \frac{\hbar c}{GeV}$ SO YOU WRITE

$$\langle r^2 \rangle^{\frac{1}{2}} = 4.1 \ \frac{\hbar c}{GeV} \cdot m = 4.1 \ \frac{[J \cdot s]\left[\frac{m}{s}\right]}{J}$$

$$= \frac{4.1 \times 1.055 \times 10^{-34} \times 2.998 \times 10^{8}}{1.602 \times 10^{-10}} \ m$$

USEFUL CONVERSION FACTORS:

$$\hbar c = 0.197 \ GeV \times 10^{-15} m$$

$$\hbar c = 0.197 \ GeV \cdot fm$$ ← FERMI

WHAT ABOUT ELECTROMAGNETISM ?

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

ε_0 = PERMETIVITY OF FREE SPACE

IN HEAVISIDE-LORENTZ $\varepsilon_0 = 1$

$$F = \frac{e^2}{4\pi r^2}$$

ε_0 ABSORBED INTO DEFINITION OF e

$$\frac{1}{\varepsilon_0 \mu_0} = c^2$$

CHOOSE $\varepsilon_0 = 1 = c = 1 \rightarrow \mu_0 = 1$

$$\hbar = c = \varepsilon_0 = \mu_0 = 1$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \longrightarrow ?$$

SI $\quad \alpha = \dfrac{e^2}{4\pi\varepsilon_0 \hbar c}$ STRENGTH OF ELECTROMAGNETIC INTERACTION

$\longrightarrow \hbar, c, \mathrm{GeV}$

NEED DIMENSIONS OF $e \longrightarrow F = \dfrac{e^2}{4\pi\varepsilon_0 r^2}$

$$[e^2] = [F][\varepsilon_0][r^2] = [\mathrm{kg}.\mathrm{m}.\mathrm{s}^{-2}][\varepsilon_0]\left[\frac{\hbar c}{\mathrm{GeV}}\right]^2$$

$$\frac{\mathrm{GeV}}{c^2} \quad \frac{\hbar c}{\mathrm{GeV}} \quad \left[\frac{\mathrm{GeV}}{\hbar}\right]^2 \left[\frac{\hbar c}{\mathrm{GeV}}\right]^2 [\varepsilon_0]$$

$$= \hbar c\,[\varepsilon_0]$$

$$\left[\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right] = \frac{\hbar c\,[\varepsilon_0]}{4\pi\,[\varepsilon_0]\,\hbar c}$$

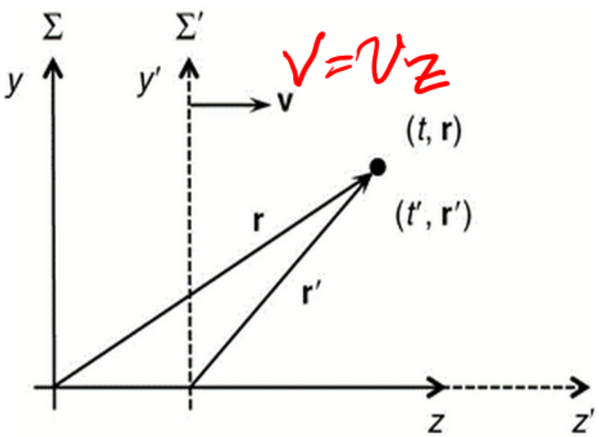
MANIFESTLY DIMESIONLESS

$$\alpha \simeq 1/137$$

$$\alpha = \frac{e^2}{4\pi} \approx 1/137$$

SPECIAL RELATIVITY – AID MÉMOIRE

LAWS OF PHYSICS SAME IN ALL INERTIAL FRAMES



LORENTZ TRANSFORM

$$t' = \gamma\left(t - \frac{v}{c^2}\cdot z\right), \quad z' = \gamma(z - vt)$$

$$\gamma = (1-\beta^2)^{-1/2}, \quad \beta = v/c$$

IN NATURAL UNITS, C=1

$$t' = \gamma(t - \beta\gamma), \quad z' = \gamma(z - \beta t)$$

$$t = \gamma(t' + \beta z), \quad z = \gamma(z' + \beta t)$$

MATRIX $X' = \Lambda X$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

+VE FOR INVERSE

4-VECTORS & LORENTZ INVARIANCE

"USEFUL" QUANTITIES → LORENTZ INVARIANT

ELEGANTLY EXPRESSED IN TERMS OF 4-VECTORS

CONTRAVARIANT 4-VECTOR IS A SET OF QUANTITIES MEASURED IN 2 INERTIAL FRAMES RELATED BY LORENTZ TRANSFORM

$$x^\mu = (t, x, y, z)$$

0^{TH} COMPONENT → TIME

$$x'^\mu = \Lambda^\mu{}_\nu \, x^\nu$$

← LORENTZ XFORM

REPEATED INDICES → SUMMATION

$$x'^\mu = \sum_\nu \Lambda^\mu{}_\nu \, x^\nu$$

3 VECTOR SCALAR PRODUCT $\underline{x} \cdot \underline{x}$ INVARIANT UNDER ROTATIONS

SPACE-TIME INTERVAL $t^2 - x^2 - y^2 - z^2$ INVARIANT UNDER LORENTZ

DEFINE COVARIANT $x_\mu = (t, -x, -y, -z)$

$$x^\mu x_\mu = x^0 x_0 + x^1 x_1 + x^2 x_2 + x^3 x_3$$

$$= t^2 - x^2 - y^2 - z^2$$

SIGNS COME OUT CORRECT

MATCHED INDICES → LORENTZ SCALAR → INVARIANT → COVARIANT

TRANSFORM OF COVARIANT 4-VECTOR

$$\begin{pmatrix} t' \\ -x' \\ -y' \\ -z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} t \\ -x \\ -y \\ -z \end{pmatrix}$$

$\rightarrow x'_\mu = \Lambda_\mu^{\ \nu} x_\nu$

cf $x'^\mu = \Lambda^\mu_{\ \nu} x^\nu$ CONTRAVARIANT

$\Lambda^\mu_{\ \nu} = \overline{\Lambda}$ ← MATRIX

$\Lambda_\mu^{\ \nu} = \overline{\Lambda}^{-1}$

COVARIANT & CONTRAVARIANT 4-VECTOR RELATED BY

$$x_\mu = g_{\mu\nu} x^\nu$$

METRIC TENSOR

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

MAKES SIGNS COME OUT CORRECT

BY DEFINITION ONLY QUANTITIES TRANSFORMING LIKE $t' = \gamma(t - \beta z)$ CONTRAVARIANT

SCALAR PRODUCT $a^\mu b_\mu = a_\mu b^\mu = g_{\mu\nu} a^\mu b^\nu$ LORENTZ INVARIANT

LORENTZ TRANSFOR $\rightarrow$ LINEAR

$\rightarrow$ $a^\mu + b^\mu + c^\mu + \ldots\ldots$

IS ALSO A CONTRAVARIAT 4-VECTOR

FOUR – MOMENTUM

PARTICLE MASS m

$$E = \gamma m c^2 \qquad \Rightarrow \qquad E = \gamma m$$

$$\vec{p} = \gamma m \vec{v} \qquad \qquad \vec{p} = \gamma m \beta$$

NATURAL UNITS

LORENTZ XFORM PROPERTIES $\vec{v} = \frac{d\vec{x}}{dt}$, $E, \vec{p}$ TRANSFORM ACCORDING TO LORENTZ XFORM

$$p^\mu = (E, p_x, p_y, p_z)$$

CONTRAVARIANT 4-VECTOR

$$\left.\begin{array}{l}\text{ENERGY CONSERVATION} \\ \qquad + \\ \text{3-MOMENTUM CONSERVATION}\end{array}\right\}$$

4-MOMENTUM CONSERVATION

$$p^\mu p_\mu = E^2 - p^2$$

LORENTZ INVARIANT

$$p^\mu p_\mu = E^2 - p^2$$ LORENTZ INVARIANT

WHAT ABOUT A PARTICLE AT REST

$$p^\mu = (E, p_x, p_y, p_z) = (m, 0, 0, 0) \; ; \; p^\mu p_\mu = m^2$$

THIS IS TRUE IN ANY INERTIAL FRAME } THAT'S WHY REST MASS IS A USEFUL QUANTITY

FOR A SYSTEM OF n PARTICLES

$$p^\mu = \sum_{i=1}^{n} p_i^\mu \; ; \; p_i^\mu \rightarrow \text{4 VECTOR} \; \therefore \; p^\mu \rightarrow \text{4-VECTOR}$$

$p^\mu p_\mu$ LORENTZ INVARIANT

$$p^\mu p_\mu = \left(\sum_i^n E_i\right)^2 - \left(\sum_{i=1}^n \vec{p}_i\right)^2 =$$ INVARIANT MASS

INVARIANT MASS IN DECAYS

A PARTICLE MASS m_a DECAYS TO m_1 & m_2

4-MOMENTUM CONSERVED

$$\underbrace{(p_1 + p_2)^\mu (p_1 + p_2)_\mu}_{p_a^\mu p_{a\mu}} = \underbrace{p^a p_a}_{m_a^2}$$

INVARIANT MASS OF DECAY PRODUCTS = MASS OF PARENT

$p_1 + p_2$ SUM OF 4-VECTORS → IS A 4-VECTOR

$$p_a^\mu p_{a\mu}$$

$$\downarrow$$

$$m_a^2$$

a - LABELS PARTICLE

μ → 4-VECTOR INDEX

FOUR - DERIVATIVES

LOOK AT LORENTZ TRANSFORMATION OF DERIVATIVES

$$z' = \gamma(z - \beta t) \quad ; \quad t' = \gamma(t - \beta z)$$

$$\frac{\partial}{\partial z'} = \frac{\partial z}{\partial z'} \cdot \frac{\partial}{\partial z'} + \frac{\partial t}{\partial z'} \frac{\partial}{\partial t}$$

$z' \equiv z'(z,t)$

$$\frac{\partial}{\partial t'} = \frac{\partial z}{\partial t'} \frac{\partial}{\partial z} + \frac{\partial t}{\partial t'} \cdot \frac{\partial}{\partial t}$$

INVERSE $\quad z = \gamma(z' + \beta t') \; ; \; t = \gamma(t' + \beta z')$

$$\frac{\partial z}{\partial z'} = \gamma \; ; \quad \frac{\partial t}{\partial t'} = \gamma \; ; \quad \frac{\partial z}{\partial t'} = \gamma\beta \; ; \quad \frac{\partial t}{\partial z'} = \gamma\beta$$

$$\left.\begin{aligned} \text{SO} \quad \frac{\partial}{\partial z'} &= \gamma \frac{\partial}{\partial z} + \gamma\beta \frac{\partial}{\partial t} \\ \frac{\partial}{\partial t'} &= \gamma\beta \frac{\partial}{\partial z} + \gamma \frac{\partial}{\partial t} \end{aligned}\right\} \text{MATRIX}$$

FROM LAST TWO EQTNS:

$$\begin{pmatrix} \partial/\partial t' \\ \partial/\partial x' \\ \partial/\partial y' \\ \partial/\partial z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} \partial/\partial t \\ \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}$$

COMPARE TO:

$$\begin{pmatrix} t' \\ -x' \\ -y' \\ -z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} t \\ -x \\ -y \\ -z \end{pmatrix}$$

SHOWS THAT $\left(\partial/\partial t \, , \, \partial/\partial x \, , \, \partial/\partial y \, , \, \partial/z \right)$

TRANSFORMS AS A COVARIANT 4-VECTOR

$$\partial_\mu = \partial/\partial x^\mu$$ ← UP INDEX MEANS NO -VES

DOWN INDEX MEANS COVARIANT.

COVARIANT $\partial_\mu = \partial/\partial x^\mu$ HAS COMPONENTS:

$$\partial_0 = \partial/\partial t\ , \quad \partial_1 = +\partial/\partial x\ , \quad \partial_2 = +\partial/\partial y\ , \quad \partial_3 = +\partial/\partial z$$

DOWN MEANS COVARIANT

+ MEANS CONTRAVARIANT

THERE IS A CORRESPONING CONTRAVARIANT 4-DERIVATIVE

$$\partial^\mu = \left(\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right)$$

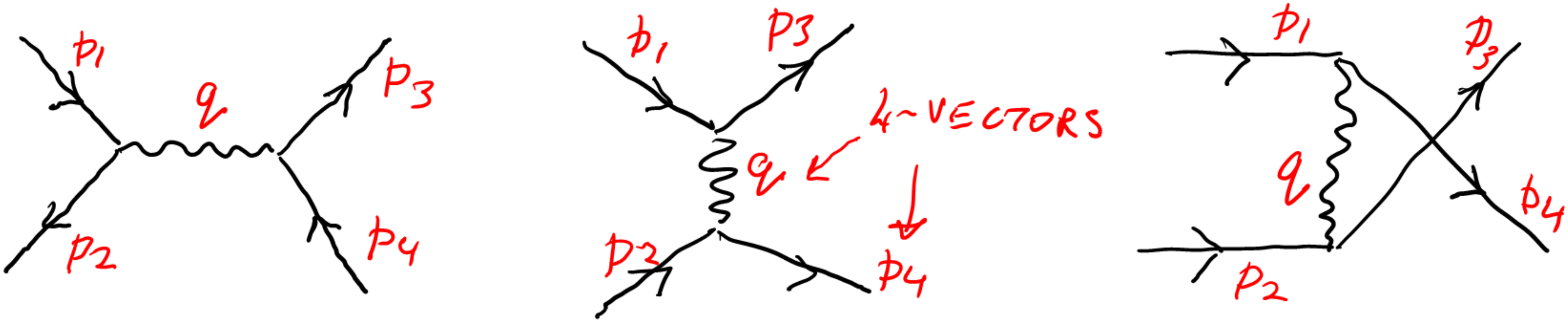
4-LAPLACIAN $\rightarrow$ D'ALEMBERTION $\Box$ OR $\Box^2$

BOX SQUARED

$$\Box = \partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

MANDELSTAM VARIABLES — LORENTZ INVARIANT

TWO PARTICLES SCATTERING $1+2 \rightarrow 3+4$



S-CHANNEL ANNIHILATION

t-CHANNEL SCATTERING

U-CHANNEL SCATTERING

NEEDED WHEN FINAL STATE PARTICLES IDENTICAL

$$\left.\begin{aligned} s &= (p_1+p_2)^2 = (p_3+p_4)^2 \\ t &= (p_1-p_3)^2 = (p_2-p_4)^2 \\ u &= (p_1-p_4)^2 = (p_2-p_3)^2 \end{aligned}\right\} = q^2$$

(4-MOMENTUM TRANSFER$)^2$

IN AN EXPERIMENT, ALL YOU SEE ARE THE INITIAL AND FINAL STATE PARTICLES → YOU DON'T SEE WHAT HAPPENS BETWEEN THE VERTICES

→ IF FINAL STATE PARTICLES ARE INDENTICAL P_3 & p_4 COULD COME FROM EITHER VERTEX.

MANDELSTAM VARIABLES ⟶ LORENTZ INVARIANT (q^2)

⟶ CAN EVALUATE IN ANY FRAME

A GOOD FRAME is CMS $\bar{p}_{TOT} = 0$ $\quad \xrightarrow{\bar{p}_1} \xleftarrow{\bar{p}_2}$

$$\left.\begin{aligned} p_1 &= (E_1^*, \bar{p}^*) \\ p_2 &= (E_2^*, -\bar{p}^*) \end{aligned}\right\} \quad \begin{aligned} s &= (p_1 + p_2)^2 = (E_1^* + E_2^*)^2 - (\bar{p}^* - \bar{p}^*)^2 \\ &= (E_1^* + E_2^*)^2 \end{aligned}$$

$\sqrt{s}$ = TOTAL ENERGY AVAILABLE IN CMS SYSTEM

AN ASIDE

$$s = (p_1 + p_2)^2 \quad \leftarrow \text{4-VECTORS !}$$

$$= p_1^2 + p_2^2 + 2 p_1 p_2$$

$$= m_1^2 + m_2^2 + 2 (E_1, \bar{p})(E_2 - \bar{p})$$

$$= m_1^2 + m_2^2 + 2E_1E_2 + 2\bar{p}^2$$

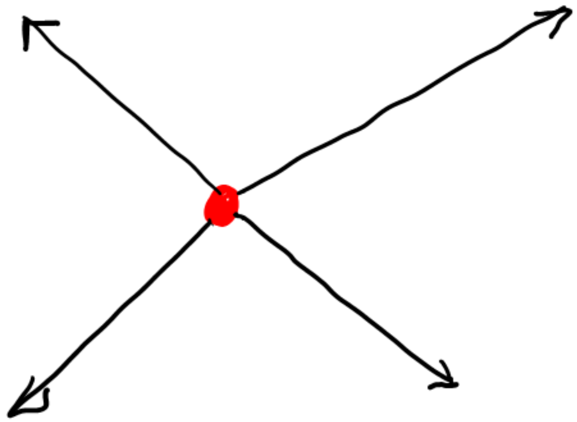
$$= \underbrace{m_1^2 + \bar{p}^2}_{E_1^2} + \underbrace{m_2^2 + \bar{p}^2}_{E_2^2} + 2E_1E_2$$

$$= (E_1 + E_2)^2$$

TWO COMMON REFERENCE FRAMES

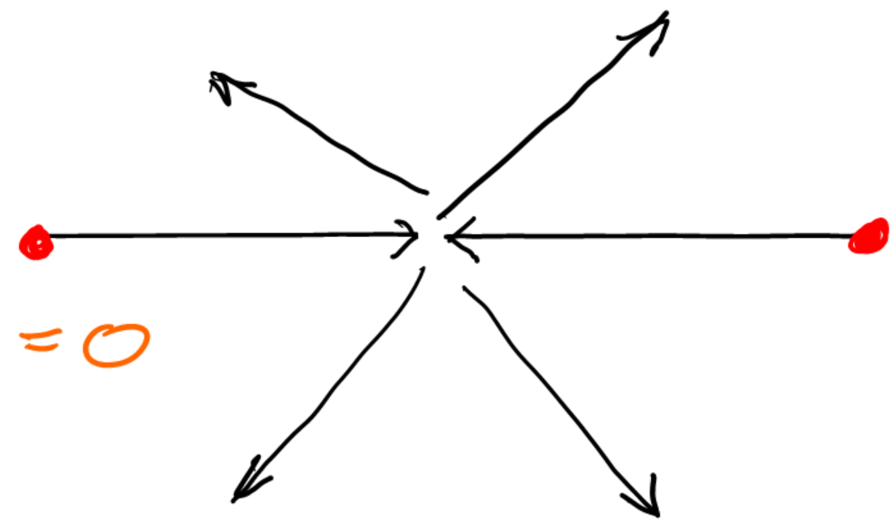
1) CM $\longrightarrow$ CENTER OF MOMENTUM (OR MASS)

PARTICLE DECAY

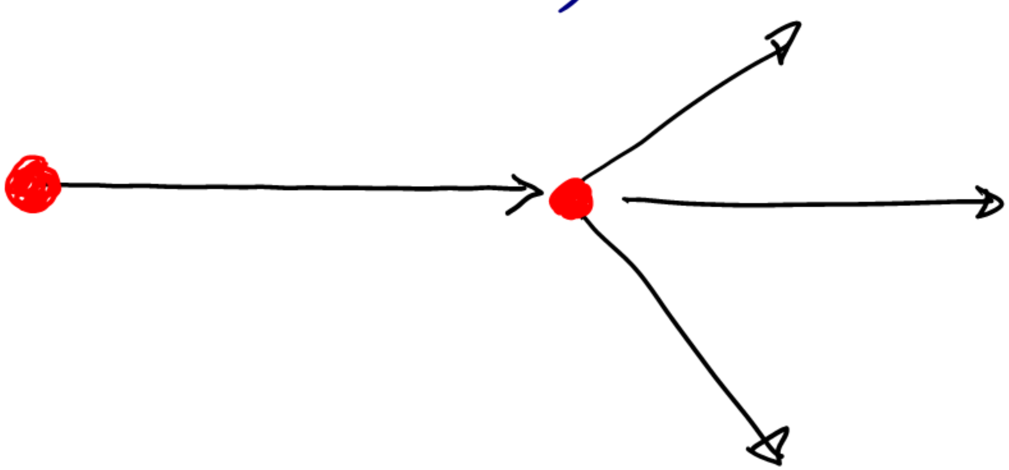


$$\sum \vec{p}_i = 0$$

COLLIDING BEAMS



2) LAB FRAME, TARGET PARTICLE AT REST.



WHEN TRANSFORMING BETWEEN FRAMES USE LORENTZ INVARIANCE OF 4-VECTOR SCALARS

$$p^{\mu} p_{\mu}$$

DOT PRODUCT OF TWO 4-VECTORS ⇒ SCALAR

$$a \cdot b = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 = a^0 b^0 - \bar{a} \cdot \bar{b}$$

$$= a^\mu b_\mu \, , \; a_\mu b^\mu , \; g_{\mu\nu} a^\mu b^\nu \quad \ldots$$

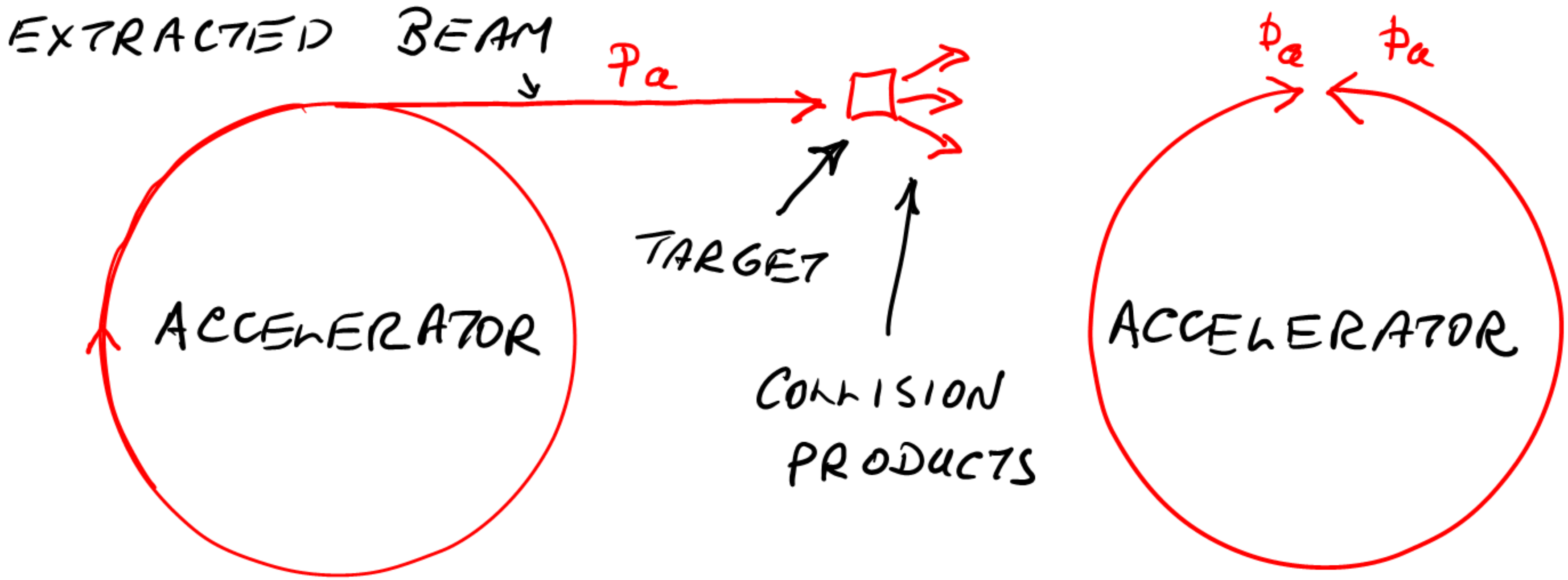
FOR 4-MOMENTUM

$$p_1 \cdot p_2 = p_1^0 p_2^0 - p_1^1 p_2^1 - p_1^2 p_2^2 - p_1^3 p_2^3 = E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2$$

$$p_1 p_1 = p_1^2 = E_1^2 - \vec{p}_1^{\,2} = m_1^2 \Longleftarrow !$$

- INVARIANTS
  - SAME IN ALL FRAMES
  - MULTI COMPONENT → SCALAR
  - CALCULATE IN MOST CONVENIENT FRAME

FIXED TARGET vs COLLIDING BEAMS



INTERESTING QUANTITY IS E_{CM}

- MOST OF p_a GOES INTO MOTION OF CM IN LAB FRAME
- ACCELERATOR IS IN LAB FRAME

- COLLIDING BEAMS ACCELERATOR IS IN CM FRAME
- ALL OF p_a GOES INTO E_{CM}

• 2 PARTICLE COLLISION IN AN ARBIRARY FRAME

$$A + B \rightarrow C + D$$

$$m_A \ (E_A, \bar{p}_A) \qquad m_B \ (E_B, \bar{p}_B)$$

• TOTAL $(4\text{-MOMENTUM})^2$ = p^2 OF SYSTEM

$$p^2 = (p_A + p_B)^2$$ ← 4-VECTORS

$$p^2 = p_A^2 + p_B^2 + 2p_A \cdot p_B = m_A^2 + m_B^2 + 2E_A E_B - 2\,\bar{p}_A \cdot \bar{p}_B$$

3 VECTORS

$$p^2 = m_A^2 + m_B^2 + 2E_A E_B - 2\,|\bar{p}_A|\,|\bar{p}_B| \cos\theta$$

SCATTERING ANGLE

IN CMS FRAME $\sum \bar{p} = 0$

$$p^{*2} = E^{*2} - |\bar{p}^*|^2 = E^{*2}$$ ← $(4 \text{ VECTOR})^2$

→ INVARIANT

$*$ MEANS CMS

$$p^{*2} = E^{*2} = m_A^2 + m_B^2 + 2E_A E_B - 2|\vec{p}_A||\vec{p}_B|\cos\theta$$

THIS IS TRUE IN ANY FRAME ∴ CAN CHOOSE CONVENIENT FRAME TO EVALUATE IN.

CHOOSE LAB FRAME $\underbrace{|\vec{p}_B| = 0 \quad E_B = m_B}_{\text{STATIONARY TARGET}}$

$$E^{*2} = m_A^2 + m_B^2 + 2m_B E_A \qquad E^* \propto \sqrt{E_A}$$

ACCELERATOR ENERGY

- COLLIDING BEAMS $|\vec{p}_A| = |\vec{p}_B| \quad \cos\theta = -1$

$$E^{*2} = m_A^2 + m_A^2 + 2E_A E_A + 2\underbrace{|p_A||p_A|}$$

FOR $E \gg m \longrightarrow 2E_A^2$

$$E^{*2} = 4E_A^2 \quad ; \quad E^* = 2E_A \rightarrow E^* \propto E_A$$

ENERGY THRESHOLD IN LAB FRAME SCATTERING

BASIC TO HEP EXPERIMENTS IS USING THE KINETIC ENERGY OF A COLLISION TO PRODUCE NEW PARTICLES IN THE FINAL STATE

SIMPLE EXAMPLE $\quad A + B \longrightarrow C$

BEAM (A)

TARGET AT REST (B)

4-VECTORS

SQUARING PRODUCES SCALARS

$$p_A + p_B = p_C \longrightarrow (p_A + p_B)^2 = p_C^2 \rightarrow p_A^2 + p_B^2 + 2(p_A \cdot p_B) = p_C^2$$

$$m_A^2 + m_B^2 + 2(p_A \cdot p_B) = m_C^2$$

. SINCE B IS AT REST $\longrightarrow$ LAB FRAME

$$\left.\begin{aligned} p_A &= (E_A, \vec{p}_A) \\ p_B &= \underbrace{(m_B, 0)}_{\text{AT REST}} \end{aligned}\right\} \rightarrow p_A \cdot p_B = (E_A, \vec{p}_A)(m_B, 0) = E_A m_B$$

$$m_A^2 + m_B^2 + 2E_A m_B = m_C^2$$

$$E_A = \frac{m_C^2 - m_A^2 - m_B^2}{2m_B}$$

$$E_A = \frac{m_C^2 - m_A^2 - m_B^2}{2 m_B^2}$$

DOESN'T SEEM TO DEPEND ON MOMENTUM OF C?

→ WE IMPOSED MOMENTUM CONSERVATION THRU 4-MOMENTUM CONSERVATION $P_C = P_A + P_B$

PROCESS $p + \bar{p} \rightarrow p + p + \bar{p} + \bar{p}$

LOWEST ENERGY CONFIG IN CMS → ALL FOUR PARTICLES AT REST

s → LORENTZ INVARIANT SAME IN LAB & CMS

CMS ENERGY $\sqrt{s} > 4 m_p$

IN LAB $s = E_{TOT}^2 - \vec{p}_{TOT}^{\,2}$

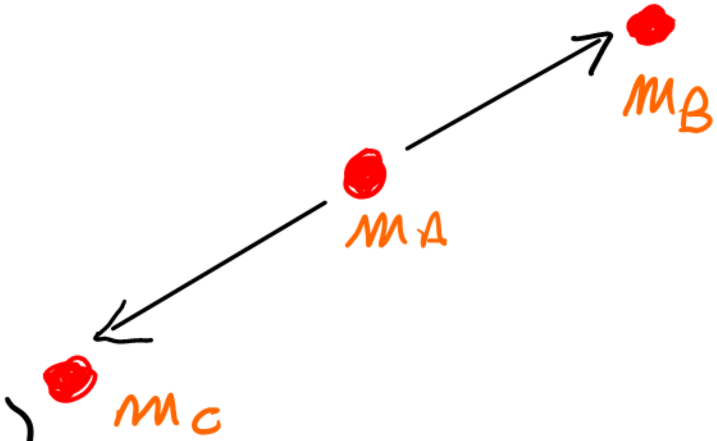
$$= (E + m_p)^2 - (\vec{p} + \vec{0})^2 > 16 m_p^2$$

$$E^2 + 2 E m_p + m_p^2 - p^2 > 16 m_p^2$$

$$2 E m_p + m_p^2 + m_p^2 > 16 m_p^2$$

$$E > 7 m_p$$

2 BODY DECAY IN CMS



$$p_A = p_B + p_C$$

$$p_A^2 = p_B^2 + p_C^2 + 2(E_c \;\; \vec{p})(E_B \;\; -\vec{p})$$

$$m_A^2 = m_B^2 + m_c^2 + 2E_c E_B + 2|p|^2 \cos\theta$$

$$= m_B^2 + m_C^2 + 2(E_A - E_B)E_B + 2(E_B^2 - m_B^2)$$

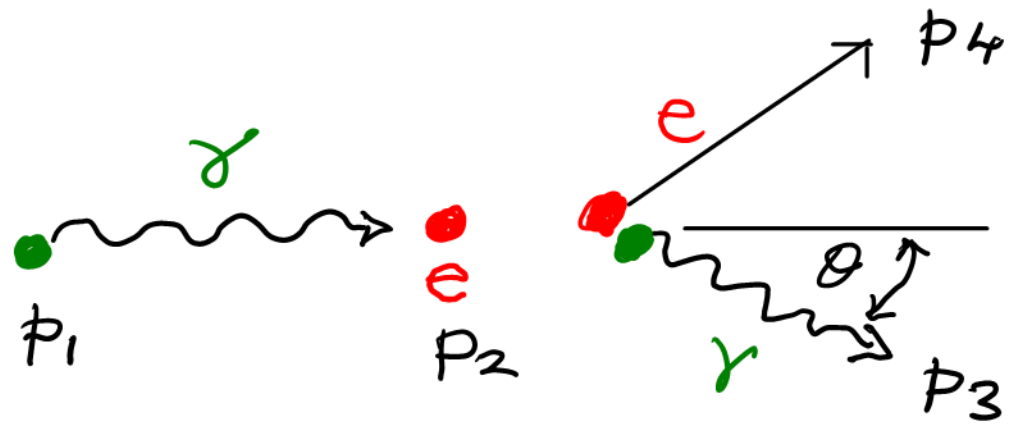
$$= m_B^2 + m_C^2 + 2E_A E_B - 2E_B^2 + 2E_B^2 - 2m_B^2$$

$$= m_B^2 + m_C^2 \;\; 2 m_A E_B \;\; - 2m_B^2$$

$$E_B = \frac{m_A^2 - m_C^2 + m_B^2}{2m_A}$$

ENERGY & MOMENTUM CONSERVATION WENT IN WHEN WE INVOKED 4-MOMENTUM CONSERVATION

COMPTON SCATTERING



4-VECTORS

$$p_1 + p_2 = p_3 + p_4 \quad \rightarrow (p_1 + p_2 - p_3) = p_4$$

$$p_4^2 = \underbrace{p_1^2}_{0} + \underbrace{p_2^2}_{m_e^2} + \underbrace{p_3^2}_{0} + 2(p_1 \cdot p_2 - p_1 \cdot p_3 - p_2 \cdot p_3)$$

$$m_e^2 = m_e^2 + 2(p_1 \cdot p_2 - p_1 \cdot p_3 - p_2 \cdot p_3)$$

$$p_1 \cdot p_3 = p_1 \cdot p_2 - p_2 \cdot p_3$$

$$p_1 \cdot p_3 = p_1 \cdot p_2 - p_2 \cdot p_3$$

SCATTERING ANGLE θ IS IN THE LAB FRAME

IN LAB

$$p_1 = (E_1, \bar{p}_1) \quad , \quad p_2 = (m_e, 0) \quad p_3 = (E_3, \bar{p}_3)$$

$$p_1 \cdot p_2 = (E_1, \bar{p}_1)(m_e, 0) = E_1 m_e$$

$$p_1 \cdot p_3 = (E_1, \bar{p}_1)(E_3, \bar{p}_3) = E_1 E_3 - \bar{p}_1 \bar{p}_3 = E_1 E_3 (1 - \cos\theta)$$

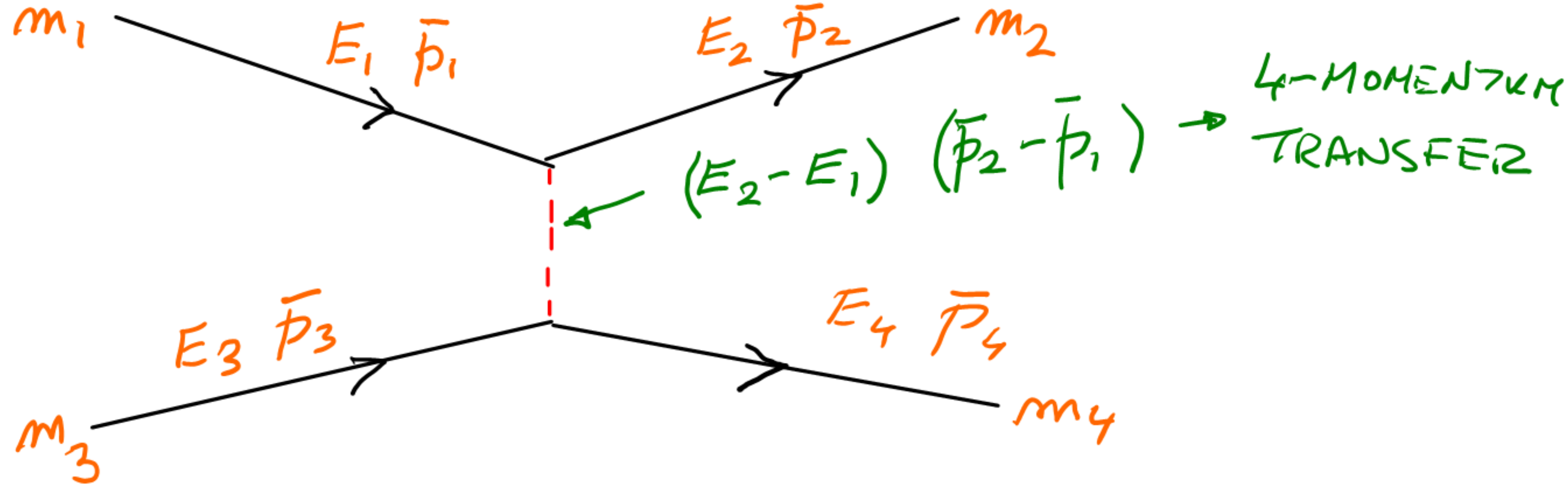
$E >> m$

$$p_2 \cdot p_3 = (m_e, 0)(E_3, \bar{p}_3) = E_3 m_e$$

$$E_1 E_3 (1 - \cos\theta) = E_1 m_e - E_3 m_e$$

$$(1 - \cos\theta) = \frac{m_e (E_1 - E_3)}{E_1 E_1}$$

SCATTERING ANGLE & 4-MOMENTUM TRANSFER



4-VECTORS

$$(\Delta p)^2 = (p_2 - p_1)^2 = p_2^2 + p_1^2 - 2 p_1 \cdot p_2$$

$$= m_2^2 + m_1^2 - 2(E_2, \bar{p}_2)(E_1, \bar{p}_1)$$

$$= m_2^2 + m_1^2 - 2E_2E_1 + 2\bar{p}_2\bar{p}_1$$

$$= m_2^2 + m_1^2 - 2E_2E_1 + 2|\bar{p}_2||\bar{p}_1| \cos\theta$$

$$(\Delta p)^2 = q^2 = m_2^2 + m_1^2 - 2E_2E_1 + 2|\vec{p}_2||\vec{p}_1|\cos\theta$$

IF $E_1 \gg m_1$, $|\vec{p}_1|^2 = E_1^2 - m_1^2 \approx E_1^2$

$$q^2 = 2E_1E_2(\cos\theta - 1)$$

LORENTZ INVARIANT $\longrightarrow$ TRUE IN ANY FRAME

IN LAB FRAME

$$q^2 = 2E_1E_2(\cos\theta - 1) \quad \text{IN LAB}$$

4-MOMENTUM TRANSFER GIVEN BY LAB KINEMATICS

$E_1 \rightarrow$ INCOMING BEAM ENERGY

$E_2 \rightarrow$ OUTGOING ENERGY

$\theta \rightarrow$ LAB SCATTERING ANGLE