NON-RELATIVISTIC QUANTUM MECHANICS

THIS IS GOING TO BE ESSENTIALLY A LIST OF THINGS TO REMEMBER

FREE PARTICLES DESCRIBED BY PACKETS OF PLANE WAVES. IN OUR NATURAL UNITS

$$\psi(\bar{x},t) = \underbrace{N}_{\text{NORMALIZATION}} \exp\{i(\bar{p}\cdot\bar{x} - Et\}$$

OPERATORS

$$\hat{p} = -i\bar{\nabla}, \quad \hat{E} = i\,\partial/\partial t$$

eg:

$$\hat{E}\psi = i\frac{\partial}{\partial t}\psi = i\frac{\partial}{\partial t}\left\{N\exp(i(\bar{p}\cdot\bar{x}-Et)\right\}$$

$$= i(-iE)\psi$$

$$= E\psi$$

SCHRÖDINGER

CLASSICALLY $E = H = T+V = \bar{p}^2/2m + V$

HAMILTONIAN

QM : DYNAMICAL VARIABLES $\longrightarrow$ OPERATORS

$$\hat{H}\,\psi(\bar{x},t) = E\,\psi(\bar{x},t)$$

$$i\,\partial/\partial t\;\psi(\bar{x},t) = \hat{H}\,\psi(\bar{x},t)$$

$$\hat{H} = \hat{p}^2/2m + \hat{V} = -\frac{1}{2m}\bar{\nabla}^2 + \hat{V}$$

So $$i\,\frac{\partial\psi(\bar{x},t)}{\partial t} = -\frac{1}{2m}\nabla^2\psi(\bar{x},t) + V\,\psi(\bar{x},t)$$

ONE DIMESION

$$i\,\frac{\partial\psi(\bar{x}\,t)}{\partial t} = -\frac{1}{2m}\,\frac{\partial^2\psi(\bar{x},t)}{\partial x^2} + \hat{V}\,\psi(\bar{x},t)$$

PROBABILITY DENSITY & PROBABILITY CURRENT

IMPORTANT FOR DIRAC EQUATION

PROBABILITY OF FINDING PARTICLE REPRESENTED BY $\psi(\bar{x}, t)$ IN VOLUME $d^3\bar{x}$ IS $\psi^*\psi \, d^3\bar{x}$

$$\rho(\bar{x}, t) = \psi^*(\bar{x}, t)\,\psi(\bar{x}, t)$$

PROBABILITY DENSITY

CONSERVATION OF PROBABILITY $\rightarrow$ CONTINUITY

$$\frac{\partial}{\partial t}\int_V \rho \, dV = -\int_S \bar{J} \cdot d\bar{s} \qquad (1)$$

RATE OF CHANGE OF PROBABILITY

NET FLUX LEAVING SURFACE

DIVERGENCE (GAUSS) THEOREM

$$\int_V \bar{\nabla} \cdot \bar{F} = \int_S \bar{F} \cdot \bar{n} \, ds$$

SO $$-\int_S \bar{j} \cdot d\bar{s} = -\int_V \bar{\nabla} \cdot \bar{j} \, dV$$

PUT THIS IN $$\frac{\partial}{\partial t} \int_V \rho \, dv = -\int \bar{j} \cdot d\bar{s}$$

$$\frac{\partial}{\partial t} \int \rho \, dv = -\int_V \bar{\nabla} \cdot \bar{j} \, dv$$

HOLDS FOR ANY ARBITRARY VOLUME SO:

$$\bar{\nabla} \cdot \bar{j} + \frac{\partial \rho}{\partial t} = 0$$

CONTINUITY EQUATION

PROBABILITY CURRENT.

PROBABILITY CURRENT IN SCHRÖDINGER

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi \quad \text{AND} \quad -i\frac{\partial\psi^*}{\partial t} = \frac{-1}{2m}\nabla^2\psi^*$$

$$\psi^* i\frac{\partial\psi}{\partial t} - \psi\left(-i\frac{\partial\psi^*}{\partial t}\right) = \psi^*\left(\frac{-1}{2m}\nabla^2\psi\right) - \psi\left(\frac{1}{2m}\nabla^2\psi^*\right)$$

$$i\left(\psi^*\frac{\partial\psi}{\partial t} + \psi\frac{\partial\psi^*}{\partial t}\right) = -\frac{1}{2m}\left(\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right)$$

$$-\frac{1}{2m}\bar{\nabla}\left(\psi^*\bar{\nabla}\psi - \psi\bar{\nabla}\psi^*\right) = i\frac{\partial}{\partial t}(\psi^*\psi) = i\frac{\partial\rho}{\partial t}$$

cf $\bar{\nabla}\cdot\bar{J} + \frac{\partial\rho}{\partial t} = 0$ CONTINUITY EQUATION

$$\bar{J} = \frac{1}{2im}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right)$$

$\hookrightarrow$ PROBABILITY CURRENT IN SCHRÖDINGER

PLANE WAVE $\psi(\bar{x},t) = N e^{i(\bar{p}\cdot\bar{x} - Et)}$

$\rightarrow$ CONSTANT DENSITY $\psi\psi^* = N^2$

REPRESENTS REGION OF SPACE WITH NUMBER DENSITY OF PARTICLES $n = |N|^2$

$$\nabla\psi = i\bar{p}\psi \quad , \quad \bar{\nabla}\psi^* = -i\bar{p}\psi^*$$

$$\psi^*\nabla\psi - \psi\nabla\psi^* = \psi^*(i\bar{p}\psi) - \psi(-i\bar{p}\psi^*) = 2ip\psi^*\psi$$

$$\bar{j} = \frac{1}{2im} 2i\bar{p}\,\underbrace{\psi\psi^*}_{N^2}$$

$$\bar{j} = \frac{\bar{p}}{m} N^2 \qquad \bar{p}/m = \bar{v}$$

$$\bar{j} = n\bar{v}$$

FLUX PASSING THRU UNIT AREA PER UNIT TIME.

TIME DEPENDENCE OF CONSERVED QUANTITIES

EXPECTATION VALUE $\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle = \int \psi^{+} \hat{A} \psi \, d^3x$

$$\frac{d\langle \hat{A} \rangle}{dt} = \int \left[\frac{\partial \psi^{+}}{\partial t} \hat{A} \psi + \psi^{+} \hat{A} \frac{\partial \psi}{\partial t} \right] d^3x$$

$\partial \hat{A} / \partial t = 0 \; ; \quad i \, \partial \psi_i / \partial t = \hat{H} \psi_i$

$$\frac{d\langle \hat{A} \rangle}{dt} = i \int \left[\psi^{+} \hat{H}^{+} \hat{A} \psi - \psi^{+} \hat{A} \hat{H} \psi \right] d^3x$$

$$H = H^{+} \rightarrow \frac{d\langle \hat{A} \rangle}{dt} = i \int \psi^{+} (\hat{H}\hat{A} - \hat{A}\hat{H}) \, d^3x$$

$$\frac{d\langle \hat{A} \rangle}{dt} = i \langle [\hat{H}, \hat{A}] \rangle$$

IF OPERATOR $\hat{A}$ COMMUTES WITH THE HAMILTONIAN EXPECTATION VALUE CONSTANT

CONSERVED QUANTITY

$$\frac{d\langle \hat{A} \rangle}{dt} = \int \left(\left[\frac{1}{i} \hat{H} \psi_i \right]^\dagger A \psi_i + \psi_i^\dagger \hat{A} \left[\frac{1}{i} \hat{H} \psi_i \right] \right) d^3\bar{x}$$

EIGENSTATE OF HAMILTONIAN

$$= \int \left(-\frac{1}{i} \left[E_i \psi_i \right]^\dagger \hat{A} \psi + \psi^\dagger \hat{A} (-i) E_i \psi_i \right) d^3\bar{x}$$

$$= \int \left[i E_i \psi_i^\dagger A \psi - i \psi_i^\dagger \hat{A} E_i \psi_i \right] d^3\bar{x}$$

FOR EIGENSTATES OF HAMILTONIAN $\langle \hat{A} \rangle$ IS CONSTANT FOR ANY OPERATOR

STATIONARY STATES.

COMMUTATORS & COMPATIBLE OBSERVABLES

COMMUTATOR BETWEEN OBSERVABLES, DETERMINE WHETHE THEY CAN BE SIMULTANEOUSLY KNOWN

$$[\hat{A},\hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A} = 0$$

$|\phi\rangle \rightarrow$ NON DEGENERATE EIGENSTATE OF $\hat{A}$

$$\hat{A}|\phi\rangle = a|\phi\rangle \rightarrow \hat{A}\hat{B}|\phi\rangle = \hat{B}\hat{A}|\phi\rangle = a\hat{B}|\phi\rangle$$

JUST A NUMBER

SO $\hat{B}|\phi\rangle$ IS ALSO AN EIGENSTATE OF $\hat{A} \rightarrow$ EIGENVALUE a

$$\hat{B}|\phi\rangle \propto |\phi\rangle$$

LABEL STATE BY SPECIFYING QUANTUM NUMBERS OF A COMPLETE SET OF COMPATIBLE OBSERVABLES

$$[\hat{C},\hat{A}] = [\hat{C},\hat{B}] = 0 \leftarrow$$

LABEL AS $|a, b, c\rangle$

ANGULAR MOMENTUM IN QUANTUM MECHANICS

THE "ALGEBRA" OF ANGULAR MOMENTUM IS EXTREMELY IMPORTANT IN PARTICLE PHYSICS

→ DIRAC & SPIN

→ ISOSPIN

CLASSICALLY $\bar{L} = \bar{r} \times \bar{p}$

$$= (y p_z - z p_y,\ z p_x - x p_z,\ x p_y - y p_x)$$

→ REPLACE POSITION & MOMENTUM → OPERATORS

$\hbar = 1$

$$[\hat{L}_x, \hat{L}_y] = i L_z \quad [\hat{L}_y, \hat{L}_z] = i \hat{L}_x \quad [\hat{L}_z, \hat{L}_x] = i \hat{L}_y$$

THIS COMMUTATOR ALGEBRA COMPLETELY DEFINES THE PROPERTIES OF ANGULAR MOMENTUM

$\hat{L}_x \quad \hat{L}_y \quad \hat{L}_z$ ARE INCOMPATIBLE OPERATORS. CAN ONLY DEFINE EIGENSTATE OF ONE OF THEM.

HOWEVER $\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$ COMMUTES WITH EACH.

$$[\hat{L}^2, \hat{L}_x] = [\hat{L}^2, \hat{L}_y] = [\hat{L}^2, \hat{L}_z] = 0$$

LADDER OPERATORS

$$\hat{L}_+ = \hat{L}_x + i\hat{L}_y, \quad \hat{L}_- = \hat{L}_x - i\hat{L}_y, \quad [\hat{L}^2, \hat{L}_x] = 0$$

$$[\hat{L}_z, \hat{L}_\pm] = [\hat{L}_z, \hat{L}_x] \pm i[\hat{L}_z, \hat{L}_y] = i\hat{L}_y \pm \hat{L}_x$$

$$[\hat{L}_z, \hat{L}_\pm] = \pm\hat{L}_\pm$$

$$\hat{L}^2 = \hat{L}_-\hat{L}_+ + \hat{L}_z + \hat{L}_z^2$$

SIMULTANEOUS EIGENSTATES OF $\hat{L}^2$ & $\hat{L}_z$

$|\lambda, m\rangle$ SIMULTANEOUS EIGENSTATE OF $\hat{L}^2, \hat{L}_z$

$$\hat{L}_z|\lambda, m\rangle = m|\lambda, m\rangle, \quad \hat{L}^2|\lambda, m\rangle = \lambda|\lambda, m\rangle$$

CONSIDER STATE $\quad \psi = \hat{L}_+|\lambda, m\rangle$

$$\hat{L}^2\psi = \hat{L}^2\hat{L}_+|\lambda, m\rangle = \hat{L}_+\hat{L}^2|\lambda, m\rangle = \lambda\hat{L}_+|\lambda, m\rangle$$
$$= \lambda\psi$$

ALSO $\quad \hat{L}_z\hat{L}_+ = \hat{L}_+\hat{L}_z + \hat{L}_+ \rightarrow$ COMMUTATOR

$$L_z\psi = \hat{L}_z\hat{L}_+|\lambda, m\rangle = (\hat{L}_+\hat{L}_z + \hat{L}_+)|\lambda, m\rangle$$
$$= \hat{L}_+\hat{L}_z|\lambda, m\rangle + \hat{L}_+|\lambda, m\rangle = \hat{L}_+ m|\lambda, m\rangle + \hat{L}_+|\lambda, m\rangle$$
$$= (m+1)\hat{L}_+|\lambda, m\rangle = (m+1)\psi$$

$\psi \rightarrow$ SIMULTANEOUS EIGENSTATE $\hat{L}^2, \hat{L}_z$

$\hat{L}_+$ STEPS THRU STATES WITH SAME VALUE OF $\hat{L}_z^2$

EACH STEP INCREASES Z-COMPONENT BY ONE UNIT

MAGNITUDE OF Z-COMPONENT CANNOT B > L_z

$\langle \hat{L}_z^2 \rangle \leq \langle \hat{L}^2 \rangle$ → FOR A GIVEN VALUE OF λ
THERE MUST BE A MAX, MIN m

$\hat{L}_+$ ACTING ON STATE WITH m_{MAX} RETURNS 0

SUPPOSE THIS STATE HAS $m = \ell$

$\hat{L}_+ |\lambda, \ell\rangle = 0$ FOR THIS STATE

$$\hat{L}^2 |\lambda, \ell\rangle = (\hat{L}_- \hat{L}_+ + \hat{L}_z + \hat{L}_z^2) |\lambda, \ell\rangle$$

$$\lambda |\lambda, \ell\rangle = (0 + \ell + \ell^2) |\lambda, \ell\rangle$$

FOR $m = \ell$, $\hat{L}^2$ EIGENVALUE $\lambda = \ell(\ell+1)$

"SAME" FOR $m = -\ell$ → $(2\ell+1)$ STATES

$2\ell+1$ STATES $\quad m = -\ell, -\ell+1 \cdots\cdots \ell+1, +\ell$

ℓ QUANTIZED AT INTEGRAL, $\frac{1}{2}$-INTEGRAL VALUES

$$\hat{L}_z |\ell, m\rangle = m|\ell, m\rangle \qquad \hat{L}^2|\ell, m\rangle = \ell(\ell+1)|\ell, m\rangle$$

$$\hat{L}_+ |\ell, m\rangle = \alpha_{\ell m} |\ell, m+1\rangle$$

SINCE $\hat{L}_+^+ = \hat{L}_-$ $\quad \left(\hat{L}_+|\ell, m\rangle \right)^+ = \langle \ell, m|\hat{L}_- = \alpha^*_{\ell m} \langle \ell, m+1|$

$$\langle \ell, m|\hat{L}_-\hat{L}_+|\ell, m\rangle = |\alpha_{\ell m}|^2 \left(\langle \ell, m+1|\ell, m+1\rangle\right) \to 1$$

$$|\alpha_{\ell, m}|^2 = \langle \ell, m|\hat{L}_-\hat{L}_+|\ell, m\rangle = \langle \ell, m|\hat{L}^2 - \hat{L}_z - \hat{L}_z^2|\ell, m\rangle$$
$$= \left[\ell(\ell+1) - m - m^2\right]\left(\langle \ell, m|\ell, m\rangle\right) \to 1$$

$$\hat{L}_+|\ell, m\rangle = \sqrt{\ell(\ell+1) - m(m+1)}\,|\ell, m+1\rangle$$

$$\hat{L}_-|\ell, m\rangle = \sqrt{\ell(\ell+1) - m(m-1)}\,|\ell, m-1\rangle$$

USE TO COMBINE ANGULAR MOMENTA

FERMI'S GOLDEN RULE

DECAY RATES, SCATTERING → TRANSITIONS BETWEEN STATES

RESULT OF $\hat{H}_{INT}$ → INDUCES TRANSITIONS

$$\phi_k(\bar{x},t) \rightarrow \hat{H}_0 \phi_k = E_k \phi_k \quad ; \quad \langle \phi_j | \phi_k \rangle \delta_{jk}$$

UNPERTURBED TIME INDEP H

$$\hat{H}'(\bar{x},t) \rightarrow$$ TIME DEP, INDUCES TRANSITIONS

$$i\frac{d\psi}{dt} = \left[\hat{H}_0 + \hat{H}'(\bar{x},t)\right]\psi \quad ; \quad \psi(\bar{x},t) = \sum_k c_k(t) \underbrace{\phi_k(\bar{x})\, e^{-iE_k t}}$$

TIME DEP

COMPLETE SET OF EIGENSTATES OF $\hat{H}_0$

$$i\sum_k \left[\frac{dc_k}{dt}\phi_k e^{-iE_k t} - iE_k c_k \phi_k e^{-iE_k t}\right]$$

$$= \underbrace{\sum_k c_k \hat{H}_0 \phi_k e^{-iE_k t}} + \sum \hat{H}' c_k \phi e^{-iE_k t}$$

$\hookrightarrow \hat{H}_0 \phi_k = E_k \phi \rightarrow \sum_k c_k (-iE_k) e^{-iE_k \cdot t}$

$$i\sum_k \left[\frac{dc_k}{dt}\phi_k e^{-iE_k t} - \cancel{iE_k c_k \phi_k e^{-iE_k t}}\right] = \cancel{\sum_k c_k(-iE_k)\phi_k e^{-iE_k t}} + \sum_k \hat{H}' c_k \phi_k e^{-iE_k t}$$

$$\text{so } i\sum_k \left[\frac{dc_k}{dt}\phi_k e^{-iE_k t}\right] = \sum_k \hat{H}' c_k(t)\phi_k e^{-iE_k t}$$

SYSTEM IS IN STATE $|i\rangle$

AT $t=0$, $|i\rangle = \phi_i$, $c_k(0) = \delta_{ik}$

IF $\hat{H}'$ IS CONSTANT & SMALL FOR $t>0$

SUCH THAT $c_i(t) \approx 1$, $c_{k\neq i}(t) \approx 0$

$$i\sum_k \frac{dc_k}{dt}\phi_k e^{-iE_k t} \approx \hat{H}\phi_i e^{-iE_i t}$$

CORRESPONDS TO TRANSITIONS

$$i\sum_k \frac{dc_k}{dt}\phi_k e^{-iE_k t} \approx \hat{H}'\phi_i e^{-iE_i t}$$

TAKE INNER PRODUCT $\langle f|LHS|i\rangle = \langle f|RHS|i\rangle$

$$\frac{dc_f}{dt} = -i\langle f|\hat{H}'|i\rangle e^{+i(E_f - E_i)}$$

$\langle \phi_f|\phi_k\rangle = \delta_{fk}$

$$i\sum_k \frac{dc_k}{dt}\int_V \phi_f^* e^{iE_f t}\phi_i e^{-iE_i t} d^3\bar{x}$$

$\left\{ \begin{array}{l} E_f = E_i \\ e^0 = 1 \end{array} \right.$

$$= i\, dc_f/dt$$

$$\hat{H}'\phi_i e^{-iE_i t} \Longrightarrow \int_V \phi_f^* e^{iE_f t}\hat{H}'\phi_i e^{-iE_i t}) d^3\bar{x}$$

$$= \langle f|\hat{H}'|i\rangle e^{i(E_f - E_i)t}$$

SO $\dfrac{dc_f}{dt} = -i\langle f|\hat{H}'|i\rangle e^{i(E_f - E_i)t}$

$T_{fi} = \langle f|\hat{H}'|i\rangle$

$\langle f|i\rangle$ DIMENSION LESS

$[T_{fi}] = [\hat{H}'] = [\text{ENERGY}]$

$$T_{fi} = \langle f | \hat{H}' | i \rangle$$ TRANSITION MATRIX ELEMENT.

AT TIME $t = T$, THE AMPLITUDE FOR $\rightarrow |f\rangle$

$$c_f(T) = -i \int_0^T T_{fi}\, e^{i(E_f - E_i)t}\, dt = -i\, T_{fi} \int_0^T e^{i(E_f - E_i)t}$$

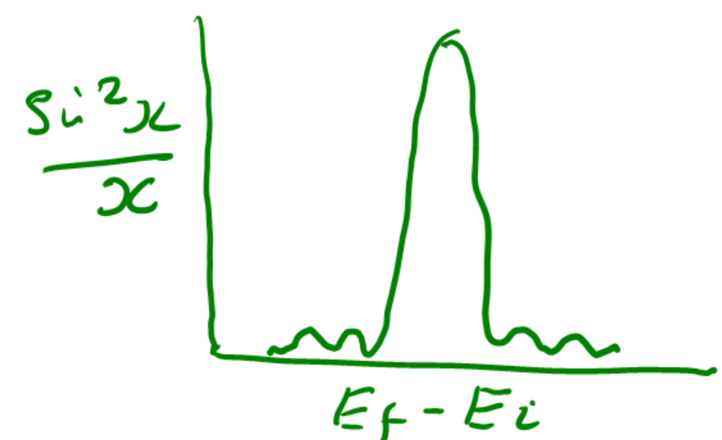
TIME INDEP

PROPABILITY FOR TRANSITION $P_{fi} = c_f(T)\, c_f^*(T)$

$$P_{fi} = |T_{fi}|^2 \int_0^T \int_0^T e^{i(E_f - E_i)t}\, e^{-i(E_f - E_i)t'}\, dt\, dt'$$

RATE OF TRANSITION $$d\Gamma_{fi} = \frac{P_{fi}}{T} = \frac{1}{T} |T_{fi}|^2 \int_{-T/2}^{+T/2} \int_{-T/2}^{+T/2} e^{i(E_f - E_i)t}\, e^{-(E_f - E_i)t'}\, dt\, dt'$$

$$\int \rightarrow \frac{\sin^2 x}{x}\ ; \quad x = \frac{(E_f - E_i)T}{2(\hbar)}$$



$$\Delta E \Delta t \sim \hbar\ ; \quad \frac{\Delta E}{\hbar} \sim \frac{1}{\Delta t} = \Gamma$$

THE FORM OF $d\Gamma_{fi}$ IS VERY NARROW, SO CAN TAKE LIMIT $\to \infty$, WHICH IS A δ-fn.

$$d\Gamma_{fi} = |T_{fi}|^2 \lim_{T\to\infty} \left\{ \frac{1}{T} \int_{-T/2}^{+T/2} \int_{-T/2}^{+T/2} e^{i(E_f - E_i)t} e^{-i(E_f - E_i)t'} \, dt \, dt' \right\}$$

USE δ-fn TO REPLACE $\int dt'$

$$d\Gamma_{fi} = 2\pi \, |T_{fi}|^2 \lim_{T\to\infty} \left\{ \frac{1}{T} \int_{-T/2}^{+T/2} e^{i(E_f - E_i)t} \delta(E_f - E_i) \, dt \right\}$$

dn ACCESIBLE STATES IN $E_f \to E_f + dE_f$

$$\Gamma_{fi} = 2\pi \int |T_{fi}|^2 \frac{dn}{dE_f} \lim_{T\to\infty} \left\{ \frac{1}{T} \int_{-T/2}^{+T/2} e^{-i(E_f - E_i)t} \delta(E_f - E_i) \, dt \right\} dE_f$$

δ fn $\to E_i = E_f$

$$\Gamma_{fi} = 2\pi \int |T_{fi}|^2 \frac{dn}{dE_f} \delta(E_f - E_i) \lim_{T\to\infty} \left\{ \frac{1}{T} \int_{-T/2}^{T+/2} dt \right\} dE_f$$

$$\Gamma_{fi} = 2\pi \int |T_{fi}|^2 \frac{dn}{dE_f} \delta(E_f - E_i) \lim_{T \to \infty} \left\{ \frac{1}{T} \int_{-T/2}^{T/2} dt \right\} dE_f$$

$$= 2\pi \int |T_{fi}|^2 \frac{dn}{dE_f} \delta(E_f - E_i)\, dE_f$$

$$= 2\pi\, |T_{fi}|^2 \left| \frac{dn}{dE_f} \right|_{E_i}$$

DENSITY OF STATES

$$\rho(E_i) = \left| \frac{dn}{dE_f} \right|_{E_i}$$

SO GOLDEN RULE

$$\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_i)$$

$T_{fi} = \langle f | \hat{H}' | i \rangle$ TO FIRST ORDER

SECOND ORDER TRANSITION MATRIX ELEMENT

IN DERIVING THE FIRST ORDER T_{fi} HAD:

$$i\sum_k \frac{dc_k}{dt}\phi_k e^{-iE_kt} = \sum_k \hat{H}' c_k(t)\phi_k e^{-iE_kt}$$

ASSUMED $c_i(t) \approx 1$, $c_{k\neq i}(t) \simeq 0$

THEN GOT $c_f(T) = -i\int_0^T T_{fi}\, e^{i(E_f-E_i)}$

INSTEAD OF ASSUMING $c_{k\neq i} \approx 0$ USE

THEN $$i\sum_k \frac{dc_k}{dt}\phi_k e^{-iE_k} = \hat{H}'\phi_i e^{-iE_it} + \sum \hat{H}'\phi_k(-i)\int_0^T T_{ki}\, e^{i(E_k-E_i)t}$$

TAKE INNER PRODUCT ϕ_f

$$i\frac{dc_f}{dt} \approx \langle f|\hat{H}'|i\rangle e^{i(E_f-E_i)t} + i\sum_{k\neq i}\langle f|\hat{H}'|i\rangle e^{i(E_f-E_k)t}\int_0^t \langle k|\hat{H}'|i\rangle e^{i(E_k-E_i)t'}dt'$$

$$i\frac{dc_f}{dt} \approx \langle f|\hat{H}'|i\rangle e^{i(E_f-E_i)t} + i\sum_{k\neq i}\langle f|\hat{H}'|i\rangle e^{i(E_f-E_k)t}\int_0^t \langle k|\hat{H}'|i\rangle e^{i(E_k-E_i)t'}\,dt'$$

$\hat{H}' = 0$ AT $t=0$, CONSTANT FOR $t>0$

$$\int_0^t \langle k|\hat{H}'|i\rangle e^{i(E_k-E_i)t'} = \langle k|\hat{H}'|i\rangle \frac{e^{i(E_k-E_i)t}}{i(E_k-E_i)}$$

$$\frac{dc_f}{dt} = -i\left(\langle f|\hat{H}'|i\rangle + \sum \frac{\langle f|\hat{H}|k\rangle\langle k|\hat{H}'|i\rangle}{E_i-E_k}\right)e^{i(E_f-E_i)t}$$

$$T_{fi} = \langle f|\hat{H}'|i\rangle + \sum_{k\neq i}\frac{\langle f|\hat{H}'|k\rangle\langle k|\hat{H}'|i\rangle}{E_i-E_k}$$

SECOND ORDER TRANSITION IS THROUGH
INTERMEDIATE STATE $|k\rangle$
HOW QUICKLY THIS CONVERGES DEPENDS ON $\hat{H}'$