DECAY RATES & CROSS SECTIONS

EXPERIMENTS MEASURE :

- RATE AT WHICH PARTICLES DECAY Γ
- PROBABILITY OF SCATTERING σ

IN NON-RELATIVISTIC PROBLEMS

$$\Gamma_{fi} = 2\pi \, |T_{fi}| \, \rho(E)$$

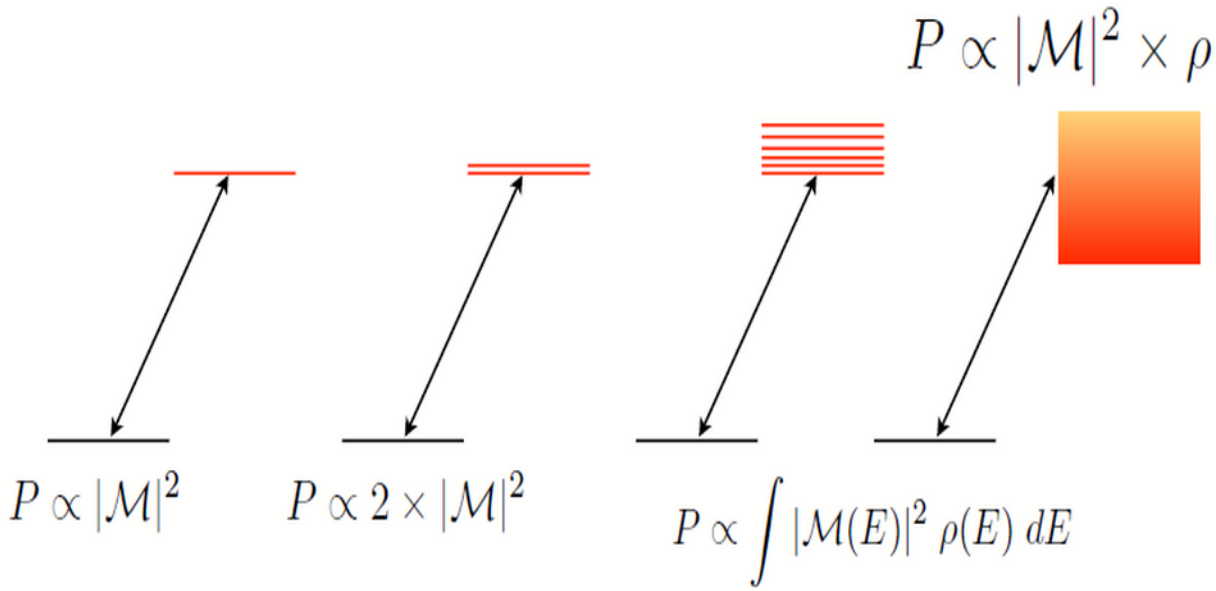
THEORY

DENSITY OF FINAL STATES

$$T_{fi} = \langle f | H' | i \rangle + \sum \frac{\langle f | H' | k \rangle \langle k | H' | i \rangle}{E_i - E_f}$$

JUST COMES FROM KINEMATICS

$H' \rightarrow$ THEORY



PROBABILITY OF TRANSITION $\propto$ DENSITY OF FINAL STATES

$$\rho(E_i) = \left| \frac{dN}{dE} \right|_{E_i}$$

CAN WRITE THIS AN A $\int$ OVER ALL FINAL STATES USING δ-FN TO FORCE ENERGY CONSERVATION

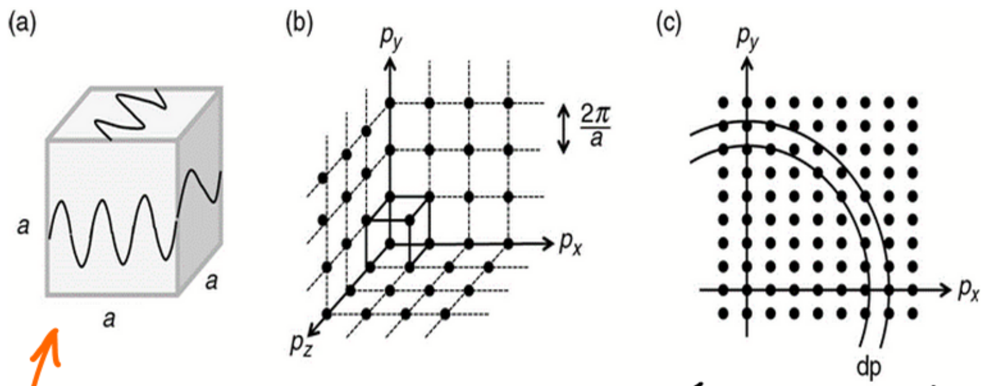
$$\left| \frac{dn}{dE} \right|_{E_i} = \int \frac{dn}{dE} \delta(E_i - E)\, dE = \int \delta(E_i - E)\, dn$$

SO

$$\Gamma_{fi} = 2\pi \int |T_{fi}|^2 \delta(E_i - E)\, dn$$

PHASE SPACE & WAVE FUNCTION NORMALIZATION

NEED TO FIND RELATIVISTIC FORMULATION OF PHASE SPACE



$$\psi(\bar{x},t) = A\, e^{i(\bar{p}\cdot\bar{x} - Et)}$$

NORMALIZATION

ON PARTICLE PER CUBE "a"

$$\int^a\!\!\int^a\!\!\int^a \psi^* \psi \, d^3\bar{x} = 1$$

$$A^2 = \frac{1}{a^3} = \frac{1}{V}$$

VOLUME OF CUBE

NON RELATIVISTICALLY

$$T_{fi} = \langle \psi_1 \psi_2 | \hat{H}' | \psi_a \rangle$$

$$= \int \psi_1^* \psi_2^* \hat{H}' \psi_a \, d^3\bar{x}$$

→ PERIODIC BOUNDARY COND

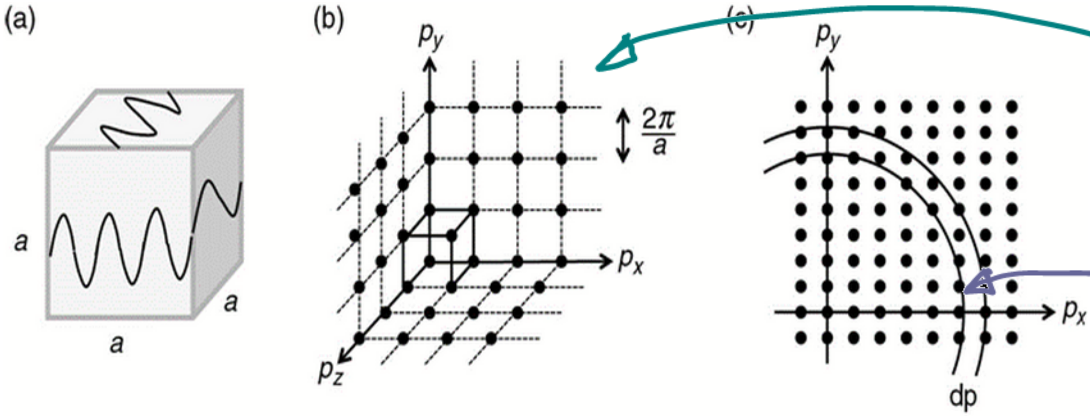
$$\psi(x+a, y, z) = \psi(x, y, z) \ldots$$

$$e^{ip_x x} = e^{ip_x(x+a)}$$

$$(p_x, p_y, p_z) = (n_x, n_y, n_z)\frac{2\pi}{a}$$

RESTRICTS MOMENTUM STATES TO DISCRETE POINTS IN PHASE SPACE

PERIODIC BOUNDARY CONDITIONS



DISCRETE MOMENTUM STATES

SPHERICAL SHELL

$p \to p + d$

HAD $(p_x, p_y, p_z) = (n_x, n_y, n_z)\frac{2\pi}{a}$

VOLUME OF EACH MOMENTUM STATE $d^3\bar{p} = dp_x dp_y dp_x = \frac{(2\pi)^3}{V}$

$$dn = \frac{\text{VOLUME OF SHELL } p \to p + dp}{\text{VOLUME OF STATE}} = 4\pi p^2 dp \times \frac{V}{(2\pi)^3}$$

$$\frac{dN}{dp} = \frac{4\pi p^2}{(2\pi)^3} \cdot V \to \rho(E) = \frac{dn}{dE} = \frac{dn}{dp}\left|\frac{dp}{dE}\right|$$

NUMER OF ACCESSIBLE MOMENTUM STATES INCREASES WITH MOMENTUM OF FINAL STATE PARTICLES

LIGHT FINAL STATE PARTICLES FAVORED OVER HEAVY

NORMALIZATION VOLUME SHOULD NOT EFFECT Γ

V (PHASE) SPACE $\to$ CANCELLED BY V(NORMALIZATION)

CHOOSE TO NORMALIZE TO $\frac{\text{1 PARTICLE}}{\text{UNIT VOL}}$ $\to$ $V = 1$

WHEN SET $V=1$ $\quad d^3\bar{p} = \frac{(2\pi)^3}{V} \to (2\pi)^3$

IF $d^3\bar{p}_i$ IS SOME SMALL VOLUME IN $\bar{p}$ SPACE

$$dn = \frac{\text{SMALL VOLUME}}{\text{VOLUME OF STATE}} \to dn_i = \frac{d^3\bar{p}_i}{(2\pi)^3}$$

FOR N FINAL STATE PARTICLES, $(N-1)$ INDEP, FROM p - CONSERVE

N PARTICLE FINAL STATE

$$dn = \prod_{i=1}^{N} dn_i = \prod_{i=1}^{N-1} \frac{d^3\bar{p}_i}{(2\pi)^3} = \prod_{i=1}^{N-1} \frac{d^3\bar{p}_i}{(2\pi)^3}\,\delta^3\left[\bar{p}_a - \sum_{i=1}^{N} \bar{p}_i\right] d^3\bar{p}_N$$

IMPOSE MOMENTUM CONSERVATION

VOLUME ELEMENT FOR N^{TH}

$$dn = \prod_{i=1}^{N-1} \frac{d^3\bar{p}_i}{(2\pi)^3}\, \delta^3\Big(\bar{p}_a - \sum_{i=1}^{N} \bar{p}_i\Big)\, d^3\bar{p}_N$$

MOMENTUM CONSERVATION

DECAYING PARTICLE

SUM OF FINAL STATE MOMENTA

N^{TH} PARTICLE

CLEARLY YOU CAN CHANGE TO PRODUCT OVER N

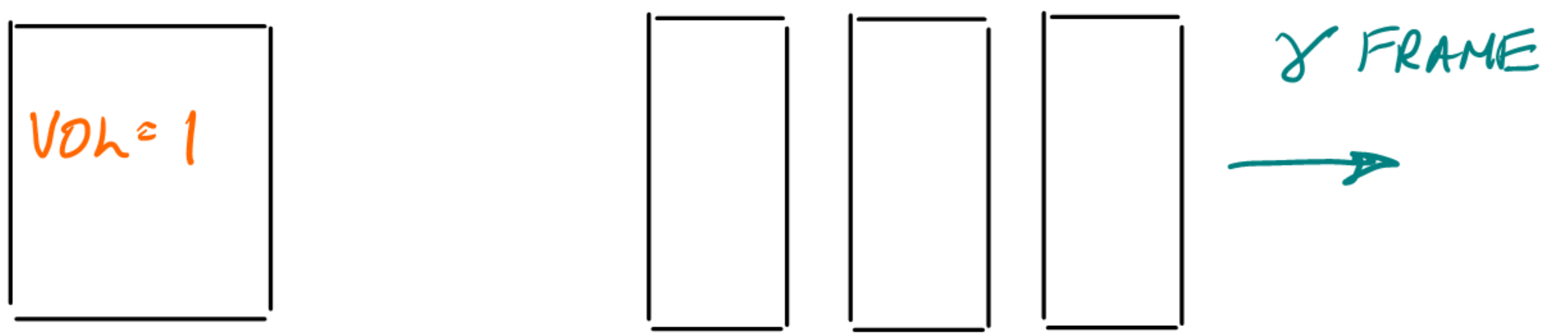
$$dn = (2\pi)^3 \prod_{i=1}^{N} \frac{d^3\bar{p}_i}{(2\pi)^3}\, \delta^3\Big(\bar{p}_a - \sum_{i}^{N} p_i\Big) \qquad (3.7)$$

GENERAL EXPRESSION FOR N-BODY PHASE SPACE IN NON-RELATIVISTIC DECAY.

LORENTZ - INVARIANT PHASE SPACE

1 PARTICLE / UNIT VOLUME REFERS TO PARTICULAR FRAME

NOT LORENTZ INVARIANT. OBSERVERS IN DIFFERENT INERTIAL FRAMES SEE VOLUME CONTRACTED BY $1/\gamma$



$\gamma = E/M$

REST FRAME

$\frac{1}{\gamma}$ $\therefore$ THERE MUST BE γ OF THESE TO HAVE SAME VOLUME AS

LORENTZ INVARIANT CHOICE IS $\propto E$ PARTICLE / UNIT VOLUME

CONVENTIONALLY CHOOSE $2E$ PARTICLES PER UNIT VOLUME

2E PARTICLES PER UNIT VOLUME

FOR NON RELATIVISTIC $\int_V \psi^* \psi \, d^3\bar{x} = 1$

RELATIVISTIC $\int_V \psi'^* \psi' \, d^3\bar{x} = 2E \Rightarrow \psi' = (2E)^{1/2} \psi$

FOR A GENERAL n-PARTICLE PROCESS $a + b \rightarrow 1 + 2 + \cdots$

LORENTZ INVARIANT ME $M_{fi} = \langle \psi_1' \psi_2' \cdots | \hat{H} | \psi_a' \psi_b' \rangle$

IN T_{fi} WE USED 1-PARTICLE PER BOX

THESE ψ' HAVE RELATIVISTIC NORMALIZATION

$$M_{fi} = \langle \psi_1' \psi_2' \cdots | \hat{H}' | \psi_a' \psi_b' \cdots \rangle = \underbrace{(2E_1 \cdot 2E_2 \cdots 2E_a \cdot 2E_b)^{1/2}}_{\text{ALL INITIAL AND FINAL STATE PARTICLES}} T_{fi}$$

LORENTZ INVARIANT MATRIX ELEMENT.

LORENTZ-INVARIANT GOLDEN RULE

2 BODY DECAY $\quad \Gamma_{fi} = 2\pi \int |T_{fi}|^2 \delta(E_a - E_1 - E_2)\, dn$ (3.7)

$a \to 1 + 2$

$$\Gamma_{fi} = (2\pi)^4 \int |T_{fi}|^2 \delta(E_a - E_1 - E_2)\, \delta^3(\bar{p}_a - \bar{p}_1 - \bar{p}_2)\, \frac{d^3\bar{p}_1}{(2\pi)^3} \cdot \frac{d^3\bar{p}_2}{(2\pi)^3}$$

RELATIVISTICALLY: (3.11)

$$\Gamma_{fi} = \frac{(2\pi)^4}{2E_a} \int |\mathcal{M}_{fi}|^2 \delta(E_a - E_1 - E_2)\, \delta^3(\bar{p}_a - \bar{p}_1 - \bar{p}_2)\, \frac{d^3\bar{p}_1}{(2\pi)^3 2E_1} \cdot \frac{d^3\bar{p}_2}{(2\pi)^2 2E_2}$$

TRANSITION RATE IN FRAME OBSERVED $\propto \frac{1}{E_a} = \frac{1}{m_a \gamma}$

TIME DILATION

LORENTZ INVARIANT NORMALIZATION

$$\int \frac{d^3\bar{p}}{(2\pi)^3} \longrightarrow \int \frac{d^3\bar{p}}{(2\pi)^3 2E}$$

DO IT!

SHOW $\frac{d^3\bar{p}'}{E'} = \frac{d^3\bar{p}}{E}$ $\leftarrow$ LORENTZ INVARIANT PHASE SPACE FACTOR

CAN EXTEND THIS 2-BODY DISCUSSION $\rightarrow$ N BODIES

$$dLIPS = \prod_{i=1}^{N} \frac{d^3 \vec{p}_i}{(2\pi)^3 2E_i}$$

$$\int \cdots dLIPS = \int \cdots\cdots \prod_{i=1}^{N} \frac{1}{(2\pi)^3} \delta^3(p_i^2 - m_i^2)\, d^4 p_i$$

$d^3\vec{p}_i\, dE_i$

4-VECTOR

GOING BACK TO 2-BODY CASE

$$\Gamma_{fi} = \frac{(2\pi)^4}{2E_a} \int (2\pi)^{-6} |\mathcal{M}_{fi}|^2 \delta^4(p_a - p_1 - p_2)\, \delta^0(p_1^2 - m_1^2)\, \delta(p_2^2 - m_2^2)$$

$$\times\, d^4 p_1\, d^4 p_2$$

4-VECTORS

p_1 & m_1 SCALARS

$\delta^4 \rightarrow$ ENERGY MOMENTUM CONSERVATION

$\delta \rightarrow E^2 = p^2 + m^2$ FOR FINAL STATE PARTICLES

GENERALITIES ON PARTICLE DECAYS

IN GENERAL, PARTICLES CAN HAVE SEVERAL DECAY MODES

$K^+ \to e^+ \nu_e$, $K^+ \to \mu^+ \nu_\mu$, $K^+ \to \pi^0 e^+ \nu_e$, $K^+ \to \pi^+ \pi^- \mu^+ \nu_\mu$

TRANSITION RATE FOR EACH MODE → CALC INDEPENDENTLY

→ PARTIAL DECAY RATES; TOTAL DECAY RATE $= \sum$ PARTIAL DECAY RATES

HAVE N PARTICLES OF A PARTICULAR TYPE

NUMBER DECAYING IN δt

$$\delta N = -N\Gamma_1 \delta t - N\Gamma_2 \delta t - \cdots\cdots$$

PARTIAL RATE

$$\delta N = -N \sum_j \Gamma_j \delta t \rightarrow -N\Gamma \delta t$$

$$\Gamma = \sum_j \Gamma_j$$

$$\delta N = -N\Gamma\delta t \rightarrow \int_{N(0)}^{N(t)} \frac{dN}{N} = \int_0^t dt$$

$$\ln N(t) - \ln N(0) = -N\Gamma t \rightarrow \ln\left\{\frac{N(t)}{N(0)}\right\} = -N\Gamma t$$

EXPONENTIATE

$$N(t) = N(0)e^{-\Gamma t} = N(0)e^{-t/\tau}$$

τ IS AVERAGE LIFETIME OF PARTICLE IN ITS REST FRAME

RELATIVE FREQUENCY OF PARTICULAR MODE BRANCHING RATIO

$$BR_j = \frac{\Gamma_j}{\Gamma_{TOT}}$$

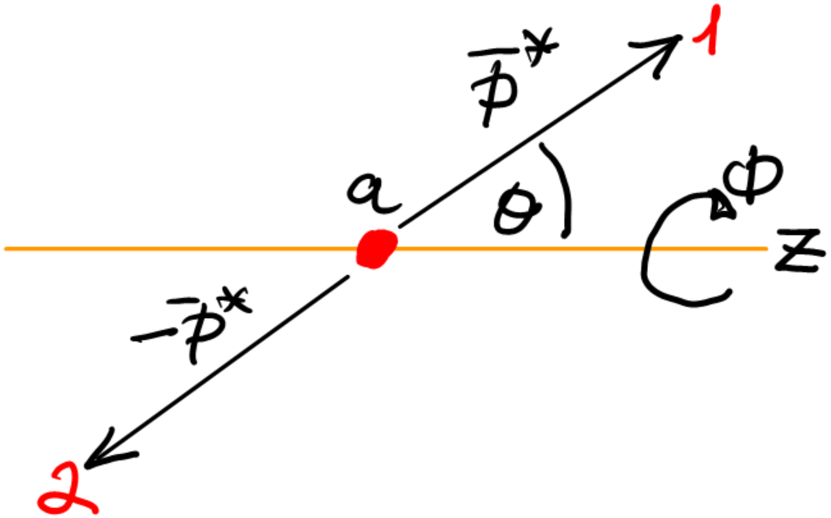
BY DEFINITION $\sum_j BR_j = 1$

KINEMATICS OF 2-BODY DECAYS

TRANSITION RATE = MATRIX ELEMENT × PHASE SPACE

FORCE INVOLVED
EG. EM, WEAK
$e^- \to \gamma e^-$, $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

NUMBER OF
PARTICLES & MASSES



(3.11)

HAD:

$$\Gamma_{fi} = \frac{(2\pi)^4}{2E_a} \int |\mathcal{M}_{fi}|^2 \delta(E_a - E_1 - E_2)\, \delta^3(\vec{p}_a - \vec{p}_1 - \vec{p}_2) \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \cdot \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2}$$

THIS IS LORENTZ INVARIANT
CAN EVALUATE IN ANY FRAME

CHOOSE
REST FRAME OF a
$\equiv$ CMS

IN CMS $\quad E_a = m_a \,,\ \bar{p}_a = 0 \,,\ \bar{p}_1 = -\bar{p}_2 = \bar{p}^*$

(3.11) BECOMES

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_A} \int |M_{fi}|^2 \delta(m_a - E_1 - E_2) \underbrace{\delta^3(\bar{p}_1 + \bar{p}_2)}_{\text{MAKES } \bar{p}_2 = -\bar{p}_1} \underbrace{\frac{d^3\bar{p}_1}{2E_1} \cdot \frac{d^3\bar{p}_2}{2E_2}}_{d^3\bar{p}_1}$$

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a} \int |M_{fi}|^2 \frac{1}{4E_1E_2} \delta(m_a - E_1 - E_2)\, d^3\bar{p}_1$$

$d^3\bar{p}_1 = p_1^2\, dp_1 \sin\theta\, d\theta\, d\phi = p_1^2\, dp_1\, d\Omega$

$p_1 \to |\bar{p}_1|$

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a} \int |M_{fi}|^2 \delta\left(m_a - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_2^2}\right) \frac{p_1^2}{4E_1E_2}\, dp_1\, d\Omega$$

INVOKE δ-fn TO DO THIS $\int$. IT HAS FORM

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a} \int |M_{fi}|^2 g(p_1)\, \delta(f(p_1))\, dp_1\, d\Omega$$

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a}\int |M_{fi}|^2 \delta\left(m_a - \sqrt{m_1^2+p_1^2} - \sqrt{m_2^2+p_2^2}\right)\frac{p_1^2}{4E_1E_2}\, dp_1\, d\Omega$$

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a}\int |M_{fi}|^2 g(p_1)\, \delta(f(p_1))\, dp_1\, d\Omega$$

$$g(p_1) = \frac{p_1^2}{4E_1E_2} \qquad f(p_1) = m_a - \sqrt{m_1^2+p_1^2} - \sqrt{m_2^2+p_2^2}$$

$\delta(f(p_1))$ IMPOSES ENERGY CONSERVATION

CAN ONLY BE DIFFERENT FROM ZERO WHEN $p_1 = p^*$

δ fn

$$\int |M_{fi}|^2 g(p_1)\, \delta(f(p_1))\, dp_1 = \underbrace{|M_{fi}|^2 g(p^*) \left|\frac{df}{dp_1}\right|^{-1}_{p^*}}_{\delta\text{- fn.}}$$

δ- fn ASIDE – READ IF YOU WANT

GENERALLY $$\int f(p_1)\,dp_1 = \int f(p_1)\,\frac{dp_1}{df}\cdot df$$

SO $$\int \delta[f(p_1)]\,dp_1 = \int \delta[f(p_1)]\,\frac{dp_1}{df}\cdot df$$

BUT $f = 1$ AT $p_1 = p^*$ ZERO EVERYWHERE ELSE

$$\int \delta[f(p_1)]\,dp_1 = 1 \times \left|\frac{dp_1}{df}\right|_{p^*}$$

SO $$\int |\mathcal{M}_{fi}|^2 g(p_1)\,\delta(f(p_1))\,dp_1$$

$$= |\mathcal{M}_{fi}|^2\, g(p^*)\left|\frac{df}{dp_1}\right|^{-1}_{\text{AT } p_1 = p^*}$$

$$f(p_1) = m_a - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_2^2}$$

$$\frac{d}{dp_1}\sqrt{m_1^2 + p_1^2} = \frac{d}{dp_1}(m_1^2 + p_1^2) \times \frac{d}{d(\ )}\sqrt{(\quad)}$$

$$= 2p_1 \times \frac{1}{2}(m_1^2 + p_1^2)^{-\frac{1}{2}} = \frac{p_1}{(m_1^2 + p_1^2)^{\frac{1}{2}}}$$

$$\text{so } \left|\frac{df}{dp_1}\right| = \frac{p_1}{(m_1^2 + p_1^2)^{\frac{1}{2}}} + \frac{p_1}{(m_2^2 + p_2^2)^{1/2}} = p_1\left\{\frac{1}{E_1} + \frac{1}{E_2}\right\}$$

$$\left|\frac{df}{dp_1}\right| = p_1\left\{\frac{E_1 + E_2}{E_1 E_2}\right\} \quad A$$

$$g(p^*)\left|\frac{df}{dp_1}\right|^{-1}_{p^*} \rightarrow \frac{p^{*2}}{4E_1E_2}\,\frac{E_1E_2}{p^*(E_1+E_2)}$$

BY ENERGY CONSERVATION $m_a = E_1 + E_2$

so $$\int |\mathcal{M}_{fi}|^2 g(p_1)\,\delta(f(p_1))\,dp_1 = \frac{p^*}{4m_a}|\mathcal{M}_{fi}|^2$$

$$\int |M_{fi}|^2 \delta(m_a - E_1 - E_2)\,\delta^3(\bar{p}_1 + \bar{p}_2)\,\frac{d^3\bar{p}_1}{2E_1}\cdot\frac{d^3\bar{p}_2}{2E_2}$$

$$= \frac{p^*}{4m_a}\int |M_{fi}|^2 d\Omega$$

THIS COULD DEPEND ON ANGLE → KEEP INSIDE

TWO BODY DECAY

$$\Gamma_{fi} = \frac{p^*}{32\pi^2 m_a^2}\int |M_{fi}|^2 d\Omega.$$

btw $$p^* = \frac{1}{2m_a}\left\{\left[m_a^2 - (m_1+m_2)^2\right]\left[m_a^2 - (m_1-m_2)^2\right]\right\}^{\frac{1}{2}}$$

INTERACTION CROSS SECTION

MANY EXPERIMENTS CONSIST OF COLLIDING PARTICLES AND LOOKING AT FINAL STATE.

IT'S QUITE SIMILAR TO DECAYS BUT HAVE TO ACCOUNT FOR INCOMING FLUX OF PARTICLES

FLUX — NUMBER OF PARTICLES CROSS UNIT AREA / UNIT TIME

HAVE BEAM OF PARTICLES TYPE a WITH FLUX ϕ_a CROSSING A REGION OF SPACE WITH n_b / UNIT VOLUME

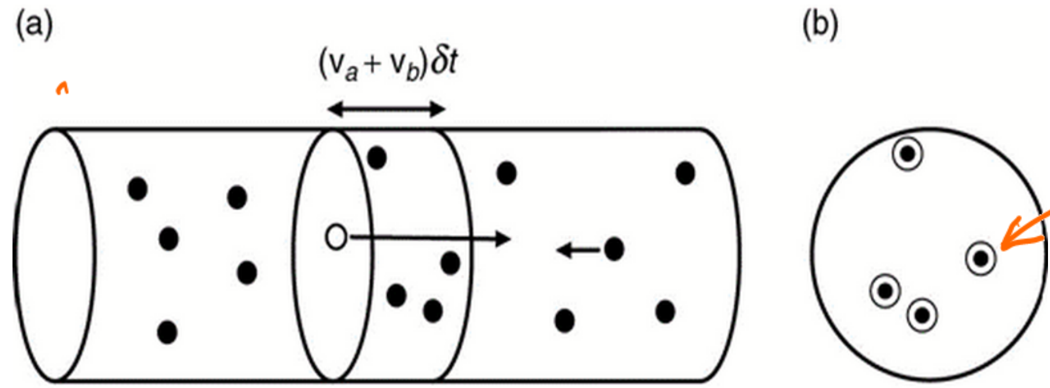
THE INTERACTION RATE IS:

$$r_b = \sigma \phi_a$$

HAS DIMENSIONS AREA

→ HENCE "CROSS SECTION"

REFLECTS PHYSICS OF QM AMPLITUDE FOR COLLISION



CROSS SECTION σ

SINGLE PARTICLE a TRAVELLING AT $\vec{v}_a$ $\rightarrow$ AREA A

REGION DEFINED BY A HAS n_b PER UNIT VOLUME TRAVELLING WITH $\vec{v}_b$

IN δt a CROSSES REGION CONTAINING $\delta N = n_b(\vec{v}_a + \vec{v}_b) A \delta t$

IN TIME δt, VOLUME $\vec{v} \times \delta t$ SWEPT OUT

↳ $= (\vec{v}_a + \vec{v}_b) A \delta t$

MULT BY NUMBER DESITY

$$\delta N = n_b(\vec{v}_a + \vec{v}_b) A \delta t$$

INTERACTION PROBABILITY

$$\delta P = \frac{\Sigma \sigma}{A} = \frac{\delta N \sigma}{A}$$

$$\delta p = \frac{\delta N \sigma}{A} = \frac{n_b (\bar{v}_a + \bar{v}_b) \cancel{A} \sigma \delta t}{\cancel{A}}$$

SINGLE PARTICLE

$$= n_b \bar{v} \sigma \delta t \; ; \quad r_a = \frac{dp}{dt} = n_b v \sigma$$

RELATIVE VELOCITY

INTERACTION RATE

FOR NUMBER DENSITY n_a IN VOLUME V

$$\text{RATE} = r_a \cdot n_a \cdot V = (n_b \sigma v) n_a V$$

$$\text{RATE} = (n_a v)(n_b V)\sigma = \text{FLUX} \cdot \text{NUMBER OF TARGETS} \cdot \sigma$$

cf. $r_b = \sigma \phi_a$

$$\sigma = \frac{\text{NUMBER OF INTERACTIONS / UNIT TIME / TARGET PARTICLE}}{\text{INCIDENT FLUX}}$$

LORENTZ INVARIANT FLUX

CALC σ — FERMI GOLDEN RULE

CALC σ — LORENTZ INVARIANT PARTICLE FLUX

$a + b \to 1 + 2$ IN REST FRAME $\vec{v}_a$ $\vec{v}_b$

n_a n_b

$$\text{RATE} = \phi_a n_b V \sigma = (v_a + v_b) n_a n_b \sigma V$$

ϕ_a = FLUX OF a THRU PLANE MOVING AT v_b

NORMALIZE WAVE FNS $\to$ ONE PARTICLE IN V

$$n_a = n_b = \frac{1}{V}$$

GOLDEN RULE

$$\text{RATE} = \Gamma_{fi} = \frac{v_a + v_b}{V} \cdot \sigma$$

V CANCELS IN FLUX AND NORM $\longrightarrow$ $\sigma = \dfrac{\Gamma_{fi}}{v_a + v_b}$

$$\sigma = \frac{(2\pi)^4}{v_a + v_b} \int |T_{fi}|^2 \, \delta(E_a + E_b - E_1 - E_2)\, \delta^3(\bar{p}_a + \bar{p}_b - \bar{p}_1 - \bar{p}_2) \frac{d^3\bar{p}_1}{(2\pi)^3} \frac{d^3\bar{p}_2}{(2\pi)^3}$$

LORENTZ-INVAR. PUT $\mathcal{M}_{fi} = (2E_1 2E_2 2E_3 2E_4)^{1/2} |T_{fi}|$

$$\sigma = \frac{(2\pi)^{-2}}{4E_a E_b (v_a + v_b)} \int |\mathcal{M}_{fi}|^2 \, \delta(E_a + E_b - E_1 - E_2)\, \delta^3(\bar{p}_a + \bar{p}_b - \bar{p}_1 - \bar{p}_2) \frac{d^3\bar{p}_1}{2E_1} \cdot \frac{d^3\bar{p}_2}{2E_1}$$

THIS $\int$ IS LORENTZ INVARIANT

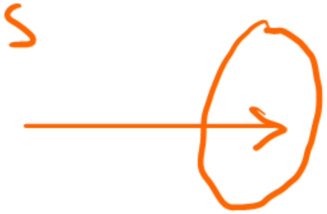
LORENT INVARIANT FLUX FACTOR, F

SHOW $F^2 = 16\left\{ (p_a \cdot p_b)^2 - m_a^2 m_b^2 \right\}$ DO IT!

4-VECTORS

SO σ IS LORENTZ INVARIANT

OBVIOUS



σ IS A "DISK" TRANVERSE TO DIRECTION OF MOTION

SCATTERING IN CMS-FRAME

σ LORENTZ INVARIANT → CALCULATE IN ANY FRAME

→ CMS OFTEN BEST.

$\xrightarrow{a} \; \xleftarrow{b}$

$$\sqrt{s} = \left(E_a^* + E_b^*\right)$$

$$\bar{p}_a = -\bar{p}_b = \bar{p}_i^* \qquad \sum_{CMS} \bar{p} = 0$$

$$\bar{p}_1 = -\bar{p}_2 = \bar{p}_f^*$$

LORENTZ INVARIANT FLUX FACTOR

$$F = 4 E_a^* E_b^* \left(v_a^* + v_b^*\right) \qquad |\bar{p}_i^*|$$

$$= 4 E_a^* E_b^* \left(\frac{p_i^*}{E_a^*} + \frac{p_i^*}{E_b^*}\right)$$

$$= 4 p_i^* \left(E_a^* + E_b^*\right) \qquad \sqrt{s}$$

$$\sigma = \frac{1}{(2\pi)^2} \frac{1}{4 p_i^* \sqrt{s}} \int |M_{fi}|^2 \, \delta\left(\sqrt{s} - E_1 - E_2\right) \delta^3\left(\bar{p}_1 + \bar{p}_2\right) \frac{d^3\bar{p}_1}{2E_1} \cdot \frac{d^3\bar{p}_2}{2E_2}$$

THIS IS SAME AS $\int$ IN 2-BODY DECAY $\qquad m_a \Leftrightarrow \sqrt{s}$

USE δ-fn TO DO $\int$ IN SAME WAY

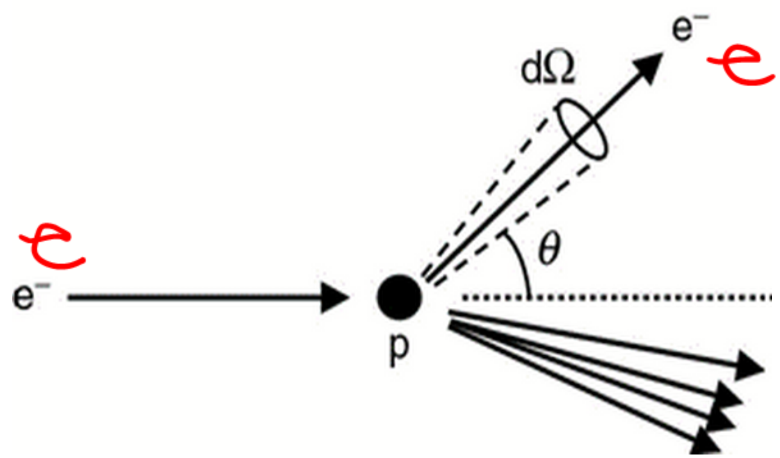
SO $\int = \frac{p_f^*}{4\sqrt{s}} \int |M_{fi}|^2 \, d\Omega^*$

AND $\sigma = \frac{1}{(2\pi)^2} \frac{1}{4 p_i^* \sqrt{s}} \cdot \frac{p_f^*}{4\sqrt{s}} \int |M_{fi}|^2 \, d\Omega^*$

$$\sigma = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \frac{p_f^*}{p_i^*} \int |M_{fi}|^2 \, d\Omega^*$$

THIS GIVES σ FOR ANY 2-BODY PROCESS

DIFFERENTIAL CROSS SECTIONS



OFTEN INTERESTED IN DISTRIBUTION OF SOME KINMATIC VARIABLE

E.G. ELECTRON IN

$ep \to e + \text{HADRONS}$

LOOK AT SCATTERING RATE INTO AN ELEMENT OF THE SOLID ANGLE $d\Omega = \underbrace{d(\cos\theta)}_{d\sin\theta\, d\theta}\, d\phi$

$$\frac{d\sigma}{d\Omega} = \frac{\#\text{ OF PARTICLES INTO } d\Omega \,/\, \text{UNIT TIME} \,/\, \text{TARGET PARTICLE}}{\text{INCIDENT FLUX}}$$

$$\sigma = \int \frac{d\sigma}{d\Omega} \cdot d\Omega$$

CAN HAVE MORE THAN ONE VARIABLE

$$\frac{d^2\sigma}{dE\, d\Omega}, \quad \frac{d\sigma}{dE}, \quad \frac{d\sigma}{dq^2} \quad \text{etc} \ldots$$

$d\sigma$ FROM DIFFERENTIAL OF $\sigma = \frac{1}{64\pi^2 s} \frac{p_f^*}{p_i^*} \int |\mathcal{M}_{fi}|^2 d\Omega^*$

SO $d\sigma = \frac{1}{64\pi^2} \frac{1}{s} \frac{p_f^*}{p_i^*} |\mathcal{M}_{fi}|^2 d\Omega^*$

EASY WHEN EXPERIMENT IN CMS

LAB = CMS
LEP, LHC, ...

JUST HAVE $\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \frac{p_f^*}{p_i^*} |\mathcal{M}_{fi}|^2$

FOR A STATIONARY TARGET eg $ep \to e + \text{HADRONS}$

BEAM

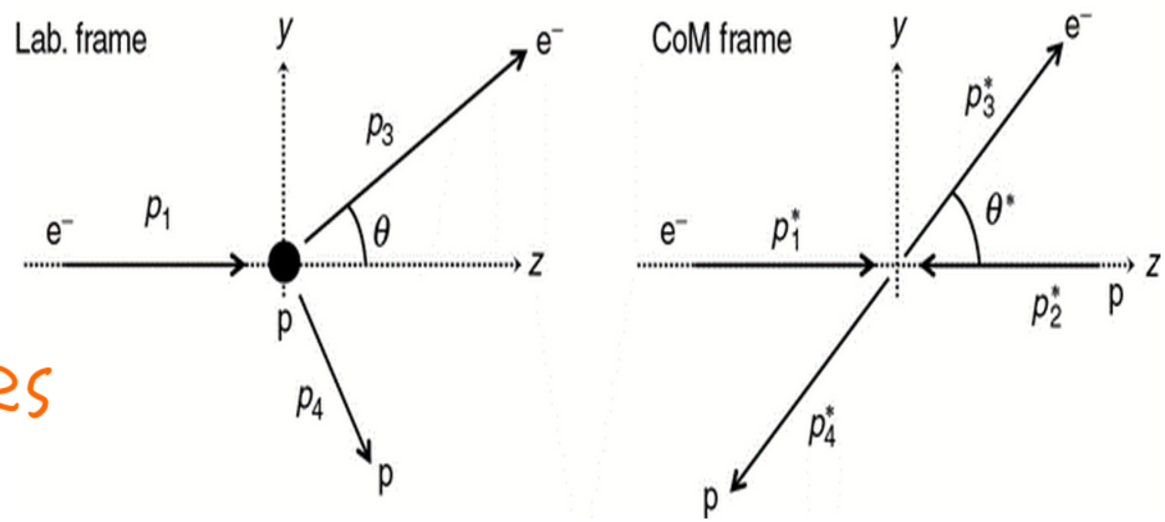
LIQUID HYDROGEN

NEED LAB QUANTITIES

NEED TO TRANSFORM $d\sigma \longrightarrow$ LAB FRAME

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s}\,\frac{p_f^*}{p_i^*}\,|M_{fi}|^2\, d\Omega^* \longrightarrow \text{LAB FRAME}$$

EXPRESS $d\Omega^*$ IN TERM OF LORENTZ INVARIANT t



4 VECTORS

$$t = (p_1^* - p_3^*) = p_1^{*2} + p_3^{*2} - 2 p_1^* \cdot p_3^*$$

$$= m_1^2 + m_3^2 - 2\left(E_1^* E_3^* - \vec{p}_1^* \cdot \vec{p}_3^*\right)$$

$$= m_1^2 + m_3^2 - 2E_1^* E_3^* + 2 p_1^*\, p_3^* \cos\theta^*$$

$= |\vec{p}_3^*|$

IN CMS ONLY VARIABLE IS θ^* $\longrightarrow$ $dt = 2 p_1^* p_3^*\, d(\cos\theta^*)$

$$d\Omega^* \equiv d(\cos\theta^*)\, d\phi^* = \frac{dt\, d\phi^*}{2 p_i^* p_3^*}$$

SUBSTITUTE THIS INTO $$d\sigma = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}\left|\mathcal{M}_{fi}\right|^2 d\Omega^*$$

$$d\sigma = \frac{1}{128\pi^2 s\, p_i^{*2}}\left|\mathcal{M}_{fi}\right|^2 d\phi^*\, dt$$

ϕ $\quad \int d\phi = 2\pi$

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s\, p_i^{*2}}\left|\mathcal{M}_{fi}\right|^2$$

THIS IS A GENERAL LORENTZ INVARIANT $\frac{d\sigma}{dt}$ FOR 2-BODY $a+b \to 1+2$

$$p_i^{*2} = \frac{1}{4s}\left[s-(m_1+m_2)^2\right]\left[s-(m_1-m_2)^2\right]$$

$\hookrightarrow$ DO IT !

LABORATORY FRAME DIFFERENTIAL CROSS SECTION

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s p_i^{*2}} |\mathcal{M}_{fi}|^2$$

IS VALID IN ANY INERTIAL FRAME

FOR $E \gg m$

$$p_1 \approx (E_1, 0, 0, E_1)$$

4-VECTORS

$$p_2 \approx (m_p, 0, 0, 0)$$

$$p_3 \approx (E_3, 0, E_3 \sin\theta, E_3 \cos\theta)$$

FOR $m_p \gg m_e$

$$p_i^{*2} = \frac{1}{4s}\left[s - (m_1 + m_2)^2\right]\left[s - (m_1 - m_2)^2\right]$$

$$\approx \frac{(s - m_p^2)^2}{4s}$$

AT REST

$$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2(E_1, |E_1|)(m_p, 0)$$

$$= m_p^2 + 2E_1 m_p$$

$$p_i^{*2} \sim \frac{(s-m_p)^2}{4s} = \frac{2E_1 m_p^2}{4s} = \frac{E_1 m_p^2}{s} \qquad (3.40)$$

WHAT WE ACTUALLY MEASURE IN THE EXPT is θ_{LAB}

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{dt} \cdot \left|\frac{dt}{d\Omega}\right| = \underbrace{\frac{1}{2\pi} \frac{dt}{d(\cos\theta)}}_{\text{NO AZIMUTHAL VARIATION}} \cdot \frac{d\sigma}{dt}$$

$$t = (p_1 - p_3)^2 = p_1^2 + p_3^2 - 2 p_1 \cdot p_2$$

$$= \cancel{m}_e^2 + \cancel{m}_e^2 - 2(E_1, |E_1|)(E_3, |E_3|)$$

$$t = -(2E_1E_3 - 2E_1E_3\cos\theta) = -2E_1E_3(1-\cos\theta)$$

THIS IS A FUNCTION OF θ

$$p_1 + p_2 = p_3 + p_4$$

CAN MAKE t OUT OF THESE

$$t = (p_2 - p_4)^2 = p_2^2 + p_4^2 - 2 p_2 \cdot p_4$$

$$= 2m_p^2 - 2(m_p, 0)(E_4, \vec{p}_4) = 2m_p^2 - 2m_p E_4$$

INVOKE ENERGY CONSERVATION $E_4 = E_1 + m_p - E_3$

$$= 2m_p^2 - 2m_p E_1 - 2m_p^2 + 2m_p E_3$$

$$t = -2m_p (E_1 - E_3)$$

EQUATE TWO EXPRESSIONS FOR t

$$2E_1 E_3 (1 - \cos\theta) = 2m_p (E_1 - E_3)$$

$$E_1 E_3 (1 - \cos\theta) + m_p E_3 = E_1 m_p$$

$$E_3 = \frac{E_1 m_p}{E_1 - E_1 \cos\theta + m_p} \qquad (3.44)$$

HAD $t = -2m_p\,(E_1 - E_3)$ FIXED INCOMING ENERGY

$$\frac{dt}{d(\cos\theta)} = 2m_p \frac{dE_3}{d(\cos\theta)}$$

FROM (3.44)

$$\frac{dE_3}{d(\cos\theta)} = \frac{E_1^2 m_p}{\underbrace{(m_p + E_1 - E_1\cos\theta)^2}_{=\frac{E_1^2 m_p^2}{E_3^2}}} = \frac{E_3^2}{m_p}$$

$$\frac{dE_3}{d(\cos\theta)} = \frac{E_3^2}{m_p}$$

HAD $\dfrac{dt}{d(\cos\theta)} = 2m_p \dfrac{dE_3}{d(\cos\theta)}$

$$\frac{dt}{d(\cos\theta)} = 2E_3^2$$ ← WHY SO SIMPLE ? ?

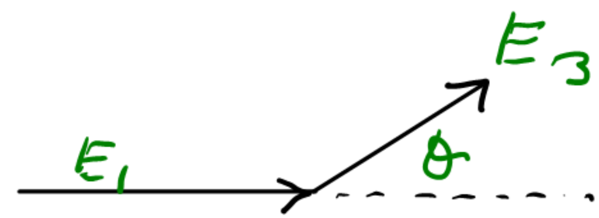
HAD $$\frac{d\sigma}{d\Omega} = \frac{1}{2\pi} \frac{dt}{d(\cos\theta)} \cdot \frac{d\sigma}{dt}$$

$$= \frac{1}{2\pi} \cdot \frac{2E_3^2}{m_p} \cdot \frac{1}{64\pi s p_i^{*2}} \left| \mathcal{M}_{fi} \right|^2$$

FROM (3·40) $$p_i^{*2} = \frac{E_1 m_p^2}{s}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{E_3}{E_1 m_p} \right)^2 \left| M_{fi} \right|^2$$

USUALLY EXPERIMENTS MEASURE INCIDENT ENERGY + θ



$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{1}{m_p + E_1 - E_1 \cos\theta} \right)^2 \left| M_{fi} \right|^2$$

SUMMARY

DECAY / INTERACTION RATE $\rightarrow$ $\Gamma_{fi} = 2\pi |T_{fi}| \rho(E)$

WE DISCUSSED HOW TO CALCULATE

DECAY RATE $a \rightarrow 1+2$

$$\Gamma = \frac{p^*}{32\pi^2 m_a} \int |\mathcal{M}_{fi}|^2 d\Omega$$

$$p^* = \frac{1}{2m_i}\sqrt{\left[(m_i^2 - (m_1+m_2)^2\right]\left[(m_i^2 - (m_1-m_2)^2\right]}$$

DIFFENTIAL σ $a+b \rightarrow c+d$
IN CMS

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2}\frac{1}{s}\frac{p_f^*}{p_i^*}\left|\mathcal{M}_{fi}\right|^2$$

FOR $ep \rightarrow ep$
NEGLECT m_e
IN THE LAB

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{E_3}{m_p E_1}\right)^2 \left|\mathcal{M}_{fi}\right|^2$$