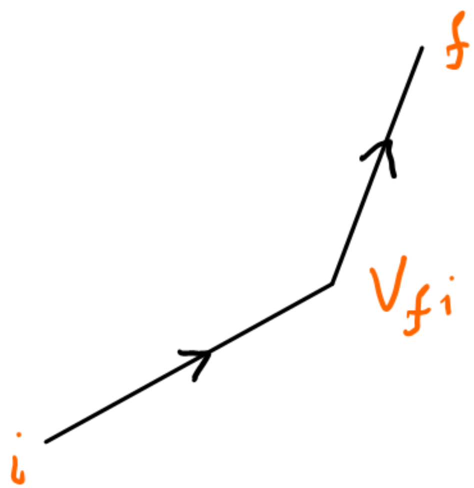
INTERACTION BY PARTICLE EXCHANGE "PSEUDO" QFT

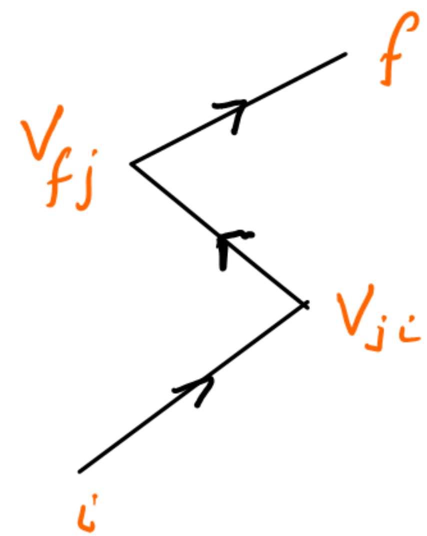
1ST, 2ND ORDER PERTURBATION

TRANSITION RATE $\Gamma_{fi} = 2\pi \left| T_{fi} \right|^2 \rho(E_f)$

$$T_{fi} = \langle f | V | i \rangle + \sum_{j \neq k} \frac{\langle f | V | j \rangle \langle j | V | i \rangle}{E_i - E_j} + \ldots$$



SCATTERING IN A POTENTIAL



SCATTERING VIA AN INTERMEDIATE STATE

THE IDEA OF SCATTERING IN A POTENTIAL → PROBLEMS

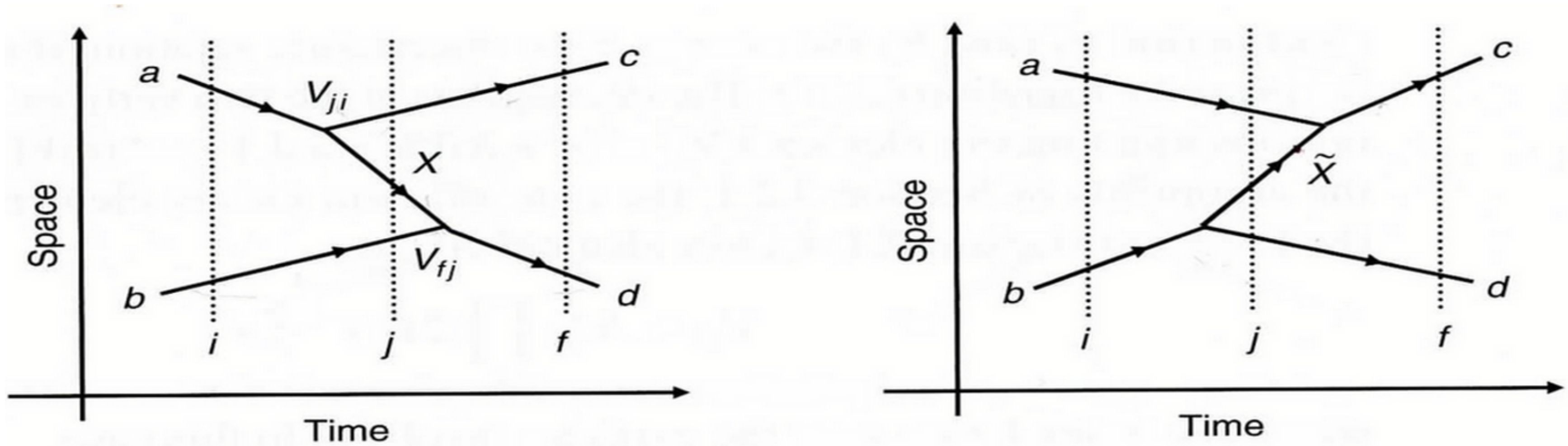
- PARTICLE SCATTERS IN THE POTENTIAL OF ANOTHER PARTICLE → MOMENTUM TRANSFER WITHOUT INTERVENING MEDIATOR → CLASSICAL FIELD
  IN QFT WE QUANTIZE THE FIELD.

FORCES PRODUCED BY POTENTIAL SEEM TO APPEAR INSTANTLY OVER ALL SPACE
→ VIOLATES SPECIAL RELATIVITY.

QFT → INTERACTIONS MEDIATED BY PARTICLE EXCHANGE
→ NO ACTION AT A DISTANCE
→ FORCES RESULT FROM MOMENTUM TRANSFER

FORCE → $F = ma = \frac{dp}{dt}$ ← MOMENTUM CHANGE

TIME ORDERED PERTURBATION THEORY



TWO POSSIBLE TIME ORDERINGS FOR $a+b \rightarrow c+d$

FIRST ONE:

$$T_{fi}^{ab} = \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_f} = \frac{\langle d|V|x+b\rangle\langle c+X|V|a\rangle}{(E_a+E_b)-(E_c+E_X+E_b)}$$

$E_j \neq E_i \longrightarrow \Delta E \Delta t \sim \hbar$

TWO VERTICES $\longrightarrow$ NON INVARIANT MATRIX ELEMENTS

$$V_{ji} = \langle c+X|V|a\rangle \quad , \quad V_{fj} = \langle d|V|X+b\rangle$$

$V_{ji} = \langle c + X | V | a \rangle$, $V_{fj} = \langle d | V | X + b \rangle$ NON INVARIANT

RELATED TO LORENTZ INVARIANT :

$$V_{ji} = M_{ji} \prod_k (2E_k)^{-1/2}$$

INVARIANT ME

RUNS OVER PARTICLES INVOLVED

IN THIS CASE:

$$V_{ji} = \langle c + X | V | a \rangle = \frac{\mathcal{M}_{a \to cX}}{(2E_a 2E_c 2E_X)^{1/2}}$$

REQUIREMENT THAT $\mathcal{M}$ IS LORENTZ INVARIANT PUTS STRONG CONSTRAINTS ON ITS FORM

ASSUME SCALAR → WE WILL SEE VECTOR, AXIAL VECTOR ETC....

$$\mathcal{M}_{a \to cX} = g_a \rightarrow V_{ji} = \langle c + X | V | a \rangle = \frac{g_a}{(2E_a 2E_c 2E_X)^{1/2}}$$

g_a → STRENGTH OF INTERACTION

→ COUPLING CONSTANT

$$V_{fi} = \langle d | V | X + b \rangle = \frac{g_b}{(2E_b 2E_d 2E_X)^{\frac{1}{2}}}$$

SECOND ORDER PERTURBATION TERM:

$$T_{fi}^{ab} = \frac{\langle d|V|x+b\rangle\langle c+x|V|a\rangle}{(E_a+E_b)-(E_c+E_x+E_b)}$$

$$= \frac{1}{2E_x}\frac{1}{(2E_a 2E_b 2E_c 2E_d)^{1/2}} \times \frac{g_a g_b}{(E_a-E_c-E_x)}$$

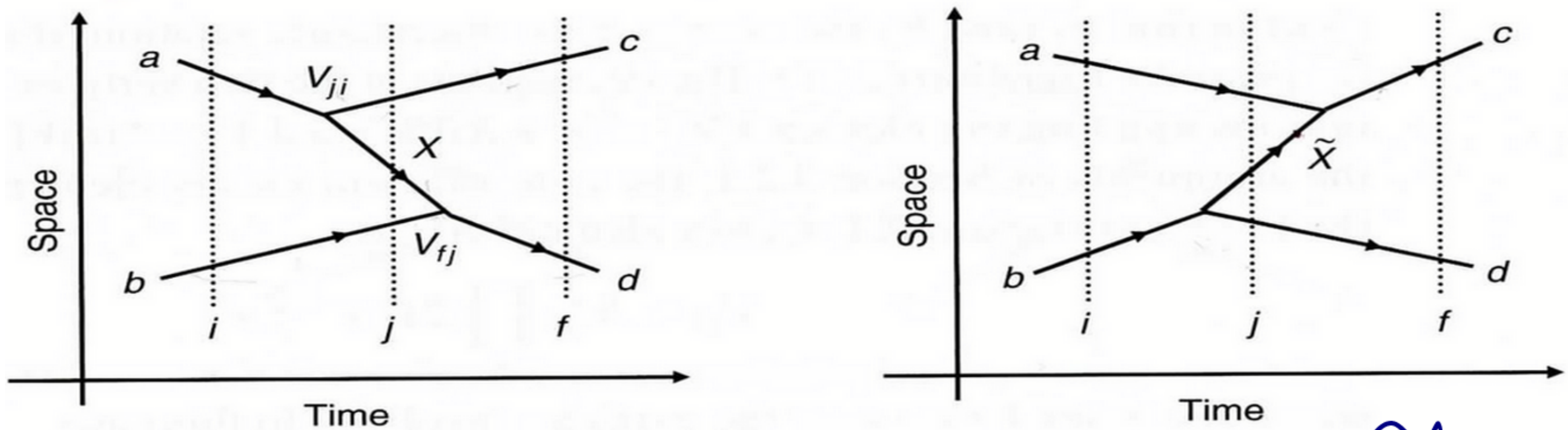
$$\mathcal{M}_{fi}^{ab} = (2E_a 2E_b 2E_c 2E_d)^{1/2}\, T_{fi}^{ab} = \frac{1}{2E_x}\frac{g_a g_b}{(E_a-E_c-E_x)}$$

DEFINED IN TERMS OF WAVE FUNCTIONS HAVING LORENTZ INVARIANT NORMALIZATION

$g \rightarrow$ SCALAR $\rightarrow$ ALSO LORENTZ INVARIANT

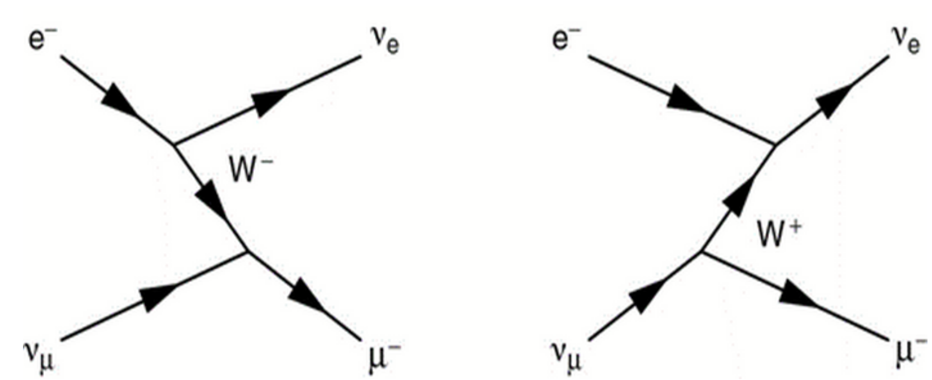
- MOMENTUM CONSERVED AT VERTICES
- $E_j \neq E_i \longrightarrow \Delta E \Delta t \sim \hbar$
- EXCHANGED PARTICLE IS ON MASS SHELL

$$E_x^2 = p_x^2 + m_x^2$$



SECOND POSSIBLE TIME ORDERING $\tilde{X}$ IN THIS TIME ORDERING IS ASSUMED TO HAVE SAME MASS AS X, BUT OPPOSITE CHARGE → CHARGE CONSERVATION

$e^- \nu_\mu \to \nu_e \mu^-$



IN QED EXCHANGED γ IS NEUTRAL, SO THERE IS NO DISTINCTION.

SECOND TIME ORDERING — THE INTERMEDIATE STATE IS THE ONE THE DOTTED LINE GOES THRU

$$b \to X\, d$$

$$X + a \to c$$

So
$$T_{fi}^{ba} = \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j} = \frac{\langle c|V|X+a\rangle\langle X+d|V|b\rangle}{(E_a+E_b)-(E_a+E_X+E_d)}$$

$$= \frac{\langle c|V|X+a\rangle\langle X+d|V|b\rangle}{E_b - E_d - E_X}$$

GOING THROUGH SAME PROCEDURE AS BEFORE

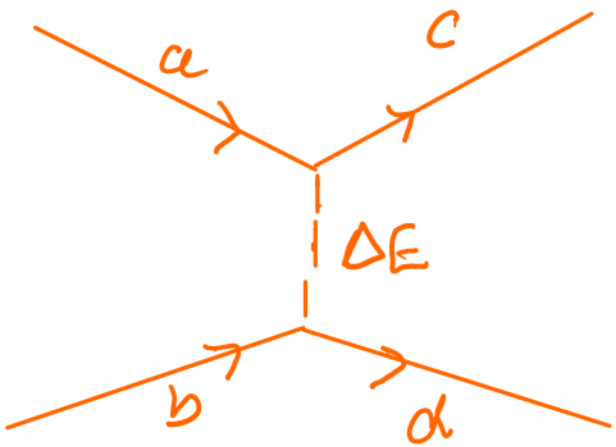
$$\mathcal{M}_{fi}^{ba} = \frac{1}{2E_X}\frac{g_a\, g_b}{(E_b - E_d - E_X)}$$

TOTAL AMPLITUDE $= \sum_i (\text{AMPLITUDES})_i$

DIFFERENT PATHS

$|i\rangle \rightarrow |f\rangle$

$$\mathcal{M}_{fi} = \mathcal{M}^{ab}_{fc} + \mathcal{M}^{ba}_{fi} = \frac{g_a g_b}{2E_x}\left(\frac{1}{E_a - E_c - E_x} + \frac{1}{E_b - E_d - E_x}\right)$$



$$E_a - E_c = \Delta E$$

$$E_b + \Delta E = E_d$$

$$\Delta E = E_d - E_b = E_a - E_c$$

$$E_b - E_d = E_c - E_a$$

$$M_{fi} = \frac{g_a g_b}{2E_x}\left(\frac{1}{E_a - E_c - E_x} - \frac{1}{E_a - E_c + E_x}\right)$$

$$M_{fi} = \frac{g_a g_b}{(E_a - E_c)^2 - E_x^2}$$

FOR BOTH OF THE TIME ORDERED DIAGRAMS THE ENERGY OF THE EXCHANGED PARTICLE IS

$$E_x^2 = \vec{p}_x^{\,2} + m_x^2$$

MOMENTUM CONSERVED AT EACH VERTEX

1ST DIAGRAM $\vec{p}_x = (\vec{p}_a - \vec{p}_c)$

2ND DIAGRAM $\vec{p}_x = (\vec{p}_b - \vec{p}_d) = (\vec{p}_a - \vec{p}_c)$

FOR BOTH DIAGRAMS:

$$E_x^2 = \vec{p}_x^{\,2} + m_x^2 = (\vec{p}_a - \vec{p}_c)^2 + m_x^2$$

$$\mathcal{M}_{fi} = \frac{g_a g_b}{(E_a - E_c)^2 - (\vec{p}_a - \vec{p}_c)^2 - m_x^2} = \frac{g_a g_b}{(p_a - p_c)^2 - m_x^2}$$

4-MOMENTA

$$\mathcal{M}_{fi} = \frac{g_a g_b}{(p_a - p_c)^2 - m_X^2} = \frac{g_a g_b}{q^2 - m_X^2}$$

4-MOMENTUM OF EXCHANGED VIRTUAL PARTICLE

SUM OVER TWO TIME ORDERINGS HAS PRODUCED A MATRIX ELEMENT WHICH DEPENDS ON q^2

$q^2 \rightarrow$ LORENTZ SCALAR

$m_X^2 \rightarrow$ LORENTZ SCALAR

$$\mathcal{M}_{fi} = \frac{g_a g_b}{q^2 - m_X^2}$$

MANIFESTLY COVARIANT

$$\frac{1}{q^2 - m_X^2} \longrightarrow \text{PROPAGATOR}$$

WHAT HAS BEEN DONE HERE IS A "MOTIVATIONAL" OR "HEURISTIC" APPROACH

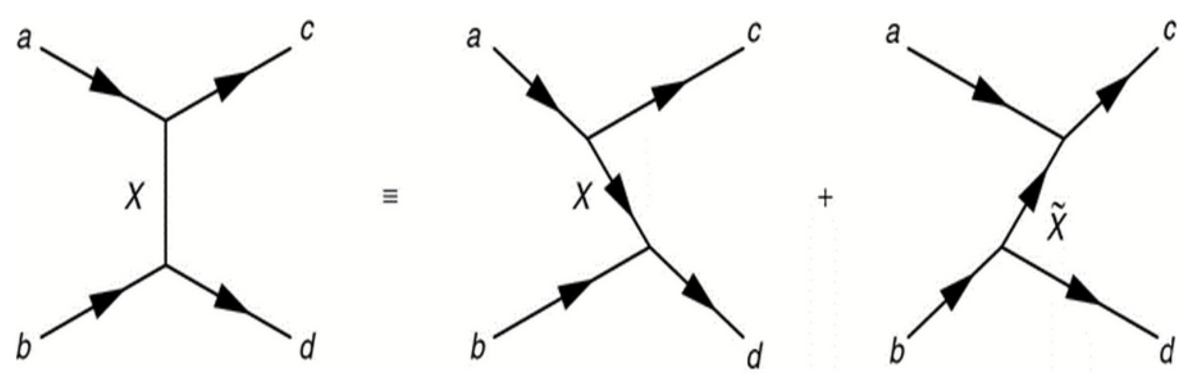
FORMAL PROPAGATOR THEORY

→ USE GREEN'S FUNCTION METHOD FOR SOLVING INHOMOGENEOUS DIFFERENTIAL EQUATIONS

→ USE TO SOLVE DIRAC EQUATION

SEE HALZEN & MARTIN P.146

FEYMAN DIAGRAMS & VIRTUAL PARTICLES



FEYNMAN DIAGRAM = TIME ORDERED PROCESSES $|i\rangle \to |f\rangle$

$q = p_a - p_c = p_a - p_b$ 4-MOMENTUM CONSERVED

NOT TRUE OF INDIVIDUAL TIME ORDERED DIAGRAMS

VIRTUAL PARTICLE ⟶ SUM OVER ALL POSSIBLE EXCHANGES

"MATHEMATICAL CONSTRUCT"

q^2 DETERMINED BY IN + OUT 4-MOMENTA

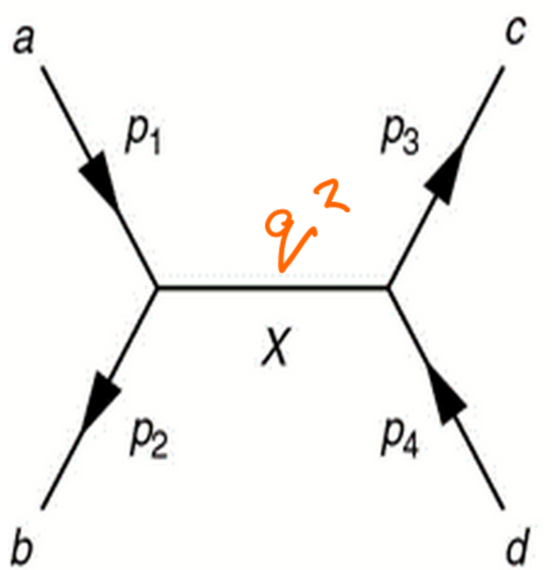
$q^2 \neq m_X^2$ ⟶ OFF MASS SHELL

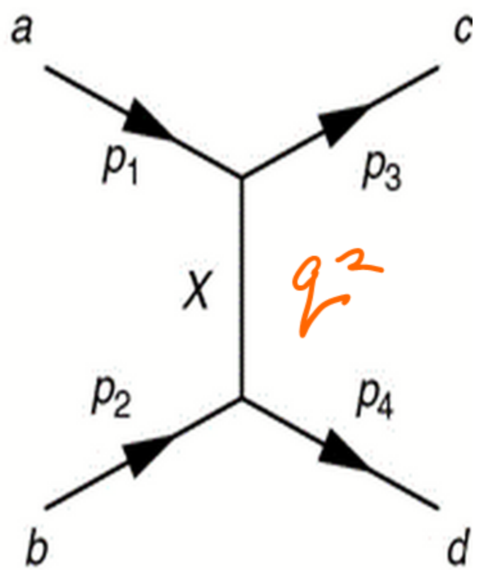
EFFECT OF VIRTUAL PARTICLE OBSERVED VIA FORCES ACTING.

COULD ONLY BE OBSERVED BY INTERACTING AGAIN & FORCING IT ON MASS SHELL

→ DIFFERENT PROCESS / FEYMAN DIAGRAM.

4-MOMENTUM IN ANNIHILATION & SCATTERING





ANNIHILATION S-CHANNEL

SCATTERING t-CHANNEL

$$s = (E_1^* + E_2^*)^2 , \quad q^2 = p_1^2 + p_2^2 + 2 p_1 p_2$$

$$= m_1^2 + m_2^2 + 2 (E_1 \ \bar{p}_1)(E_2 \ \bar{p}_2)$$

$$= m_1^2 + m_2^2 + 2E_1E_2 - 2p^2 \cos\theta$$

θ

$\cos\theta = -1$

TIMELIKE $\rightarrow$ $q^2 > 0$

t - CHANNEL → SCATTERING

$$q^2 = (p_1 - p_3)^2 = m_1^2 + m_3^2 - 2p_1 \cdot p_3$$

$$= m_1^2 + m_3^2 - 2(E_1\ \vec{p}_1)(E_3, \vec{p}_3)$$

$$= m_1^2 + m_3^2 - 2E_1 E_3 + |\vec{p}_1||\vec{p}_3| \cos\theta$$

EASY TO SEE IF $E \gg m$

$$\approx -2E_1 E_3 + 2E_1 E_3 \cos\theta$$

$$\approx -2E_1 E_3 (1 - \cos\theta)$$

$\cos\theta_{MAX} = 1$

$\therefore q^2 < 0 \longrightarrow$ SPACE LIKE