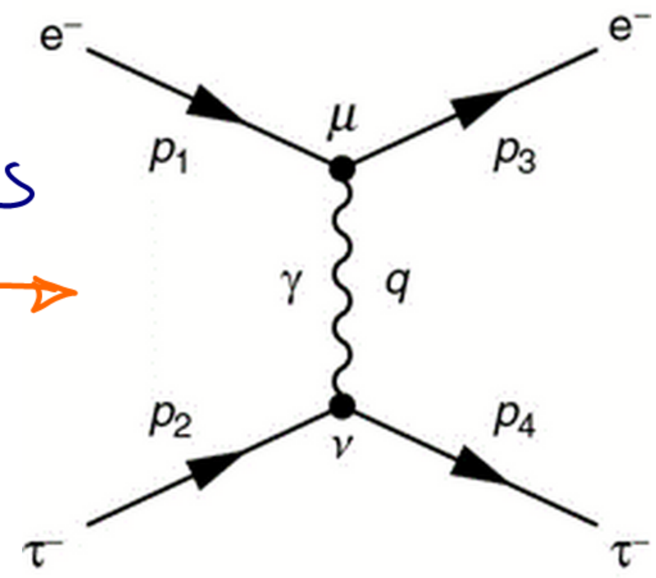
ELEMENTS OF QUANTUM ELECTRODYNAMICS

WE ARE GETTING CLOSE TO BEING ABLE TO CALCULATE REAL INTERESTING PROCESSES

$e^- \tau^- \rightarrow e^- \tau^-$ $\longrightarrow$



QED IS THE PROTOTYPE QUANTUM FIELD THEORY THAT ACTED AS THE TEMPLATE FOR

- ELECTROWEAK
- QUANTUM CHROMODYNAMICS

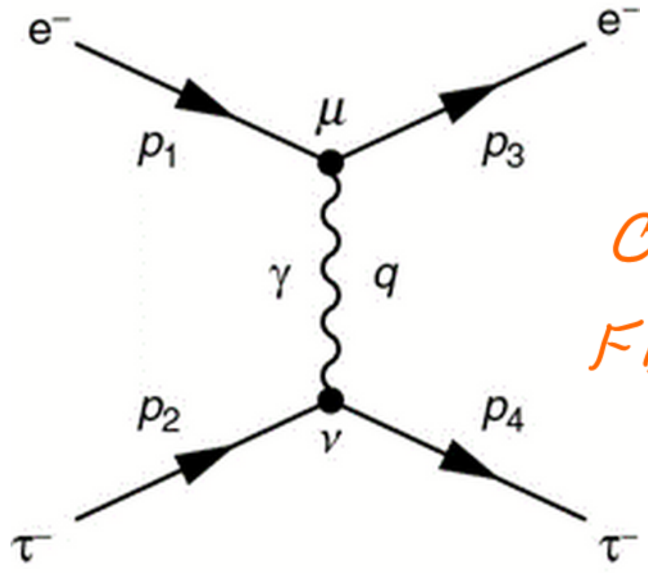
WE SHALL SEE THAT ALL THREE ARE FINITE RENORMALIZABLE GAUGE THEORIES

THIS SECTION QUOTES MANY RESULTS FROM APPENDIX D

- CLASSICAL ELECTROMAGNETISM
- GAUGE INVARIANCE → I WILL COVER LATER
- γ POLARIZATION STATES → QED
- POLARIZATION STATES OF MASSIVE SPIN-1 PARTICLES → $W^{\pm}$ Z^{0}
- POLARIZATION SUMS FOR MASSIVE GAUGE BOSONS
- POLARIZATION SUMS FOR EXTERNAL γ

10 PAGES LONG

- γ PROPAGATOR



$e^- \tau^- \to e^- \tau^-$

CHARGED ELEMENTARY FERMION, ELASTIC SCATTERING

NOTE CONSERVATION OF LEPTON FLAVOR

$$\mathcal{M} = \langle \psi_c | V | \psi_a \rangle \frac{1}{q^2 - m_x^2} \langle \psi_d | V | \psi_b \rangle$$

STRENGTH OF INTERACTION AT VERTEX

PROPAGATOR FOR VIRTUAL EXCHANGE

OTHER VERTEX

WE STARTED OFF LOOKING AT A SCALAR INTERACTION

SCALAR $\to$ SPINLESS EXCHANGE

γ^μ – SPIN 1 $\to$ VECTOR EXCHANGE

HAVE TO SUM OVER ALL POLARIZATIONS OF VIRTUAL γ^μ

FREE PHOTON FIELD

$$A_\mu = \varepsilon_\mu^{(\lambda)} e^{i(\bar{p}\cdot\bar{x} - Et)}$$

PLANE WAVE

POLARIZATION

FREE γ's ALWAYS TRANSVERSE → NOT TRUE OF VIRTUAL γ

FREE γ PROPAGATING IN Z-DIRECTION

2 ORTHOGONAL POLARIZATIONS

$$\varepsilon^{(1)} = (0,1,0,0) \quad ; \quad \varepsilon^{(2)} = (0,0,1,0)$$

INTERACTION CHARGED FERMION AND A_μ

$$\partial_\mu \to \partial_\mu + iq A_\mu$$

DID THIS IN E&M?

$(\partial/\partial t, +\bar{\nabla})$

CHARGE

$(\phi, -\bar{A})$

VECTOR

SCALAR POTENTIAL

DIRAC BECOMES

$$\gamma^\mu \partial_\mu \psi + \underbrace{iq\gamma^\mu A_\mu \psi}_{\text{INTERACTION}} + im\psi = 0$$

MULTIPLY THIS BY $i\gamma^0$ ← DIRAC γ MATRIX

$$i\gamma^0\gamma^\mu\partial_\mu\psi - q\gamma^0\gamma^\mu A_\mu\psi - \gamma^0 m\psi = 0$$

WRITE OUT COMPONENTS OF DIFFERENTIAL

$$i\underbrace{\gamma^0\gamma^0}_{I}\frac{\partial\psi}{\partial t} + i\gamma^0\bar{\gamma}\cdot\bar{\nabla}\psi - q\gamma^0\gamma^\mu A_\mu - m\gamma^0 = 0$$

NUMBER

$$\bar{\gamma}\cdot\bar{\nabla} = \gamma^1\frac{\partial}{\partial x} + \gamma^2\frac{\partial}{\partial x} + \gamma^3\frac{\partial}{\partial x}$$

$$i\frac{\partial\psi}{\partial t} = \underbrace{\left(-i\gamma^0\bar{\gamma}\cdot\bar{\nabla} + q\gamma^0\gamma^\mu A_\mu + m\gamma^0\right)}\psi$$

$$i\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

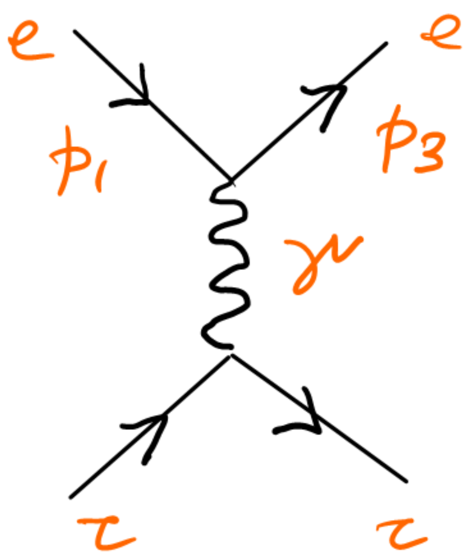
$$\hat{H}_{INT} = -i\gamma^0\bar{\gamma}\cdot\bar{\nabla} + m\gamma^0 + q\gamma^0\gamma^\mu A_\mu$$

INTERACTION HAMILTONIAN

$$\hat{H}_{INT} = \underbrace{-i\gamma^0\bar{\gamma}\cdot\bar{\nabla} + m\gamma^0}_{\text{FREE PARTICLE MASS + KINETIC ENERGY}} + \underbrace{q\gamma^0\gamma^\mu A_\mu}_{\text{POTENTIAL ENERGY}}$$

POTENTIAL ENERGY OPERATOR $\hat{V}_D = q\gamma^0\gamma^\mu A_\mu$

TIME LIKE PART $q\gamma^0\gamma^0 A_0 = q\phi$ → ENERGY OF CHARGE IN SCALAR POTENTIAL



→ $\langle\psi(p_3)|\hat{V}_D|\psi(p_1)\rangle$

$$= u_e^\dagger(p_3)\, Qe\, \gamma^0\gamma^\mu\, \varepsilon_\mu^{(\lambda)}\, u_e(p_1)$$

SPINOR

CHARGE ON ELECTRONS

VIRTUAL γ POLARIZATION

BOTTOM VERTEX $u_\tau^{+}(p_4)\, Q_\tau e \gamma^0 \gamma^\mu \varepsilon_\lambda^{(\lambda)*}\, u_\tau(p_2)$

?

$$\mathcal{M}^2 = \sum_\lambda |\mathcal{M}|^2 = \sum j_\mu j^{\nu *} \varepsilon_\lambda^{\mu} \varepsilon_\lambda^{*\nu} = JJ^* \rightarrow$$ P. 532

NOW WE ARE GETTING SOMEWHERE → MATRIX ELEMENT

$$\mathcal{M} = \sum_\lambda \left[u_e^{+}(p_3)\, Q_e e \gamma^0 \gamma^\mu u_e(p_1) \right] \varepsilon_\mu^{(\lambda)} \frac{1}{q^2} \varepsilon_\nu^{(\lambda)*}$$
$$\times \left[u^{+}_\tau(p_4)\, Q_\tau e \gamma^0 \gamma^\mu u_\tau(p_2) \right]$$

AGAIN FROM APPENDIX D $\sum_\lambda \varepsilon_\mu^{(\lambda)} \varepsilon_\nu^{(\lambda)} = -g_{\mu\nu}$

$g_{\mu\nu}$ MATCHES HELICITIES

$$\mathcal{M} = \underbrace{\left[Q_e u_e^{+}(p_3) \gamma^0 \gamma^\mu u_e(p_1) \right]}_{\text{ELECTRON CURRENT}} \underbrace{\frac{-g_{\mu\nu}}{q^2}}_{\text{VIRTUAL } \gamma \text{ PROPAGATOR}} \underbrace{\left[Q_\tau e u_\tau^{+}(p_4) \gamma^0 \gamma^\nu u_\tau(p_2) \right]}_{\tau \text{ LEPTON CURRENT}}$$

CAN SIMPLIFY A BIT USING $\bar{\psi} = \psi^\dagger \gamma^0$

$$\mathcal{M} = -\left[Q_e \bar{u}_e(p_3)\gamma^\mu u_e(p_1)\right] \frac{g_{\mu\nu}}{q^2}\left[Q_\tau e\, \bar{u}_\tau(p_4)\gamma^\nu u_\tau(p_2)\right]$$

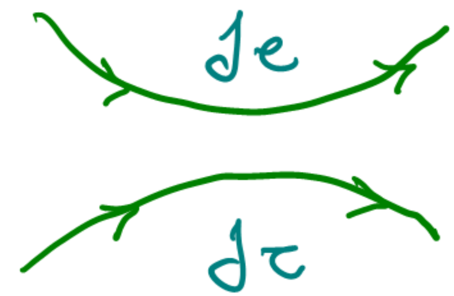
APPENDIX 3 $\rightarrow$ $j^\mu = \bar{u}(p)\gamma^\mu u(p)$ $\rightarrow$ 4 VECTOR CURRENT

$$j_e^\mu = \bar{u}_e(p_3)\gamma^\mu u_e(p_1)$$

$$j_\tau^\nu = \bar{u}_\tau(p_4)\gamma^\nu u_\tau(p_2)$$

MANIFESTLY COVARIANT

$$\mathcal{M} = -Q_e Q_\tau e^2 \, \frac{j_e \cdot j_\tau}{q^2}$$



$$= -Q_e Q_\tau e^2 \, \frac{j_e^\nu \, j_{\tau\nu}}{q^2}$$

$g_{\mu\nu}$ DROPS THIS INDEX

ELECTROMAGNETIC CURRENT X CURRENT INTERACTION

FEYNMAN RULES FOR QED

$$\mathcal{M} = -\left[Q_e \bar{u}_e(p_3)\gamma^\mu u_e(p_1)\right] \frac{g_{\mu\nu}}{q^2} \left[Q_\tau e \bar{u}_\tau(p_4)\gamma^\nu u_\tau(p_2)\right]$$

THIS CAN BE OBTAINED MORE FORMALLY IN QFT

QFT ALLOWS TREAMENT OF CREATION/ANNIHILATION

OUR DERIVATION LIMITED, NOT WRONG

SALIENT FEATURES:

$\sum$ TIME ORDER + $\sum$ POLARIZATIONS $\rightarrow$ PROPAGATOR $\dfrac{g_{\mu\nu}}{q^2}$

FERMION - PHOTON VERTEX $Q_e \bar{u}\gamma^\mu u$

FEYMAN REALIZED THAT YOU DON'T NEED TO CALCULATE EVERY NEW PROCESS FROM FIRST PRINCIPALS

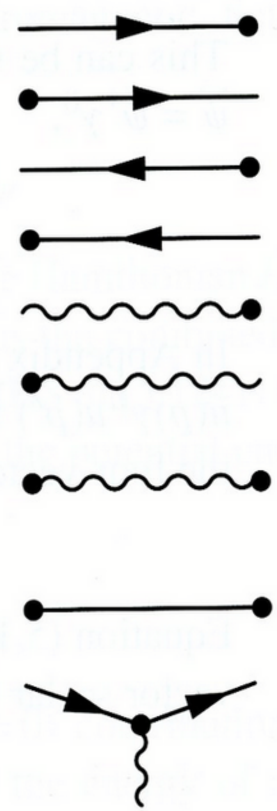
DRAW FEYMAN DIAGRAM $\longrightarrow$ APPLY RULES

(1) PROPAGATOR FOR VIRTUAL PHOTON

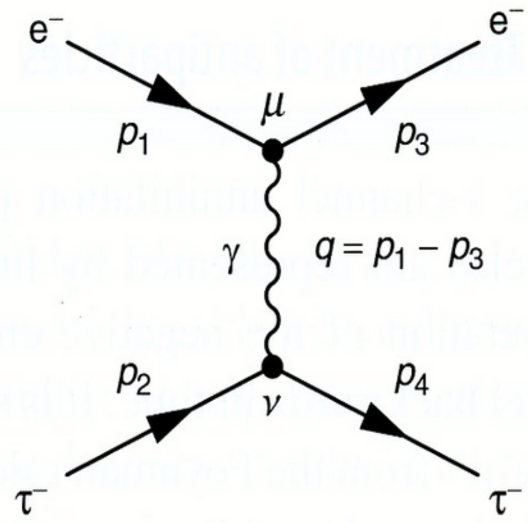
(2) SPINORS FOR FERMIONS IN AND OUT

(3) VERTEX FACTOR FOR EACH VERTEX.

initial-state particle:	$u(p)$
final-state particle:	$\bar{u}(p)$
initial-state antiparticle:	$\bar{v}(p)$
final-state antiparticle:	$v(p)$
initial-state photon:	$\varepsilon_\mu(p)$
final-state photon:	$\varepsilon^*_\mu(p)$
photon propagator:	$-\frac{ig_{\mu\nu}}{q^2}$
fermion propagator:	$+\frac{i(\gamma^\mu q_\mu + m)}{q^2 - m^2}$
QED vertex:	$-iQe\gamma^\mu$



QED ONLY HAS 3-PARTICLE VERTICES



$\overline{u}(p_3)[ie\gamma^\mu]u(p_1)$

$$\frac{-ig_{\mu\nu}}{q^2}$$

$\overline{u}(p_4)[ie\gamma^\nu]u(p_2)$

VERTEX μ $\quad \bar{u}(p_3) \cdot ie\gamma^\mu \cdot u(p_1)$

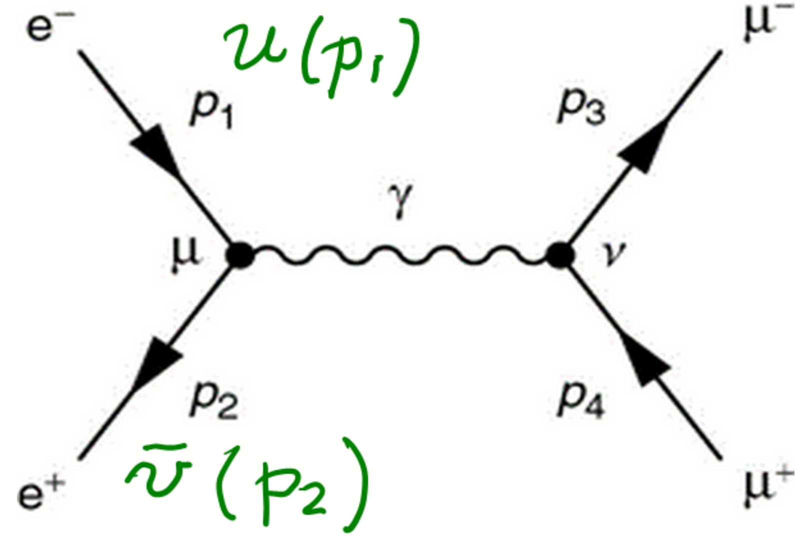
VERTEX ν $\quad \bar{u}(p_4) \cdot ie\gamma^\nu \cdot u(p_2)$

PROPAGATOR $\quad \dfrac{-ig_{\mu\nu}}{q^2}$

$$-i\mathcal{M} = \left[\bar{u}(p_3)\, ie\gamma^\mu\, u(p_1)\right] \frac{-ig_{\mu\nu}}{q^2} \left[\bar{u}(p_4)\, ie\gamma^\nu\, u(p_2)\right]$$

"i"s PUT IN TO MAKE HIGHER ORDERS CORRECT.

ANNIHILATION PROCESS INVOLVES ANTIPARTICLES



$$-i\mathcal{M} = \left[\bar{v}(p_2)\, ie\gamma^{\mu}\, u(p_1)\right] \frac{-ig_{\mu\nu}}{q^2} \left[\bar{u}(p_3)\, ie\gamma^{\nu}\, v(p_4)\right]$$

INCOMING ANTIPARTICLE

OUTGOING ANTIPARTICLE

FIRST PARTICLE IN FERMION CURRENT AGAINST ARROW $\longrightarrow$ ADJOINT SPINOR