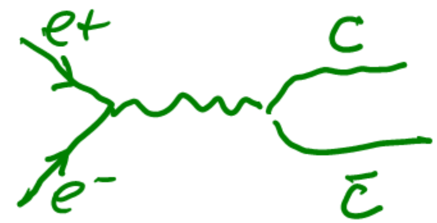
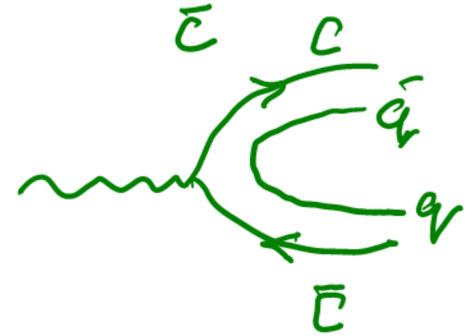
# ELECTRON POSITRON ANNIHILATION

WE ARE NOW IN THE POSITION TO CALCULATE WHAT HAS BEEN ONE OF THE MOST IMPORTANT DISCOVERY PROCESSES $e^+e^- \rightarrow f\bar{f}$ $e^+e^-$ COLLIDER

SPEAR $\sqrt{s} > 3$ GeV

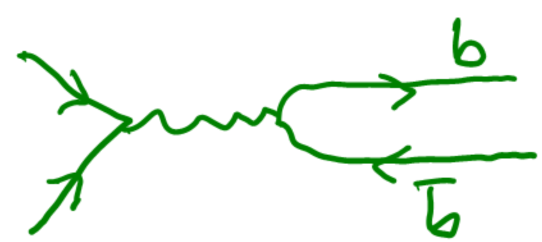



$J/\psi$

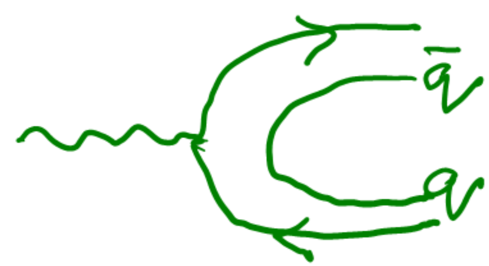



CHARM MESONS
D

DORIS $\sqrt{s} > 10$ GeV




$\Upsilon$




B-MESONS

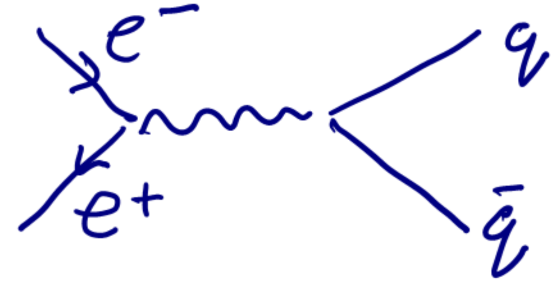
PETRA $\sqrt{s} \sim 30\,\mathrm{GeV}$

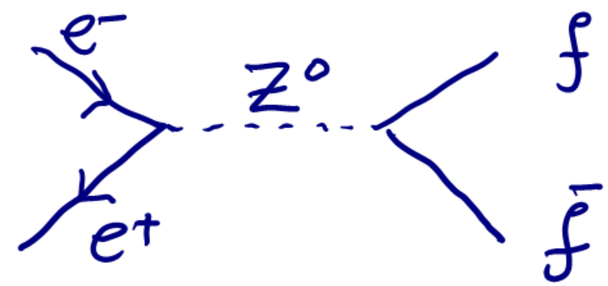

STRONG
COUPLING
CONSTANT

$$\alpha_s(q^2) = \frac{1}{B \ln\left(\frac{q^2}{\Lambda^2}\right)}$$

QCD AS
NON-ABELIAN
GAUGE THEORY

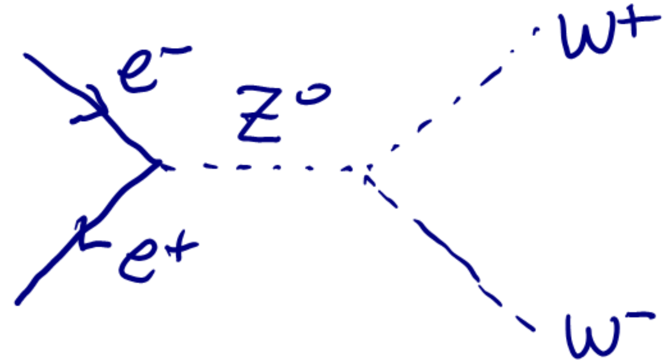
LEP $\sqrt{s} > 200\,\text{GeV}$



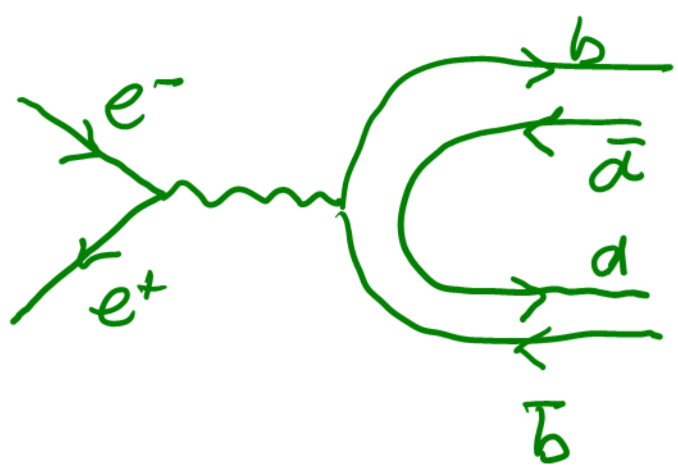




$3\nu$

ELECTROWEAK COUPLINGS →




B-FACTORIES



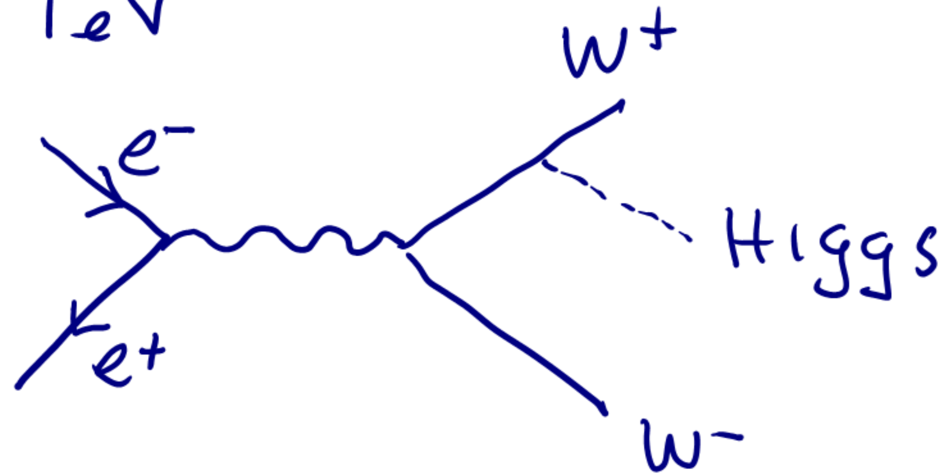

$B^0$

$\bar{B}^0$

CP VIOLATION

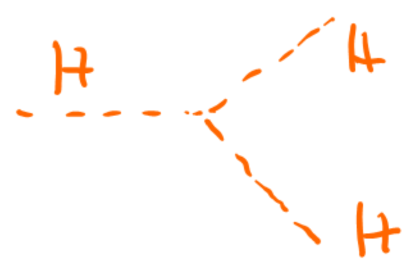
# INTERNATIONAL LINEAR COLLIDER

$\sqrt{s} > 1$ TeV




DETAILED STUDY OF Higgs

eg

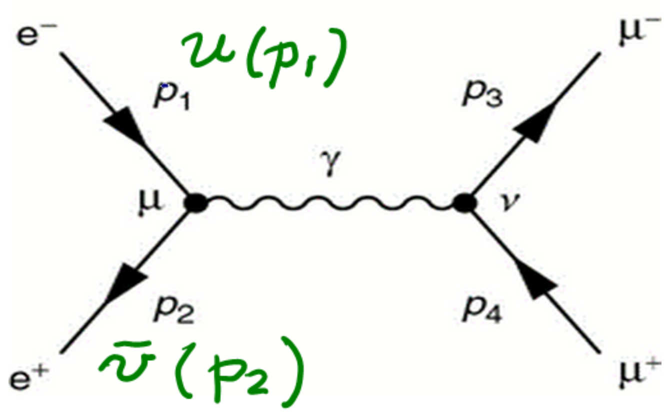



SELF COUPLING

HIGGS POTENTIAL

CENTRAL TO STANDARD MODEL

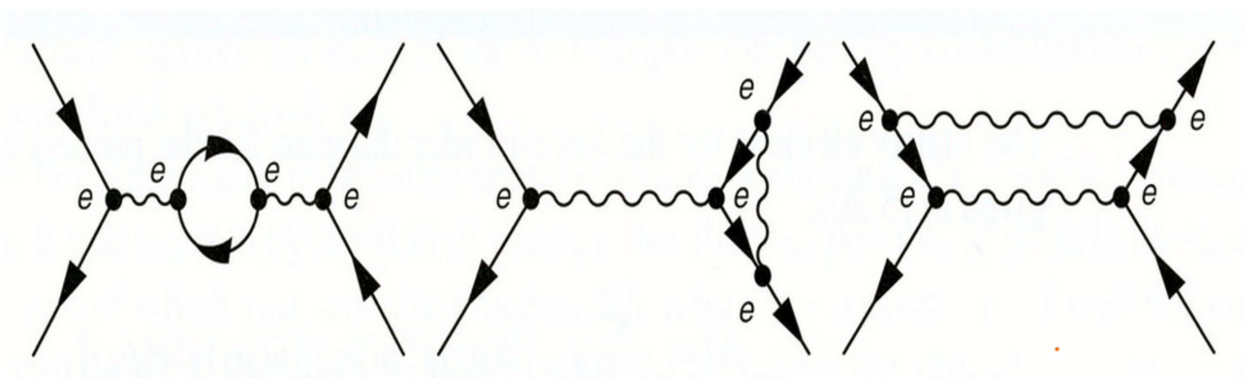
IN QED, DOMINANT CONTRIBUTION USUALLY FEYMAN DIAGRAM WITH FEWEST VERTICES




LEADING ORDER → 2 VERTICES

EACH VERTEX $ie\gamma^\mu$

$$|\mathcal{M}|^2 \sim e^4 \sim \alpha^2 \leftarrow \left(\frac{1}{137}\right)^2$$




FOR EACH DIAGRAM

4 VERTICES $ie\gamma^\mu$

$$|\mathcal{M}|^2 \sim e^8 \sim \alpha^4$$

$$\alpha^2 \sim 5 \times 10^{-5}$$

$$\alpha^4 \sim 3 \times 10^{-9}$$

$$\mathcal{M}_{fi}^{TOT} = \sum_{\substack{\text{ALL} \\ \text{ORDERS}}} \mathcal{M}_{fi}$$

WRITE $M_{fi} = \alpha M_{LO} + \alpha^2 \sum_j M_{1j}$

2 VERTICES
1 DIAGRAM

4 VERTICES

SUM OVER DIAGRAMS

PHYSICAL OBSERVABLES $\longrightarrow |M|^2$

$$|M_{fi}|^2 = \left(\alpha M_{LO} + \alpha^2 \sum_j M_{1,j} + \cdots\right)\left(\alpha M_{LO}^* + \alpha^2 \sum_j M_{1j}^* + \cdots\right)$$

$$= \alpha^2 |M_{LO}|^2 + \alpha_3 \sum_j \left( M_{LO} M_{1j}^* + M_{LO}^* M_{1j} \right)$$

$$+ \alpha_4 \sum_{jk} M_{1j} M_{1k}^* + \cdots\cdots$$

RAPIDLY CONVERGING SERIES $\rightarrow$ LO OK TO 1%

AMPLITUDES ARE COMPLEX $\rightarrow$ INTERFERENCE

# ELECTRON - POSITRON ANNIHILATION

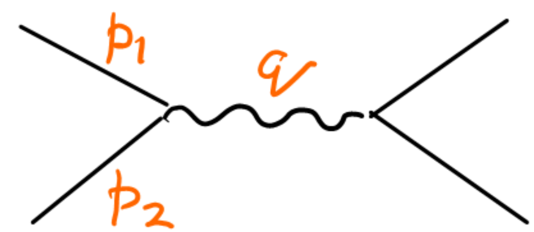
$$e^+e^- \rightarrow \mu^+\mu^-$$

$$-i\mathcal{M} = \left[\bar{v}(p_2)\, ie\gamma^\mu\, u(p_1)\right] \frac{-i g_{\mu\nu}}{q^2} \left[\bar{u}(p_3)\, ie\gamma^\nu\, v(p_4)\right]$$

ELECTRON

MUON

$$= -\frac{e^2}{q^2}\, g_{\mu\nu}\, J_e^\mu J_\mu^\nu$$




$$q = p_1 + p_2$$

$$q^2 = (p_1 + p_2)^2 = s$$

IN MOST $e^+e^-$ COLLIDERS BEAMS HAVE SAME ENERGY

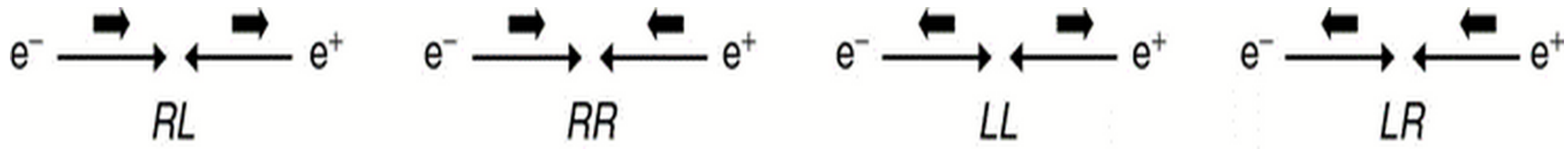
NOT IN B-FACTORIES

$$\sqrt{s} = 2E_{beam}$$

$$i\mathcal{M} = -\frac{e^2}{s} J_e \cdot J_\mu$$

# SPIN SUMS

4 POSSIBLE INITIAL STATES




MUST EVALUATE $\mathcal{M}$ TAKING ACCOUNT OF ALL POSSIBLE SPIN STATES OF $e^+, e^-, \mu^+, \mu^-$.

EACH OF THESE PARTICLES HAS 2 POSSIBLE HELICITY STATES

↳ 4 POSSIBLE INITIAL STATES

4 POSSIBLE FINAL STATES

16 ORTHOGONAL COMBINATIONS

EACH COMBINATION → DISTINCT PHYSICAL PROCESS

→ THEY DO NOT INTERFER

→ HAVE TO EVALUATE EACH OF THESE 16 PROCESSES.

FOR A GIVEN INITIAL STATE RATE = $\sum$ FINAL STATES

FOR EXAMPLE START IN RR


$$\sum |M_{RR}|^2 = |M_{RR\to RR}|^2 + |M_{RR\to RL}|^2 + |M_{RL\to LR}|^2 + |M_{RR\to LL}|^2$$

USUALLY INITIAL STATE BEAMS UNPOLARIZED

SO HELICITY STATE FOR ANY COLLISION IS EQUALLY LIKELY TO BE IN ANY OF THE 4 POSSIBLE STATES.

↳ AVERAGE M OVER THE 4 HELICITY STATES.

$$\langle |M_{fi}|^2 \rangle = \frac{1}{4}\left( |M_{RR}|^2 + |M_{RL}|^2 + |M_{LR}|^2 + |M_{LL}|^2 \right)$$

$$= \frac{1}{4}\Big( |M_{RR\to RR}|^2 + |M_{RR\to RL}|^2 +$$

$$\ldots + |M_{RL\to RR}|^2 + \ldots \Big)$$

$$\langle |M_{fi}|^2 \rangle = \frac{1}{4} \sum_{\text{SPINS}} |M|^2$$

HAVE TO CALCULATE MATRIX ELEMENT FOR 16 PROCESSES

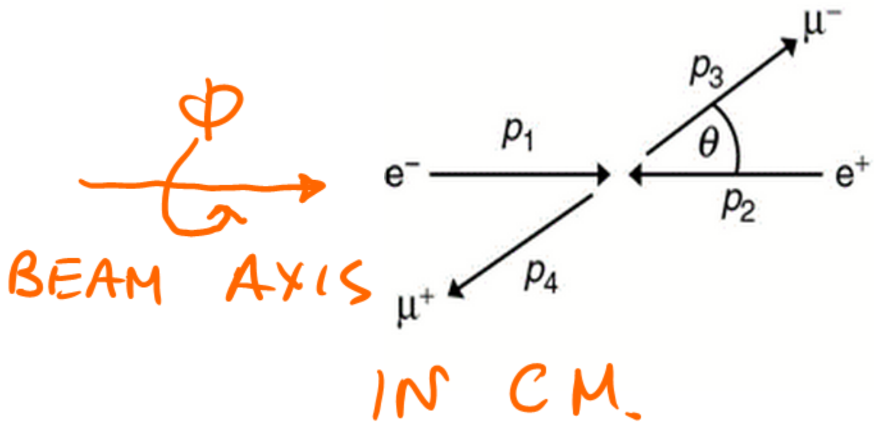
USUALLY PEOPLE USE "TRACE TECHNIQUES" – SEE TEXT

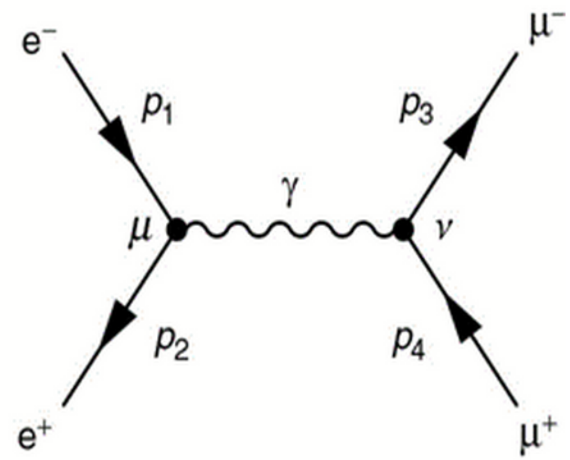
HERE WE WILL CALCULATE EACH OF THE 16 AMPLITUDES DIRECTLY → MAKES PHYSICS TRANSPARENT.

↓

UNDERSTAND HELICITY STRUCTURE OF QED

# HELICITY AMPLITUDES




NEGLECT MASSES $\sqrt{s} \gg m$

$|\vec{p}| \approx E$

$$p_1 = (E, 0, 0, E)$$

$$p_2 = (E, 0, 0, -E)$$

$$p_3 = (E, E\sin\theta, 0, E\cos\theta)$$

$$p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$$

USUALLY PHYSICS IS UNIFORM AROUND BEAM AXIS

$$\phi_{\mu^-} = 0$$

$$\phi_{\mu^+} = \pi$$

SPINORS IN DIFFERENT $\mu$S !

$$J_e^\mu = \bar{v}(p_2)\gamma^\mu u(p_1)$$

$$J_\mu^\nu = \bar{u}(p_3)\gamma^\nu v(p_4)$$

ARE IN $E \gg m$ LIMIT

$$u_\uparrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix} \quad u_\downarrow = \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix} \quad v_\uparrow = \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix} \quad v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

$$s = \sin\frac{\theta}{2}, \quad c = \cos\frac{\theta}{2}$$

INITIAL ELECTRON $\theta = 0, \phi = 0$ INITIAL POSITRON $\theta = \pi, \phi = \pi$

INITIAL HELICITY STATES:

$$u_\uparrow(p_1) = \sqrt{E}\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}, v_\uparrow(p_1) = \sqrt{E}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, v_\downarrow(p_1) = \sqrt{E}\begin{pmatrix} 0 \\ -1 \\ 0 \\ -1 \end{pmatrix}$$

FOR FINAL STATE $\mu^-(\theta, 0)$ $\mu^+(\pi-\theta, \pi)$

$$\sin\left(\frac{\pi-\theta}{2}\right) = \cos\frac{\theta}{2}, \quad \cos\left(\frac{\pi-\theta}{2}\right) = \sin\frac{\theta}{2} \qquad e^{i\pi} = -1$$

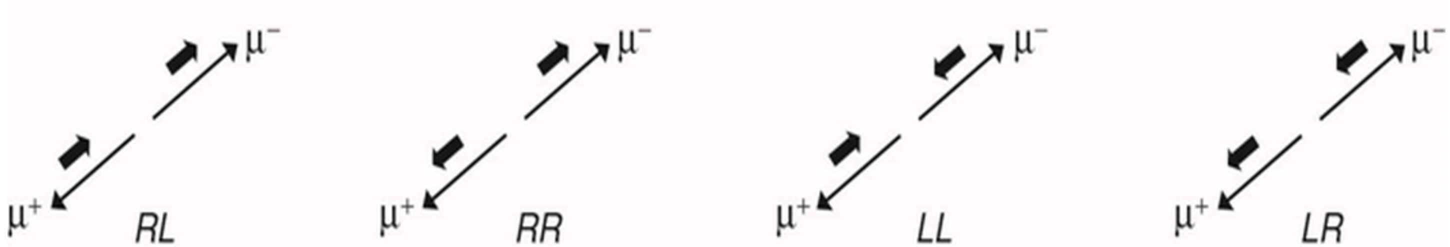
SO THE SPINORS FOR THE TWO POSSIBLE $\mu^+\mu^-$ HELICITY STATES ARE:

$$u_\uparrow(p_3) = \sqrt{E}\begin{pmatrix} c \\ s \\ c \\ s \end{pmatrix} \quad u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix} -s \\ c \\ s \\ -c \end{pmatrix} \quad v_\uparrow(p_4) = \sqrt{E}\begin{pmatrix} c \\ s \\ -c \\ -s \end{pmatrix} \quad v_\downarrow(p_4) = \sqrt{E}\begin{pmatrix} s \\ -c \\ s \\ -c \end{pmatrix}$$

$$j_e^\mu = \bar{v}(p_2)\gamma^\mu u(p_1)$$
$$j_\mu^\nu = \bar{u}(p_3)\gamma^\nu v(p_4)$$

$$\mathcal{M} = -\frac{e^2}{s} j_e \cdot j_\mu$$

JUST HAVE TO FIGURE OUT CORRECT LR COMBINATIONS




$$\mathcal{M} = -\frac{e^2}{s}\, j_e \cdot j_\mu$$

THIS HAS TO BE EVALUATED FOR ALL THESE CONFIGURATIONS

USE DIRAC-PAULI TO SHOW THAT FOR ANY TWO SPINORS $\psi, \phi$

$$\bar{\psi}\gamma^\mu\phi = \psi^\dagger\gamma^0\gamma^\mu\phi$$

DO IT !

$$\bar{\psi}\gamma^0\phi = \psi^\dagger\gamma^0\gamma^0\phi = \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4$$

$$\bar{\psi}\gamma^1\phi = \psi^\dagger\gamma^0\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1$$

$$\bar{\psi}\gamma^2\phi = \psi^\dagger\gamma^0\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1)$$

$$\bar{\psi}\gamma^3\phi = \psi^\dagger\gamma^0\gamma^3\phi = \psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2$$

$j_\mu$ CAN BE DETERMINED FOR ANY HELICITY COMBINATION

FOR EXAMPLE RL $\quad \mu^-$ is R, $\mu^+$ is L

$$u_\uparrow(p_3) \qquad v_\downarrow(p_4)$$

COMPONENTS OF $j_\mu$ ARE

$$j_\mu^0 = \bar{u}_\uparrow(p_3)\gamma^0 v_\downarrow(p_4)$$

$$= u^*_{\uparrow 1}(p_3)v_{\downarrow 1}(p_4) + u^*_{\uparrow 2}(p_3)v_{\downarrow 2}(p_4) + u^*_{\uparrow 3}(p_3)v_{\downarrow 3}(p_4) + u^*_{\uparrow 4}(p_3)v_{\downarrow 4}(p_4)$$

$$= E\,(cs - cs + cs - cs) = 0$$

$$j_\mu^1 = E(-c^2 + s^2 - c^2 + s^2) = 2E(s^2 - c^2) \quad \leftarrow \gamma^1$$

$c^2 + s^2 = 1$

$$j_\mu^2 = -i(-c^2 - s^2 - c^2 - s^2)E = +iE(c^2 + s^2 + c^2 + s^2) = 2iE \quad \leftarrow \gamma^2$$

$$j_\mu^3 = E(cs + cs + cs + cs) = 4E\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 2E\sin\theta \quad \leftarrow \gamma^3$$

So $j_{\mu,RL} = 2E(0, -\cos\theta, i, \sin\theta)$

$\rightarrow$ CAN DO THE OTHER RL COMBINATIONS

$$J_{\mu RR} = \bar{u}_{\uparrow}(p_3)\gamma^{\nu}v_{\uparrow}(p_4) = (0,0,0,0)$$

$$J_{\mu LL} = \bar{u}_{\downarrow}(p_3)\gamma^{\nu}v_{\downarrow}(p_4) = (0,0,0,0)$$

$$J_{\mu LR} = \bar{u}_{\downarrow}(p_3)\gamma^{\nu}v_{\uparrow}(p_4) = 2E(0,-\cos\theta,-i,\sin\theta)$$

IN THE LIMIT OF $E \gg m$ ONLY TWO OF THE FOUR $\mu^+\mu^-$ HELICITY COMBINATIONS ARE NON-ZERO

QED IS A CHIRAL THEORY

$\hookrightarrow$ GREEK FOR "HAND"

WE COULD WORK THROUGH THE FOUR POSSIBLE HELICITY STATES FOR THE INCOMING $e^+e^-$ IN THE SAME WAY, BUT.......

LOOK AT

$$\mathcal{M} = -\frac{e^2}{q^2} g_{\mu\nu} \underbrace{\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right]}_{j_e^\mu} \underbrace{\left[\bar{u}(p_3)\gamma^\nu v(p_4)\right]}_{j_\mu^\nu}$$

← JUST DID THIS

THE ELECTRON AND MUON CURRENTS ONLY DIFFER IN THE ORDER OF THE PARTICLE AND ANTIPARTICLE SPINORS

SO WE CAN GO $\mu \to e$ BY TAKING HERMITIAN CONJUGATE

$$\underbrace{\left[\bar{u}(p_3)\gamma^\mu v(p_4)\right]}_{j_\mu}{}^\dagger = \left[u(p_3)^\dagger \gamma^0 \gamma^\mu v(p_4)\right]^\dagger$$

$$= v(p_4)^\dagger \gamma^{\mu\dagger} \gamma^{0\dagger} u(p_3) \qquad (AB)^\dagger = B^\dagger A^\dagger$$

$$= v(p_4)^\dagger \gamma^{\mu\dagger} \gamma^0 u(p_3) \qquad \gamma^{0\dagger} = \gamma^0$$

$$= v(p_4)^\dagger \gamma^0 \gamma^\mu u(p_3) \qquad \gamma^{\mu\dagger}\gamma^0 = \gamma^0\gamma^\mu$$

$$= \bar{v}(p_4)\gamma^\mu u(p_3)$$

CAN USE THIS TO GET $j_e$

$$\left[\bar{u}(p_3)\gamma^\mu v(p_4)\right]^\dagger = v(p_4)\gamma^\mu u(p_3)$$

- SO → HERMITIAN CONJUGATE → SWAP ORDER OF SPINORS
  EACH ELEMENT OF 4-VECTOR CURRENT → COMPLEX NUMBER

ELEMENTS OF $\bar{v}\gamma^\mu u$ ARE JUST THE COMPLEX CONJUGATES OF ELEMENTS OF $\bar{u}\gamma^\mu v$

THIS IS JUST $A^\dagger = (\tilde{A})^*$

$$\bar{v}_\downarrow(p_4)\gamma^\mu u_\uparrow(p_3) = \left[\bar{u}_\uparrow(p_3)\gamma^\mu v_\downarrow(p_4)\right]^* = 2E(0, -\cos\theta, -i, \sin\theta)$$

$$\bar{v}_\uparrow(p_4)\gamma^\mu u_\downarrow(p_3) = \left[\bar{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)\right]^* = 2E(0, -\cos\theta, i, \sin\theta)$$

INITIAL STATE $\longrightarrow\longleftarrow$ → $\theta = 0$

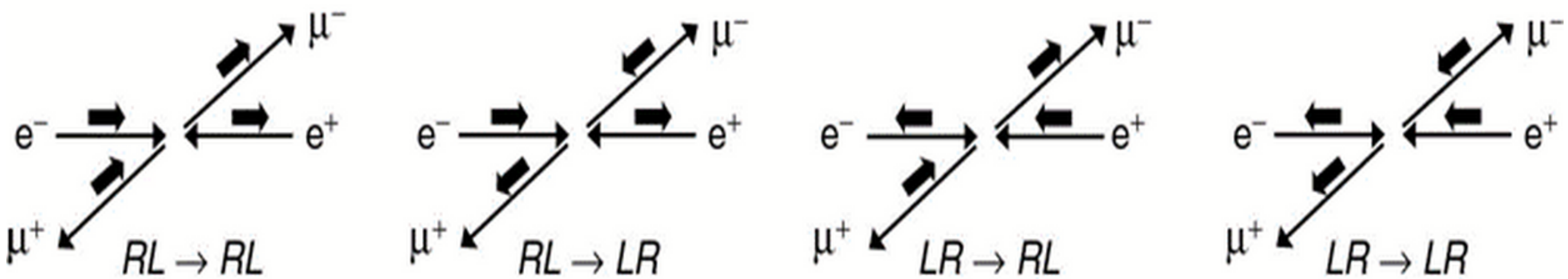
ONLY 4-MOMENTUM LABELS CHANGE

$$j_{eRL} = \bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1) = 2E(0, -1, i, 0)$$

$$j_{eLR} = \bar{v}_\uparrow(p_2)\gamma^\mu u_\downarrow(p_1) = 2E(0, -1, i, 0)$$

FROM $j_{\mu LL} = j_{\mu RR} = 0$ → $j_{eLL} = j_{eRR} = 0$

# $e^- e^+ \to \mu^- \mu^+$ CROSS SECTION




IN LIMIT $E \gg m$ ONLY 2 OUT OF 4 HELICITY COMBINATIONS NON-ZERO

FOR EACH COMBINATION, GET MATRIX ELEMENT FROM

$$\mathcal{M} = -\frac{e^2}{s} j_e \cdot j_\mu$$

EXAMPLE $\quad e^-_\uparrow e^+_\downarrow \to \mu^-_\uparrow \mu^+_\downarrow$

$$j^\mu_{eRL} = \bar{v}_\downarrow(p_2)\gamma^\mu u_\uparrow(p_1) = 2E\,(0,-1,-i,0)$$

$$j^\nu_{\mu RL} = \bar{u}_\uparrow(p_3)\gamma^\mu v_\downarrow(p_4) = 2E\,(0,-\cos\theta, i, \sin\theta)$$

$$\mathcal{M}_{RL\to RL} = j_e \cdot j_\mu = -\frac{e^2}{s}\left[2E\,(0,-1,-i,0)\right]\left[2E\,(0,-\cos\theta,i,\sin\theta)\right]$$

$$= -\frac{e^2}{s}\cdot s\left[0,-\cos\theta,-1,0\right] = e^2\left[1+\cos\theta\right] = 4\pi\alpha\,(1+\cos\theta)$$

EVALUATE ALL 4 MATRIX ELEMENTS DO IT!

$$|M_{RL\to RL}|^2 = |M_{LR\to LR}|^2 = (4\pi\alpha)^2(1+\cos\theta)^2$$

$$|M_{RL\to LR}|^2 = |M_{LR\to RL}|^2 = (4\pi\alpha)^2(1-\cos\theta)^2$$

AVERAGE OVER INCOMING SPINS

$$\langle|\mathcal{M}|^2_{fi}\rangle = \frac{1}{4}\left[|\mathcal{M}_{RL\to RL}|^2 + |\mathcal{M}_{RL\to LR}|^2 + |\mathcal{M}_{LR\to RL}|^2 + |\mathcal{M}_{LR\to LR}|^2\right]$$

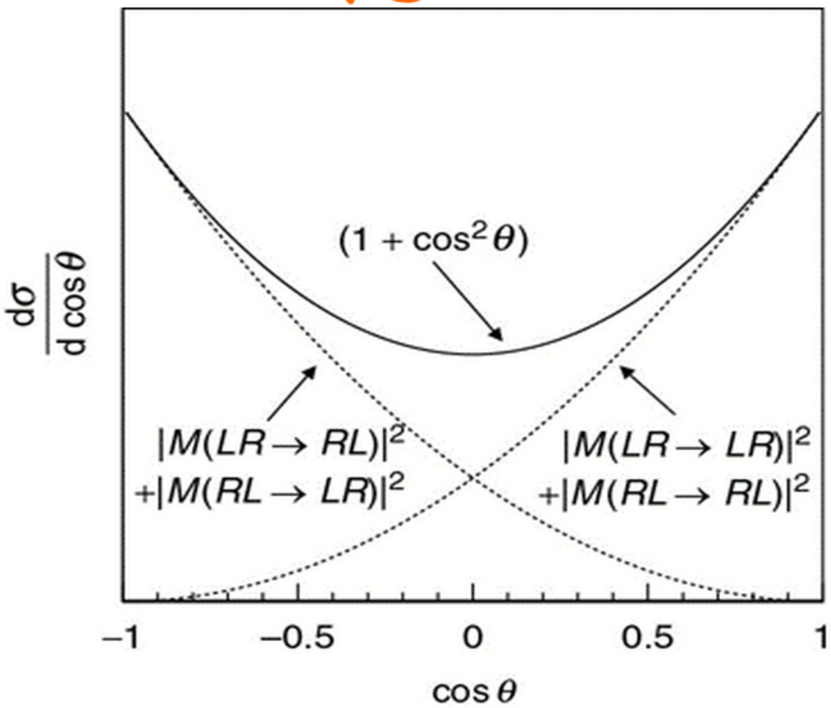
$$= \frac{1}{4}e^2\left[2(1+\cos\theta)^2 + 2(1-\cos\theta)^2\right] = e^4(1+\cos^2\theta)$$

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}|\mathcal{M}_{fi}|^2$$ CMS $p_f^* = p_i^* = E$

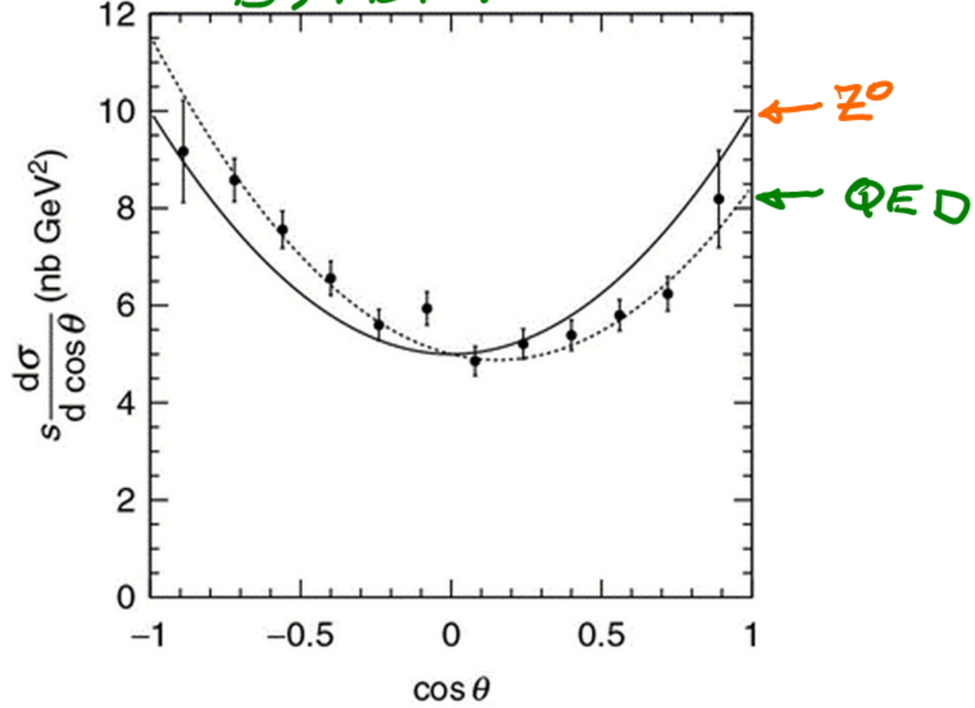
$$\frac{d\sigma}{d\Omega^*} = \frac{16\pi^2\alpha^2}{64\pi^2 s}(1+\cos^2\theta) = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

CAN MEASURE AND COMPARE

QED



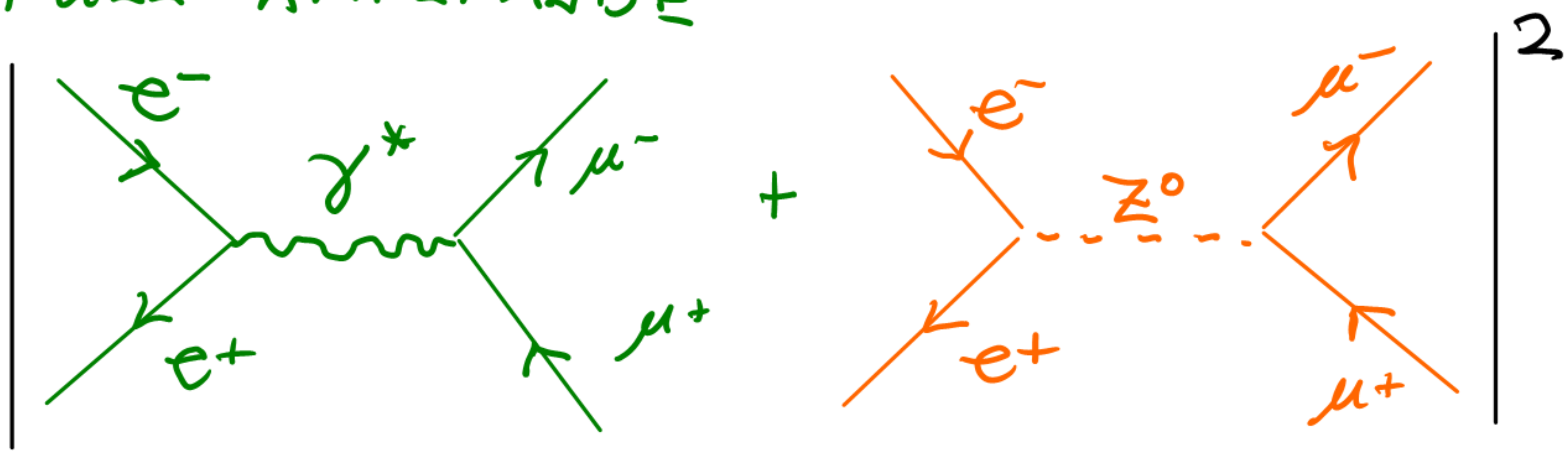

EXPERIMENT




RESULTS FROM PETRA $\sqrt{s} = 30\,\text{GeV}$ $e^+e^-$ COLLIDER

$\cos\theta$ SYMMETRY → PARITY CONSERVED IN QED

FULL AMPLITUDE

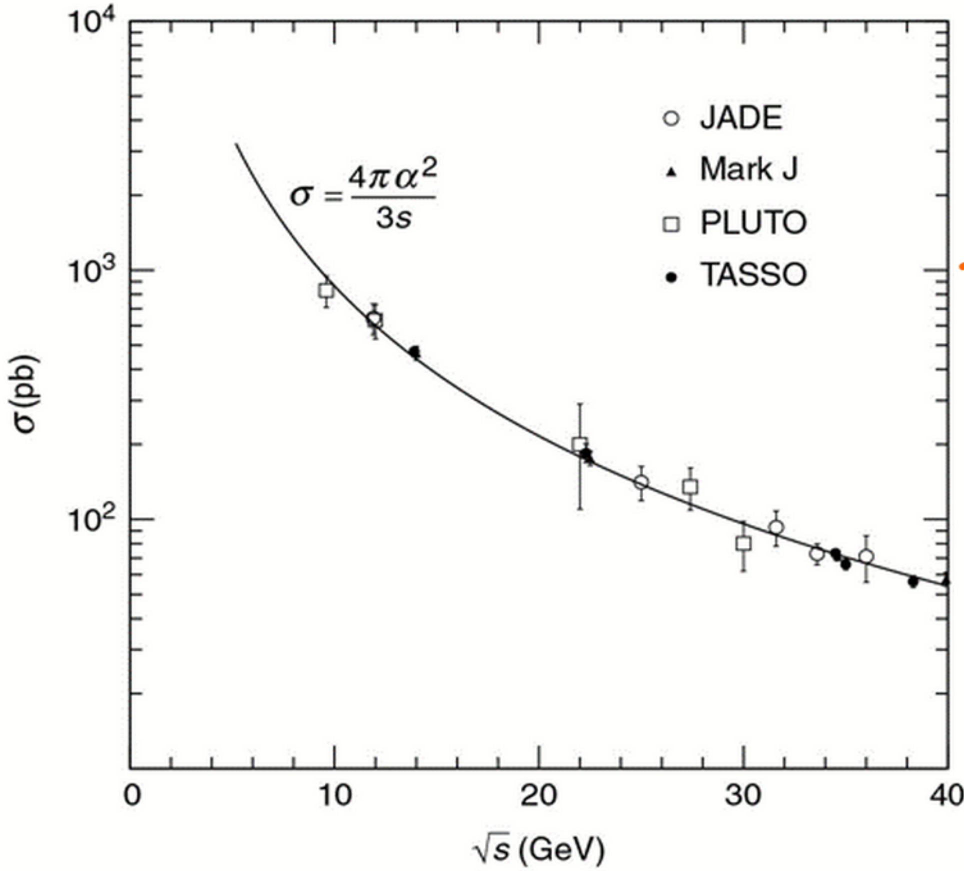



INTERFERENCE

ASSYMETRY

WEAK INTERACTION DOES NOT CONSERVE PARITY

$\sigma_{TOT}$ $e^+e^- \to \mu^+\mu^-$




VARIOUS EXPERIMENTS AT PETRA

DESY HH

$$\sigma_{TOT} = \int \frac{d\sigma}{d\Omega} \cdot d\Omega = \frac{\alpha^2}{4s}\int (1+\cos^2\theta)\, d\Omega = \frac{\alpha^2}{4s}\, 2\pi \int_{-1}^{+1} (1+\cos^2\theta)\, d\theta$$

$$\sigma_{TOT} = \frac{4\pi\alpha^2}{3s}$$

CM ENERGY OF COLLIDER

## $\langle |\mathcal{M}_{fi}|^2 \rangle \rightarrow$ LORENTZ INVARIANT FORM

$$\langle |\mathcal{M}_{fi}|^2 \rangle = e^4 (1 + \cos^2\theta)$$

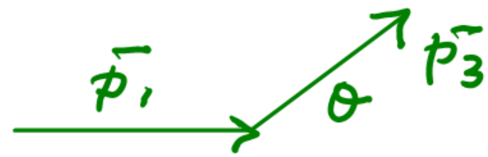
SCATTERING ANGLE IN CMS

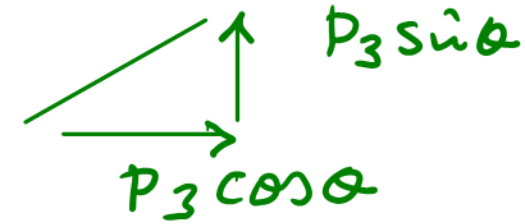
LET'S MAKE IT MANIFESTLY COVARIANT

4 VECTOR SCALAR PRODUCTS

$$p_1 \cdot p_2 = 2E^2, \quad p_1 \cdot p_3 = E^2(1-\cos\theta), \quad p_1 \cdot p_4 = E^2(1+\cos\theta)$$

EG $p_1 \cdot p_3$




$$p_1 \cdot p_3 = (E, 0, 0, E)(E, 0, p_3 \sin\theta, p_3 \cos\theta)$$

$$= (E, 0, 0, E)(E, 0, E\sin\theta, E\cos\theta)$$

$$= E^2 - E^2\cos\theta = E^2(1-\cos\theta)$$

DO THE OTHERS !

$$\frac{(p_1\cdot p_3)^2 + (p_1\cdot p_4)^2}{(p_1\cdot p_2)^2} = \frac{E^4(1-\cos\theta)^2 + E^4(1+\cos\theta)^2}{4E^4}$$

$$= \frac{1}{2}(1+\cos^2\theta)$$

$$\langle |M_{fi}|^2\rangle = e^4(1+\cos^2\theta)$$

$$= \frac{2e^4\left((p_1\cdot p_3)^2 + (p_1\cdot p_4)^2\right)}{(p_1\cdot p_2)^2}$$

THESE ARE ALL 4-VECTOR SCALAR PRODUCTS → LORENTZ INVARIANT

MANDELSTAM

$$\left.\begin{aligned} s &= (p_1+p_2)^2 = \cancel{m_1^2} + \cancel{m_2^2} + 2p_1\cdot p_2 \\ &= 2p_1\cdot p_2 \\ t &= (p_1-p_3)^2 = -2p_1\cdot p_3 \\ u &= (p_1-p_4)^2 = -2p_1\cdot p_4 \end{aligned}\right\}$$

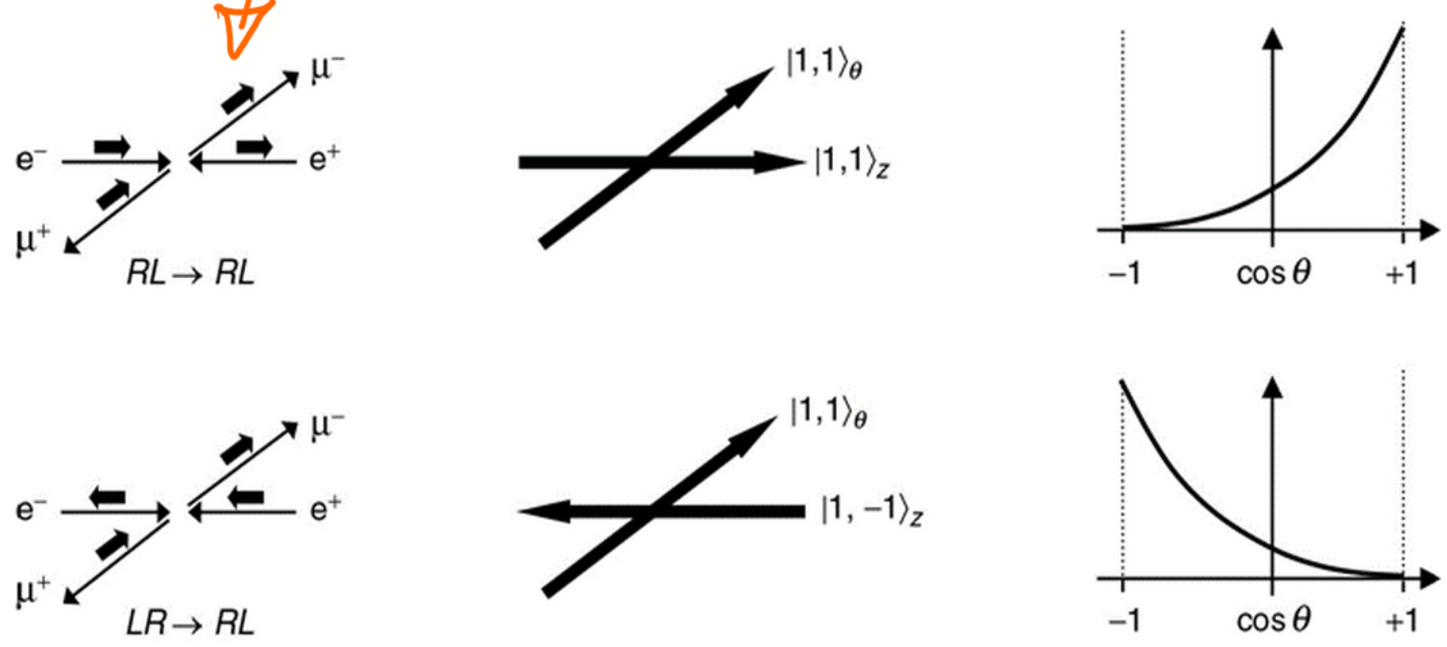
$$\langle |M_{fi}|^2\rangle = 2e^4\left\{\frac{t^2+u^2}{s^2}\right\}$$

# MORE ON $e^+e^-$ SPIN STRUCTURE

WE ARE SEEING THAT SPIN PLAYS A VERY IMPORTANT ROLE IN FERMION SCATTERING

4 HELICITY STATES $\longrightarrow$ NON ZERO MATRIX ELEMENTS

SPINS OF INITIAL AND FINAL STATE PARTICLES $\rightarrow$ ALIGNED



DEFINE Z-AXIS AS INCOMING ELECTRON

COMBINED SPIN OF $e^+e^-$

$$RL = |1,+1\rangle_z \qquad LR = |1,-1\rangle_z$$

NON ZERO COMBINATIONS FOR $\mu^+\mu^-$ WITH RESPECT TO $\mu^+\mu^-$ DIRECTION OF MOTION

$$RL = |1,+1\rangle_\theta \quad , \quad LR = |1,-1\rangle_\theta$$

OPERATOR CORRESPONDING TO COMPONENT OF SPIN ALONG AXIS DEFINED BY UNIT VECTOR $\hat{n}$ AT AN ANGLE $\theta$ TO Z-AXIS

$$\hat{S}_n = \frac{1}{2}\,\hat{n}\cdot\hat{\underline{\sigma}}$$

USE THIS TO EXPRESS SPIN STATES OF $\mu^+\mu^-$ IN TERMS OF EIGENSTATES OF $\hat{S}_z$

$$\hat{S}_n = \frac{1}{2}\hat{n}\cdot\bar{\sigma}$$

RL $\mu^+\mu^-$

$$|1,+1\rangle_\theta = \frac{1}{2}(1-\cos\theta)|1,-1\rangle + \frac{1}{\sqrt{2}}\sin\theta|1,0\rangle + \frac{1}{2}(1+\cos\theta)|1,+1\rangle$$

ANGULAR DISTRIBUTION → INNER PRODUCT $|e\rangle_{IN}$ WITH $|\mu\rangle_{OUT}$

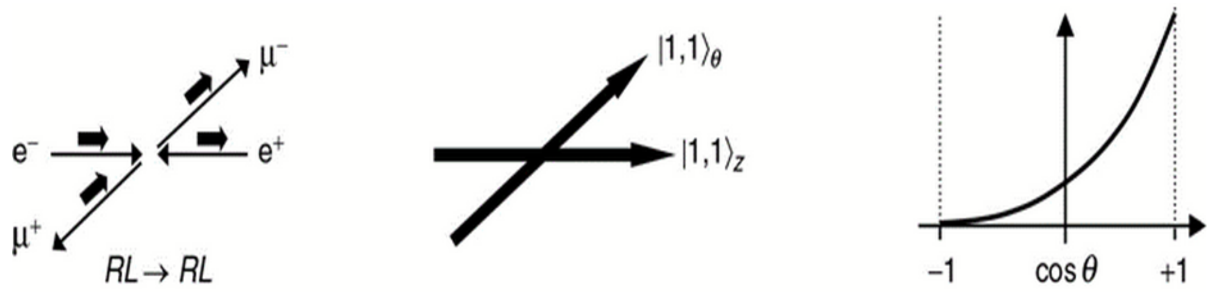
$$\mathcal{M}_{RL\to RL} \propto \langle \underset{e}{1,+1} | \underset{\mu}{1,+1}\rangle_\theta$$

$$= \langle 1+1||1-1\rangle\frac{1}{2}(1-\cos\theta) + \frac{1}{\sqrt{2}}\langle 1,+1||1,0\rangle\sin\theta + \langle 1,+1||1,+1\rangle(1+\cos\theta)$$

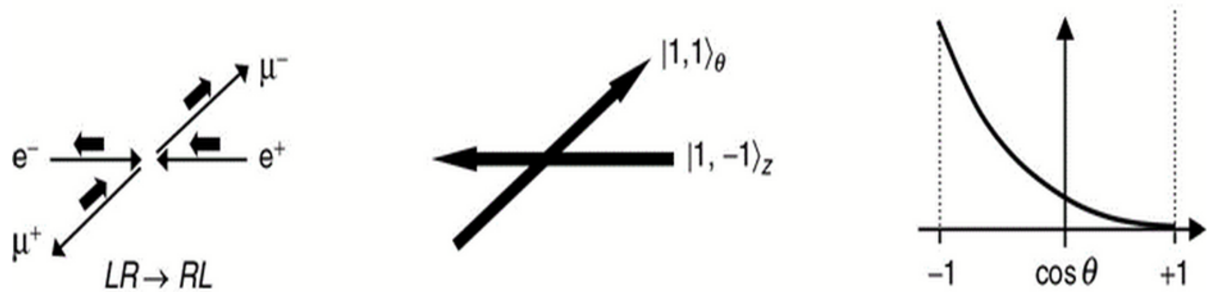
$$\langle 1,+1|1,-1\rangle = 0 \qquad \langle 1,+1|1,+1\rangle = 1$$

$$\mathcal{M}_{RL\to RL} \propto \frac{1}{2}(1+\cos\theta)$$

$$\mathcal{M}_{LR\to RL} \propto \langle \underset{e}{1,-1} | \underset{\mu}{1,+1}\rangle_\theta \propto \frac{1}{2}(1-\cos\theta)$$




$$\mathcal{M}_{RL\to RL} \propto \frac{1}{2}(1+\cos\theta)$$




$$\mathcal{M}_{LR\to RL} \propto \frac{1}{2}(1-\cos\theta)$$

IN THE LIMIT $E \gg m$, NON-ZERO MATRIX ELEMENTS CORRESPOND TO TOTAL SPIN STATES = 1

SPIN POINTS ALONG DIRECTION OF MOTION

ANGULAR DISTRIBUTION → SPIN 1

INTERACTION $\bar{\psi}\gamma^{\mu}\psi$ → EXCHANGE OF SPIN 1 PARTICLE

VECTOR INTERACTION

# CHIRALITY

$E \gg m \rightarrow$ 4 NONZERO HELICITY STATES $\rightarrow$ CHIRAL STRUCTURE OF QED

CHIRALITY $\rightarrow$ IMPORTANT CONCEPT IN STANDARD MODEL

DEFINE $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

WEAK INTERACTION

CAN DERIVE PROPERTIES OF $\gamma^5$ FROM COMMUTATORS AND HERMITIAN PROPERTIES OF $\gamma^0\gamma^1\gamma^2\gamma^3$

$$(\gamma^5)^2 = 1, \quad (\gamma^5)^\dagger = \gamma^5, \quad \gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$$

DO IT!

IN THE LIMIT $E \gg m$ THE HELICITY EIGENSTATES

$$u_\uparrow \approx \sqrt{E}\begin{pmatrix} c \\ s e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix}, \quad u_\downarrow \approx \sqrt{E}\begin{pmatrix} -s \\ c e^{i\phi} \\ s \\ -c e^{i\phi} \end{pmatrix} \quad v_\uparrow \approx \sqrt{E}\begin{pmatrix} s \\ -c e^{i\phi} \\ -s \\ c e^{i\phi} \end{pmatrix} \quad v_\downarrow = \sqrt{E}\begin{pmatrix} c \\ s e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix}$$

ARE ALSO EIGENSTATES OF $\gamma^5$

DO IT !

$$\gamma^5 u_\uparrow = +u_\uparrow, \quad \gamma^5 u_\downarrow = -u_\downarrow, \quad \gamma^5 v_\uparrow = -v_\uparrow, \quad \gamma^5 v_\downarrow = +v_\downarrow$$

IN GENERAL THE EIGENSTATES OF $\gamma^5$ ARE DEFINED AS LEFT HANDED AND RIGHT HANDED CHIRAL STATES

THEY ARE DISTINCT FROM HELICITY EIGENSTATES

$$\gamma^5 u_R = +u_R, \quad \gamma^5 u_L = -u_L, \quad \gamma^5 v_R = -v_R, \quad \gamma^5 v_L = +v_L$$

$$\gamma^5 u_R = +u_R, \quad \gamma^5 u_L = -u_L, \quad \gamma^5 v_R = -v_R, \quad \gamma^5 v_L = +v_L$$

WITH THIS CONVENTION, WHEN $E \gg m$, THE CHIRAL EIGENSTATES FOR BOTH PARTICLE AND ANTIPARTICLE SPINORS ARE THE SAME AS HELICITY SPINORS

$$u_\uparrow \rightarrow u_R$$

SOLUTIONS OF DIRAC EQUATION WHICH ARE ALSO EIGENSTATES OF $\gamma^5$ $\rightarrow$ IDENTICAL TO MASSLESS HELICITY EIGENSTATES

$$u_R \approx \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}, \quad u_L \approx \sqrt{E}\begin{pmatrix} -s \\ ce^{i\phi} \\ s \\ -ce^{i\phi} \end{pmatrix} \quad v_R \approx \sqrt{E}\begin{pmatrix} s \\ -ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix} \quad v_L = \sqrt{E}\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}$$

# CHIRAL PROJECTION OPERATORS

ANY DIRAC SPINOR CAN BE DECOMPOSED INTO LEFT HANDED AND RIGHT HANDED CHIRAL COMPONENTS USING CHIRAL PROJECTION OPERATORS

WHEN WE COME TO WEAK INTERACTIONS, WE WILL SEE HOW IMPORTANT THIS IS:

$$P_R = \frac{1}{2}(1+\gamma_5) \quad , \quad P_L = \frac{1}{2}(1-\gamma_5)$$

PROPERTIES OF $\gamma^5 \rightarrow P_R, P_L$ SATISFY ALGEBRA OF QM PROJECTION OPERATORS

$$P_R + P_L = 1, \quad P_R P_R = P_R, \quad P_L P_L = P_L, \quad P_L P_R = 0.$$

$$P = |\psi\rangle\langle\psi| \rightarrow P^2 = |\psi\rangle\underbrace{\langle\psi|\psi\rangle}_{1}\langle\psi| = P$$

$$P_L P_R = |L\rangle\underbrace{\langle L|R\rangle}_{0}\langle R| = 0$$

## ASIDE — READ IF YOU WANT TO

PROJECTION USED TO EXPAND A VECTOR IN A COMPLETE ORTHONORMAL BASIS $|\psi_n\rangle$

$$|\psi\rangle = \sum_n c_n |\psi_n\rangle \quad (1)$$

$$\langle\psi_m|\psi\rangle = \sum_n c_n \underbrace{\langle\psi_m|\psi_n\rangle}_{\delta_{mn}}$$

$$c_n = \langle\psi_m|\psi\rangle \quad (2)$$

IN TRANSITION AMPLITUDES $|\psi\rangle \to |\psi_n\rangle$

PUT (2) $\to$ (1)

$$|\psi\rangle = \sum_n \underbrace{|\psi_n\rangle\langle\psi_n|}_{P_n}\psi\rangle$$

FOR A COMPLETE SET OF ORTHONORMAL BASIS VECTORS THE ORTHOGONAL PROJECTION OPERATORS SATISFY THE COMPLETENESS RELATION

$$\sum_n P_n = \sum_n |\psi_n\rangle\langle\psi_n| = 1$$

PROJECTION OPERATOR ACTING ON AN ARBITRARY STATE $|\psi\rangle$ PROJECTS $|\psi\rangle$ TO $|\psi_n\rangle$ WITH A PROBABILITY $|\langle\psi_n|\psi\rangle|^2$

$$P_n P_m = \delta_{nm} \qquad P_n^2 = P_n \qquad \sum P_n = 1$$

AFTER THE ASIDE ...... IN DIRAC–PAULI

$$P_R = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \quad ; \quad P_L = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

$$P_R u_R = \frac{1}{2}(1+\gamma_5)u_R = \frac{1}{2}(u_R + u_R) = u_R$$

$$P_R u_L = \frac{1}{2}(1+\gamma_5)u_L = \frac{1}{2}(u_L - u_L) = 0$$

SIMILARLY $\quad P_R v_R = 0$

$\quad P_R v_L = 0$

$P_R$ PROJECTS OUT RH CHIRAL STATES FOR PARTICLES, AND LH FOR ANTIPARTICLES

LEFT HANDED PROJECTION

$P_L u_R = 0$, $P_L u_L = u_L$, $P_L v_R = v_R$, $P_L v_L = 0$

$P_L \rightarrow$ LH FOR PARTICLE, RH FOR ANTIPARTICLE

SINCE $P_R$, $P_L$ PROJECT OUT CHIRAL STATES, ANY SPINOR CAN BE DECOMPOSED INTO RH, LH

$$u = a_R u_R + a_L u_L = \frac{1}{2}(1+\gamma^5)u + \frac{1}{2}(1-\gamma_5)u$$

THIS IS JUST DOING

$$|\psi\rangle = \sum_n c_n |\psi_n\rangle$$

$$|\psi\rangle = \sum_n \underbrace{|\psi_n\rangle\langle\psi_n|}_{P_n}\psi\rangle$$

# CHIRALITY IN QED

FUNDEMENTAL INTERACTION BETWEEN FERMION $\leftrightarrow \gamma$

$$i Q_f e \overline{\psi} \gamma^\mu \phi$$

ANY 4-VECTOR CURRENT CAN BE DECOMPOSED INTO RH, LH $\rightarrow$ PROJECTION OPERATORS

E.G. $\overline{\psi} \gamma^\mu \phi = (a_R^* \overline{\psi}_R + a_L^* \overline{\psi}_L) \gamma^\mu (b_R \phi_R + b_L \phi_L)$

$$= a_R^* b_R \overline{\psi}_R \gamma^\mu \phi_R + a_R^* b_L \overline{\psi}_R \gamma^\mu \phi$$
$$+ a_L^* \overline{\psi}_L \gamma^\mu \phi_R + a_L^* b_L \overline{\psi}_L \gamma^\mu \phi_L$$

QED CHIRAL STRUCTURE $\longrightarrow$ TWO OF THESE ALWAYS ZERO.

LOOK AT $\overline{u}_L(p) \gamma^\mu u_R(p')$

$$\bar{u}_L(p)\gamma^\mu u_R(p') =$$

NOW $\bar{u}_L(p) \equiv [u_L(p)]^\dagger \gamma^0$ DEFINITION OF ADJOINT

$$= [P_L u_L(p)]^\dagger \gamma^0 = \left[\tfrac{1}{2}(1-\gamma_5)u_L(p)\right]^\dagger \gamma^0$$

USE $\gamma^5 = \gamma^{5\dagger}$ $\quad = u_L^\dagger(p)\,\tfrac{1}{2}(1-\gamma^5)\gamma^0$

USE $\gamma^0\gamma^5 = -\gamma^5\gamma^0$ $\quad = \frac{u_L^\dagger}{2}(\gamma^0 - \gamma^5\gamma^0) = \tfrac{1}{2}u_L^\dagger(\gamma^0 + \gamma^0\gamma_5)$

$$= \tfrac{1}{2}\,u_L^\dagger\gamma_0(1+\gamma_5)$$

$$= \bar{u}_L(1+\gamma_5)$$

$P_R u_R = u_R$

SO $\bar{u}_L(p)\gamma^\mu u_R(p') = \bar{u}_L(p)\,P_R\gamma^\mu P_R\,u_R(p')$

$$\bar{u}_L(p)\gamma^\mu u_R(p') = \bar{u}_L(p) P_R \gamma^\mu P_R u_R(p')$$

$$\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$$

so $\quad P_R\gamma^\mu = \frac{1}{2}(1+\gamma_5)\gamma^\mu = \frac{1}{2}\gamma^\mu(1-\gamma_5) = \gamma^\mu P_L$

$$\bar{u}_L(p)\gamma^\mu u_R(p') = \bar{u}_L(p)\underbrace{\gamma^\mu}_{P_R\gamma^\mu}\underbrace{P_L P_R}_{0} u_R(p)$$

$\bar{\psi}\gamma^\mu\phi$ FORM $\rightarrow$ ONLY CERTAIN COMBINATIONS OF CHIRAL EIGENSTATES NON-ZERO

$$\bar{u}_L\gamma^\mu u_R = \bar{u}_R\gamma^\mu u_L = \bar{v}_L\gamma^\mu v_R = \bar{v}_R\gamma^\mu v_L = \bar{v}_L\gamma^\mu u_L$$

$$= \bar{v}_R\gamma^\mu u_R$$

$$= 0 \qquad \text{ALWAYS !}$$

# HELICITY AND CHIRALITY

HELICITY $\not\leftrightarrow$ CHIRALITY

HELICITY EIGENSTATES $\rightarrow$ PROJECTION OF SPIN ONTO DIRECTION OF MOTION

RELATIONSHIP BETWEEN HELICITY AND CHIRALITY

$\rightarrow$ DECOMPOSE HELICITY SPINORS INTO CHIRAL COMPONENTS

$$u_\uparrow(p,\theta,\phi) = N\begin{pmatrix} c \\ se^{i\phi} \\ \kappa c \\ \kappa se^{i\phi} \end{pmatrix} \qquad \kappa = \frac{p}{E+m}, \quad N = \sqrt{E+m}$$

$$P_R u_\uparrow = \frac{1}{2}(1+\kappa)N\begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}, \quad P_L u_\uparrow = \frac{1}{2}(1-\kappa)N\begin{pmatrix} c \\ se^{i\phi} \\ -c \\ -se^{i\phi} \end{pmatrix}$$

DO IT!

$$P_R u_\uparrow = \frac{1}{2}(1+K)N \begin{pmatrix} c \\ se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix} , \quad P_L u_\uparrow = \frac{1}{2}(1-K)N \begin{pmatrix} c \\ se^{i\phi} \\ -c \\ -se^{i\phi} \end{pmatrix}$$

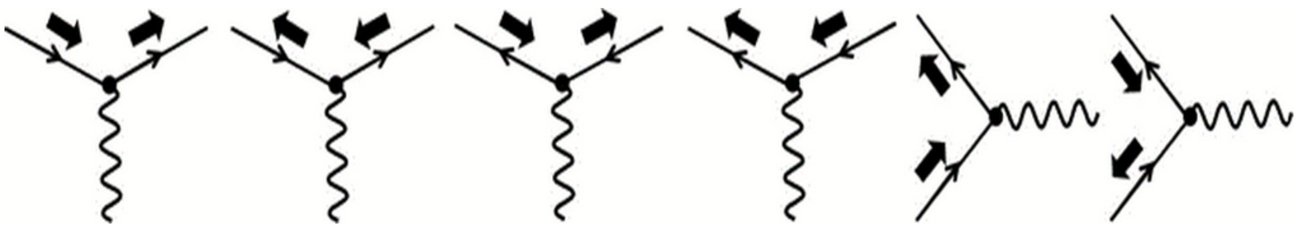
ADDING THESE GIVES BACK $u_\uparrow(p,\theta,\phi)$

SO $u_\uparrow \propto \frac{1}{2}(1+K)\,u_R + \frac{1}{2}(1+K)\,u_L$

$\gamma^5 u_R = u_R$ $\quad\quad$ $\gamma^5 u_L = -u_L$

FOR $E \gg m$, $K \to 1$ $\quad$ HELICITY $\longrightarrow$ CHIRALITY

SO NOT ONLY $\bar{u}_R \gamma^\mu u_L$ etc $= 0$ $\quad$ ALSO $\bar{u}_\uparrow \gamma^\mu u_\downarrow \cdots = 0$



IN QED HELICITY OF PARTICLE ENTERING VERTEX SAME AS HELICITY LEAVING VERTEX

HELICITY CONSERVATION