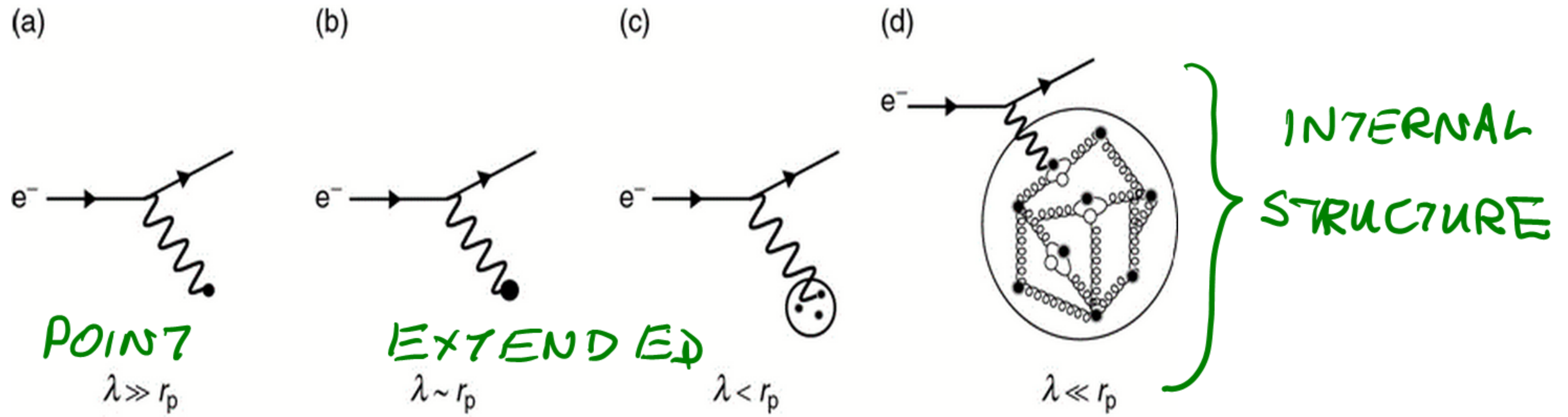
PROBING PROTON STRUCTURE

SO FAR WE HAVE LOOKED AT SCATTERING OF POINT LEPTONS

OUR KNOWLEDGE OF THE STRUCTURE OF THE PROTON LARGELY COMES FROM $ep \to ep$, $ep \to eX$



PROTON IS AN EXTENDED OBJECT — WHAT EXPERIMENT SEES DEPENDS ON "WAVELENGTH" OF PROTON

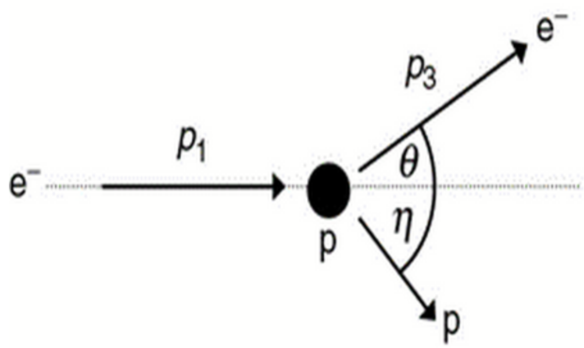
$\lambda \to$ DEPENDS ON 4-MOMENTUM TRANSFER

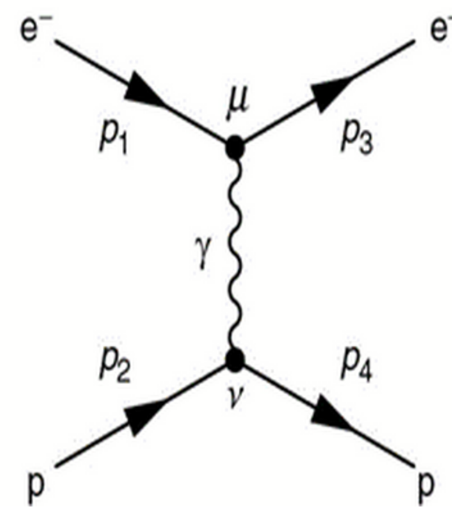
AS $q^2 \uparrow$, RESOLVED DISTANCE SCALE $\downarrow$

RUTHERFORD AND MOTT SCATTERING

LOW ENERGY $\rightarrow$ E_p RECOIL $\ll m_p$

CAN TREAT AS POINT DIRAC PARTICLE





$$\mathcal{M}_{fi} = \frac{Q_q^2 e^2}{q^2} \cdot \left[\bar{u}(p_3)\gamma^\mu u(p_1)\right] g_{\mu\nu} \left[\bar{u}(p_4)\gamma^\mu u(p_2)\right]$$

FEYNMAN RULES $\rightarrow$ NO ANTIPARTICLES

DIRAC SPINORS FOR TWO POSSIBLE ELECTRON HELICITY STATES

$$u_\uparrow = N_e \begin{pmatrix} c \\ s e^{i\phi} \\ kc \\ k s e^{i\phi} \end{pmatrix} \qquad u_\downarrow = N_e \begin{pmatrix} -s \\ c e^{i\phi} \\ ks \\ -kc e^{i\phi} \end{pmatrix}$$

DISTINGUISHES RELATIVISTIC & NON RELATIVISTIC

$$N_e = \sqrt{E + m_e}\,, \quad s = \sin\theta/2\,, \quad c = \cos\theta/2$$

$$k = \frac{|\bar{p}|}{E + m_e} \equiv \frac{\beta_e \gamma_e}{\gamma_e + 1}$$

β_e SPEED OF ELECTRON

γ_e LORENTZ BOOST

VELOCITY OF SCATTERED PROTON LOW — KE NEGLECTED

→ ELECTRON ENERGY DOES NOT CHANGE

k SAME FOR ELECTRON IN AND OUT

$$u_\uparrow(p_1) = N_e \begin{pmatrix} 1 \\ 0 \\ k \\ 0 \end{pmatrix}, u_\downarrow(p_1) = N_e \begin{pmatrix} 0 \\ 1 \\ 0 \\ -k \end{pmatrix}, u_\uparrow(p_3) = N_e \begin{pmatrix} c \\ s \\ kc \\ ks \end{pmatrix}, u_\downarrow(p_3) = N_e \begin{pmatrix} -s \\ c \\ ks \\ -kc \end{pmatrix}$$

JUST AS WE DID FOR $e^+e^- \rightarrow \mu^+\mu^-$ CAN WRITE DOWN THE FOUR POSSIBLE HELICITY COMBINATIONS

$$j_{e\uparrow\uparrow} = \bar{u}_\uparrow(p_3)\gamma^\mu u_\uparrow(p_1) = (E+m_e)\left[(k^2+1)c,\ 2ks,\ +2iks,\ 2kc\right]$$

$$j_{e\downarrow\downarrow} = \bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1) = (E+m_e)\left[(k^2+1)c,\ 2ks,\ -2iks,\ 2kc\right]$$

$$j_{e\downarrow\uparrow} = \bar{u}_\uparrow(p_3)\gamma^\mu u_\downarrow(p_1) = (E+m_e)\left[(1-k^2)s,\ 0,\ 0,\ 0\right]$$

$$j_{e\uparrow\downarrow} = \bar{u}_\downarrow(p_3)\gamma^\mu u_\uparrow(p_1) = (E+m_e)\left[(k^2-1)s,\ 0,\ 0,\ 0\right]$$

RELATIVISTIC $k \approx 1$ ONLY TWO HELICITY STATES

LOW ENERGY $k < 1$ $\rightarrow$ 4 HELICITY STATES

HELICITY $\neq$ CHIRALITY

HELICTY NOT CONSERVED

RECOILING PROTON SLOW $\beta_p \ll 1$, $k \sim 0$ LOWER 2 COMPONENTS OF SPINOR $= 0$

TAKE $\theta_p = \eta$, $\phi_p = \pi$

INITIAL STATE PROTON
$$u_\uparrow(p_2) = \sqrt{2m_p}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad u_\downarrow(p_2) = \sqrt{2m_p}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

FINAL STATE PROTON
$$u_\uparrow(p_4) \approx \sqrt{2m_p}\begin{pmatrix}c_\eta\\-s_\eta\\0\\0\end{pmatrix} \qquad u_\downarrow(p_4) \approx \sqrt{2m_p}\begin{pmatrix}-s_\eta\\-c_\eta\\0\\0\end{pmatrix}$$

YET AGAIN CALC FOUR POSSIBLE 4-VECTOR CURRENTS

INITIAL $$j_{p\uparrow\uparrow} = -j_{p\downarrow\downarrow} = 2m_p[c_\eta, 0, 0, 0]$$

FINAL $$j_{p\uparrow\downarrow} = j_{p\downarrow\uparrow} = -2m_p[s_\eta, 0, 0, 0]$$

$\rightarrow$ ALL FOUR CONTRIBUTE.

PLUG THESE INTO $$\mathcal{M}_{fi} = \frac{e^2}{q^2} j_e \cdot j_p$$

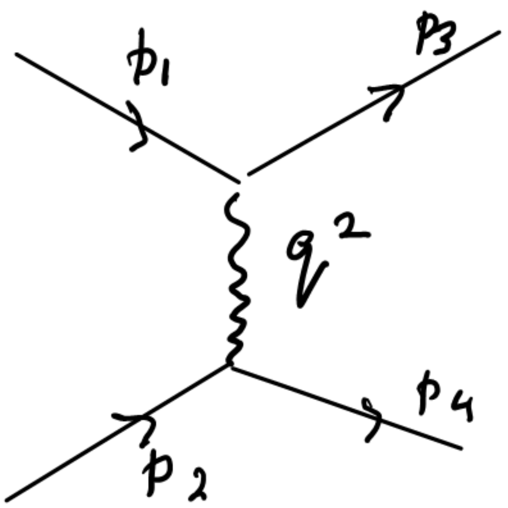
$$\langle |\mathcal{M}_{fi}^2| \rangle = \frac{1}{4} \frac{e^4}{q^4} \cdot 4 m_p^2 (E + m_e)^2 \cdot [c_\eta^2 + s_\eta^2]$$
$$\times \left[4 (1 + \kappa^2)^2 c^2 + 4 (1 - \kappa^2)^2 s^2 \right]$$
$$= 4 m_p^2 m_e^2 e^4 (\gamma_e + 1)^2 \left[(1 - \kappa^2)^2 + 4 \kappa^2 c^2 \right]$$

$E = \gamma_e m$

$$\kappa = \frac{\beta_e \gamma_e}{\gamma_e + 1} \qquad (1 - \beta_e^2) \gamma_e^2 = 1$$

$$\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{16 m_p^2 m_e^2 e^4}{q^4} \left[1 + \beta_e^2 \gamma_e^2 \cos^2 \frac{\theta}{2} \right]$$

$$\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{16 m_p^2 m_e^2 e^4}{q^4} \left[1 + \beta_e^2 \gamma_e^2 \cos^2 \frac{\theta}{2} \right]$$



NEGLECT PROTON RECOIL

$E_1 = E_3 = E$

$|\bar{p}_1| = |\bar{p}_3|$

$$q^2 = (p_1 - p_3)^2 = p_1^2 + p_3^2 - 2 p_1 \cdot p_2$$

$$= m^2 + m^2 - 2(E, \bar{p})(E, \bar{p})$$

$$= 2m^2 - 2E^2 + 2p^2 \cos\theta$$

$$= -2p^2 + 2p^2 \cos\theta = -2p^2(1 - \cos\theta)$$

$$= -4p^2 \sin^2 \frac{\theta}{2}$$

$$\langle |M_{fi}^2| \rangle = \frac{m_p^2 m_e^2 e^4}{p^4 \sin^4\left(\frac{\theta}{2}\right)} \left[1 + \beta_e^2 \gamma_e^2 \cos^2 \frac{\theta}{2} \right]$$

THIS IS TRUE RELATIVISTICALLY OR NON RELATIVISTICALLY

AS LONG AS NEGLECT PROTON RECOIL $m_p \gg m_e$

RUTHERFORD SCATTERING

PROTON RECOIL NEGLECTED $\quad \beta_e \gamma_e \ll 1$

$$\langle |M_{fi}^2| \rangle = \frac{m_p^2 m_e^2 e^4}{p^4 \sin^4\left(\frac{\theta}{2}\right)} \left[1 + \beta_e^2 \gamma_e^2 \cos^2 \frac{\theta}{2} \right] \rightarrow \frac{m_p^2 m_e^2 e^4}{p^4 \sin^4 \frac{\theta}{2}}$$

HAD $\quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{1}{m_p + E_1 - E_1 \cos\theta} \right) \langle |M_{fi}|^2 \rangle$

$E_1 \sim m_e \ll m_p \quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 m_p^2} \langle |M_{fi}|^2 \rangle$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 p^2 \sin^4\frac{\theta}{2}}$$

PUT $E_k = \frac{p^2}{2m_e}$, $e^2 = 4\pi\alpha$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16 E_k^2 \sin^4 \theta/2}$$

STATIC ELECTRIC FIELD

NO SPIN-SPIN INTERACTION

MAGNETIC

ANGULAR DISTRIBUTION $\sim \frac{1}{q^2}$

FORM FACTORS – SIZE & SHAPE OF NUCLEI & PARTICLES

SCATTERING EXPERIMENTS

- DETERMINE NUCLEAR SIZE
- DETERMINE NUCLEAR SHAPE
  - DISTRIBUTION OF "MATTER"
  - DISTRIBUTION OF ELECTRIC CHARGE
- DETERMINE SIZE OF PROTON
- DETERMINE SHAPE OF PROTON
  - DISTRIBUTION OF ELECTRIC CHARGE

    → DIRECT OBSERVATION OF QUARKS

RUTHERFORD DIFFERENTIAL CROSS SECTION IS:

A THEORETICAL MODEL

VALIDITY DEPENDS ON VALIDITY OF ASSUMPTIONS

- COULOMB POTENTIAL – WEAK PERTURBATION
- ONLY HEAVY TARGET — NO RECOIL
- SPIN 0 PROJECTILE & TARGET
- NO STRUCTURE, OR SPATIAL EXTENT OF TARGET OR BEAM PROJECTILE

DEVIATIONS FROM RUTHERFORD GIVE INFORMATION ON:

- DIFFERENT POTENTIAL ACTING
- NON POINT PARTICLES
- INTERNAL STRUCTURE
- SPIN OF PARTICLES

RUTHERFORD & MOTT IGNORE SPATIAL EXTENT OR INTERNAL STRUCTURE OF TARGET

HOWEVER, WE DO EXPERIMENTS TO DETERMINE EXPERIMENTALLY:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{EXPERIMENT}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{MOTT}} \cdot \left| F(q^2) \right|^2$$

MEASURE

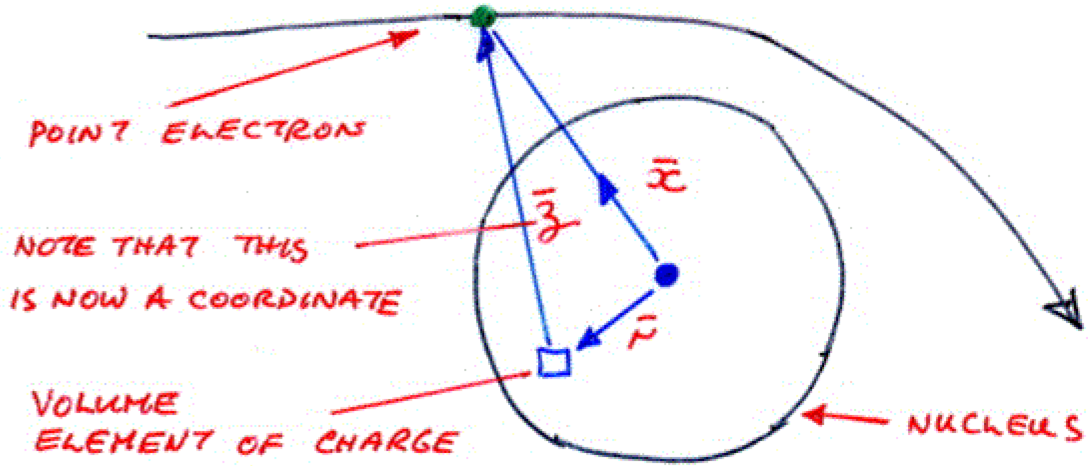
CALCULABLE
POINT SCATTERING

FORM FACTOR
INCORPORATES
SPATIAL EXTENT

Eg. DISTRIBUTION
OF ELECTRIC
CHARGE

EXPERIMENT + THEORY → INFER →

RUTHERFORD SCATTERING FROM EXTENDED TARGET



CHARGE DENSITY IN NUCLEUS $\rho(r)$

IN VOLUME ELEMENT $d^3r = r^2 \sin\theta \, d\theta \, d\phi \, dr$

CHARGE IS $Ze\,\rho(r)\,d^3r$ CONTRIBUTION OF THIS ELEMENT TO COULOMB POTENTIAL AT ELECTRON

$$dV(x) = \underbrace{\frac{-Ze^2}{z}}_{\text{COULOMB}} \underbrace{\exp\left(\frac{-z}{a}\right)}_{\text{SHIELDING}} \underbrace{\rho(r)}_{\text{CHARGE DENSITY}} dr$$

$$dV(x) = \frac{-Ze^2}{z} \exp\left(\frac{-z}{a}\right) \rho(r)\,dr$$

POTENTIAL AT x DUE TO WHOLE NUCLEUS

$$V(x) = -Ze^2 \int d^3r\, \frac{\rho(r)}{z} \exp\left(\frac{-z}{a}\right)$$

PUT THIS POTENTIAL INTO BORN APPROXIMATION

$$f(q^2) = \frac{-m}{2\pi\hbar^2} \int V(x)\, e^{-i\bar{q}\cdot x/\hbar}\, d^3\bar{x}$$

NEW COORDINATE SYSTEM $\bar{x} = \bar{r} + \bar{z}$

$$f(q^2) = \frac{mZe^2}{2\pi\hbar^2} \int d^3r\, e^{i\bar{q}\cdot\bar{r}/\hbar} \rho(r) \int d^3z\, \frac{e^{-z/a}}{z}\, e^{i\bar{q}\cdot\bar{z}/\hbar}$$

$$f(q^2) = \frac{m Z e^2}{2\pi\hbar^2} \underbrace{\int d^3r\, e^{i\vec{q}\cdot\vec{r}} \rho(r)}_{} \underbrace{\int d^3\xi\, \frac{e^{-\xi/a}}{\xi} e^{i\vec{q}\cdot\vec{\xi}/\hbar}}_{}$$

INTEGRAL OVER VOLUME OF NUCLEUS

INTEGRAL OVER ELECTRON ORBIT OF COULOMB INTERACTIONS

FORM FACTOR OF NUCLEUS

$$F(q^2) = \int d^3r\, e^{i\vec{q}\cdot\vec{r}/\hbar} \rho(r)$$

FOURIER TRANSFORM OF SPATIAL CHARGE DENSITY

CHARGE DENSITY IN MOMENTUM SPACE

$$= \frac{4\pi\hbar^2}{q^2 + \left(\frac{\hbar}{a}\right)^2} \rightarrow \frac{4\pi\hbar^2}{q^2}$$

$$\sim \frac{1}{q^2} \rightarrow \text{MOMENTUM TRANSFER}$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \left|f(q^2)\right|^2 = \left|F(q^2)\right|^2 \left(\frac{d\sigma}{d\Omega}\right)_{\text{POINT TARGET}}$$

EXPERIMENTAL DETERMINATION OF TARGET STRUCTURE

MEASURE FORM FACTOR $\left(\frac{d\sigma}{d\Omega}\right)_{\text{EXPT}} = |F(q^2)|^2 \frac{d\sigma}{d\Omega}$ POINT THEORY

DO A FOURIER TRANSFORM TO GET CHARGE DENSITY IN REAL SPACE

$$\rho(r) = \frac{1}{(2\pi)^3} \int d^3q \; F(q^2)\, e^{-i\vec{q}\cdot\vec{r}/\hbar}$$

FORM FACTOR

HAVE TO MEASURE OVER ALL q^2 — DIFFICULT.

USUALLY HYPOTHESIZE FORM OF $\rho(r)$

PUT INTO $F(q^2) = \int d^3r \, e^{i\vec{q}\cdot\vec{r}/\hbar} \rho(r)$ AND

DETERMINE WHICH $\rho(r)$ GIVES BEST FIT TO THE EXPERIMENTAL $F(q^2)$

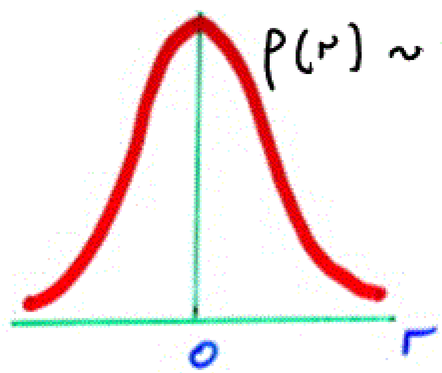
ELECTRIC CHARGE DISTRIBUTIONS IN NUCLEI

TWO SIMPLE MODELS

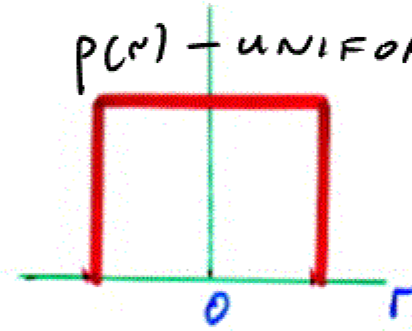
GAUSSIAN

$$F(q^2) = \exp\left[\frac{-q^2 b^2}{4\hbar^2}\right]$$

$$\rho(r) \sim e^{-(r/b)^2}$$

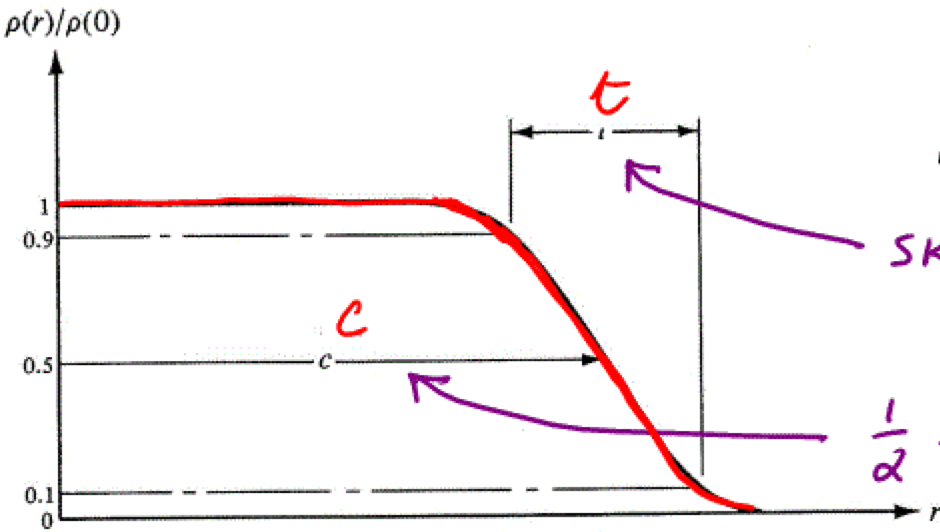


$\rho(r)$ — UNIFORM



$F(q^2)$ COMPLEX

STANDARD NUCLEAR MODEL



$$\rho(r) = \frac{1}{1 + e^{(r-c)a}}$$

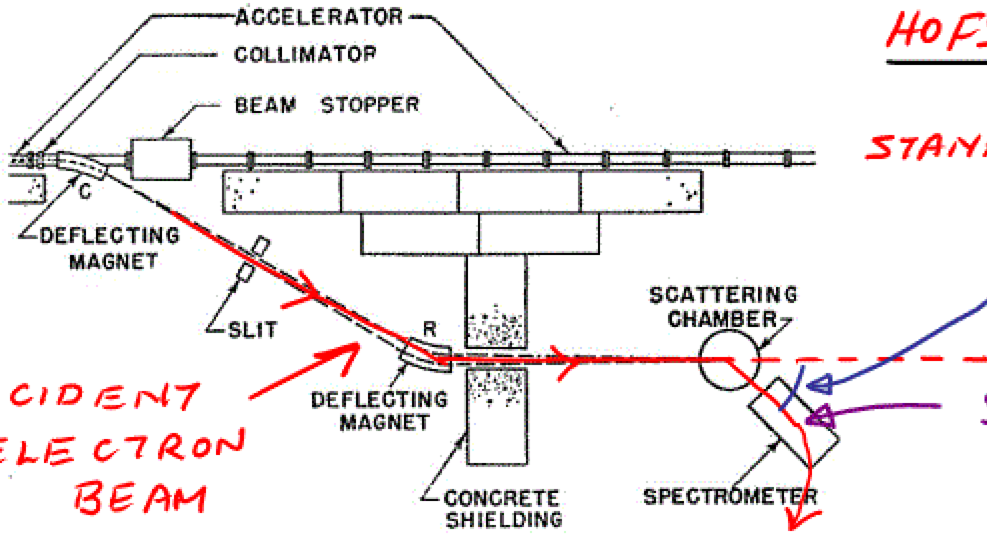
SKIN THICKNESS

$$t = (4 \ln 3)\, a$$

$\frac{1}{2}$ DENSITY RADIUS OF NUCLEUS

HOFSTADTER 1953

STANFORD 550 MeV LINAC



SCATTERING ANGLE

SCATTERED ELECTRON BEAM

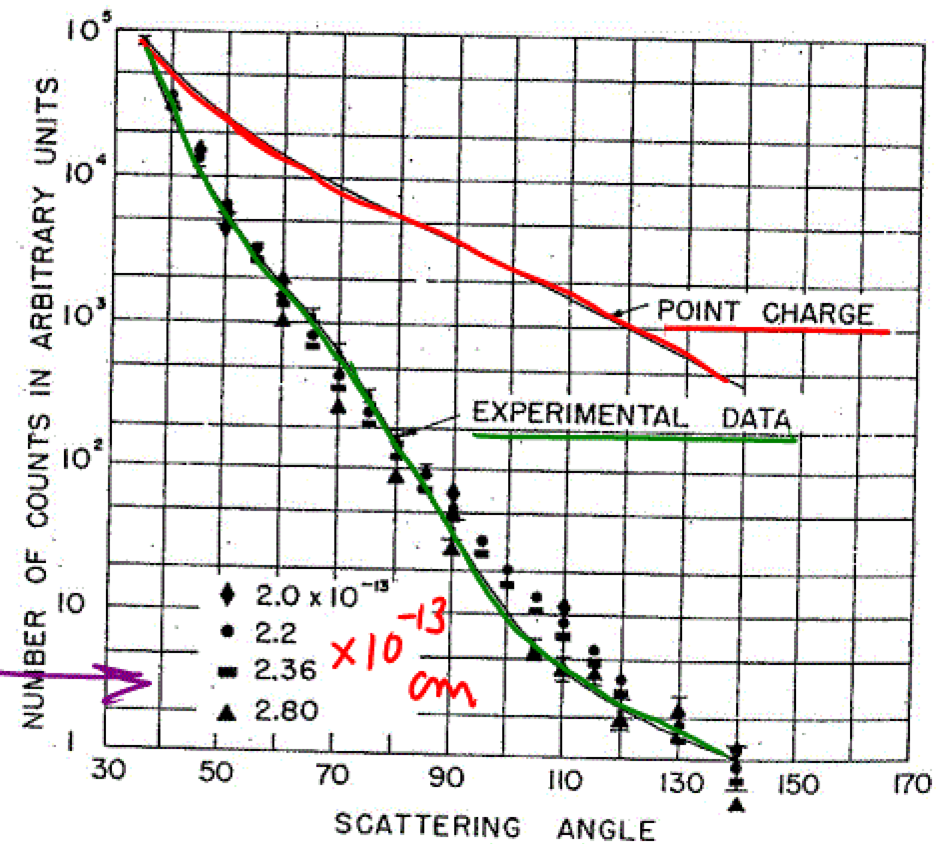
INCIDENT ELECTRON BEAM

NUCLEAR SIZE

$eAu \to eAu$

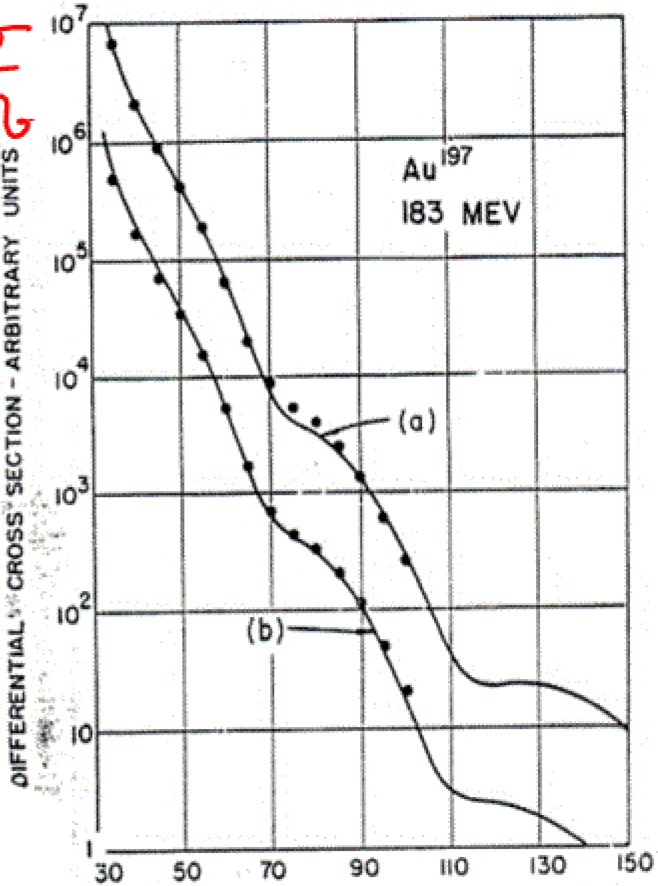
NUCLEUS NOT A POINT PARTICLE

NUCLEAR RADIUS HYPOTHESIZED IN GAUSSIAN FORM FACTOR

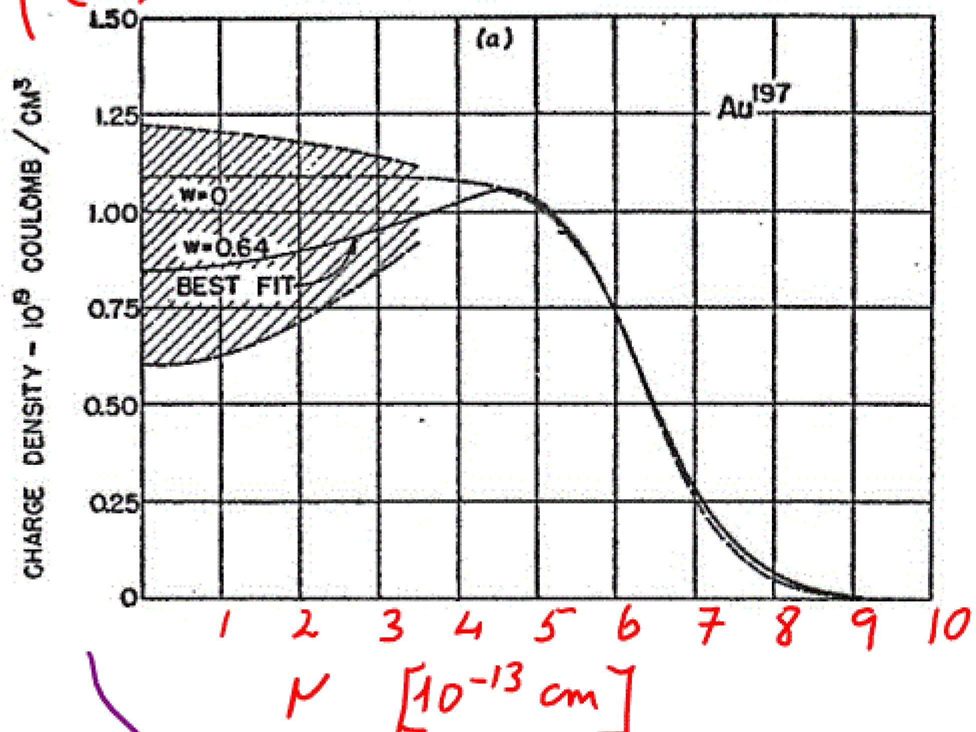


DETERMINATION OF CHARGE DISTRIBUTION IN NUCLEI

$\frac{d\sigma}{d\Omega}$



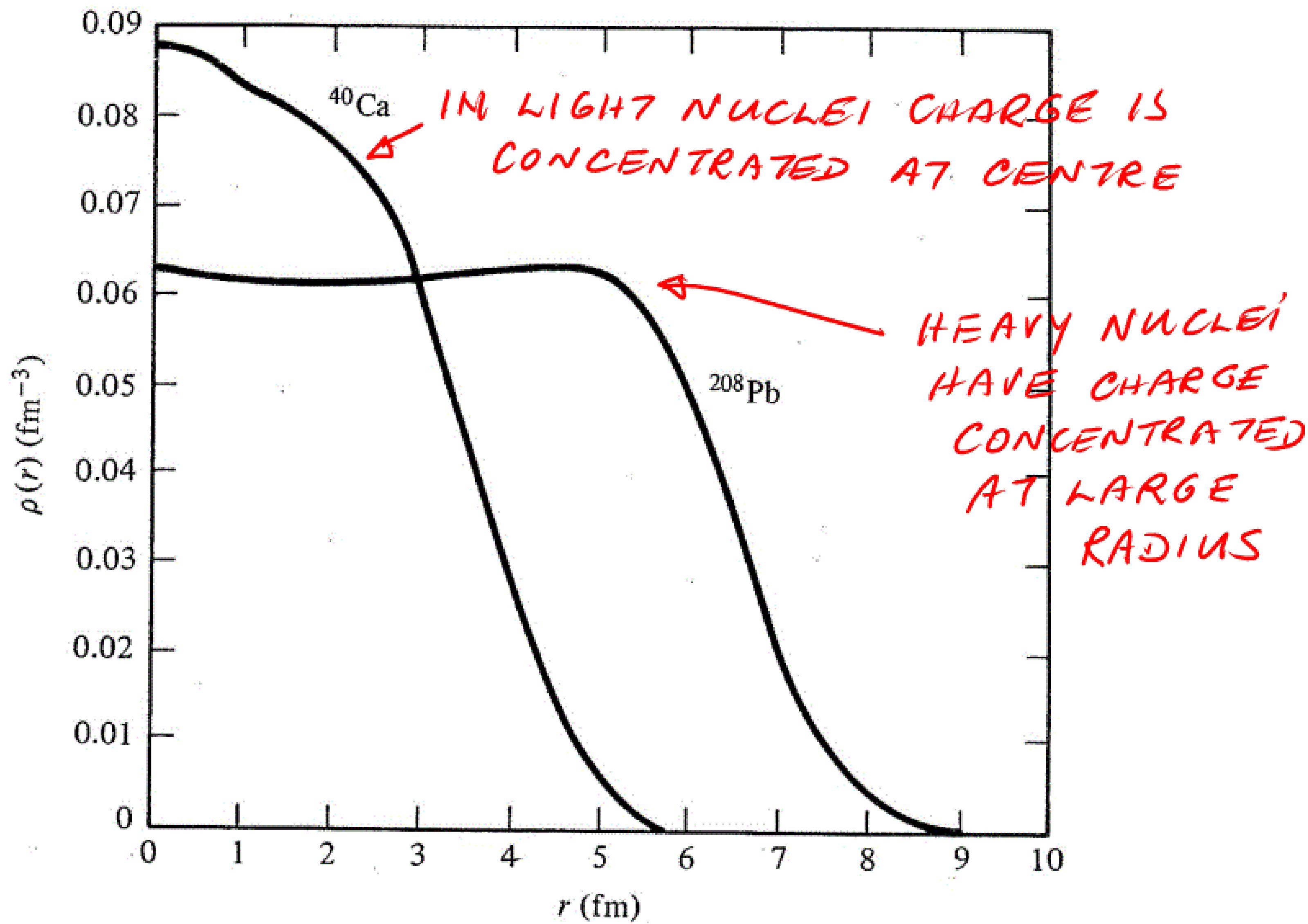
$\rho(r)$



$$\rho(r) = \frac{1}{(2\pi)^3} \int d^3q \, F(q^2) \, e^{-i\vec{q}\cdot\vec{r}/\hbar}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\substack{\text{MOTT} \\ \text{(POINT)}}} \left| F(q^2) \right|^2$$

COMPARISON OF LIGHT & HEAVY NUCLEI



MOTT SCATTERING

ELECTRON RELATIVISTIC, STILL NEGLECT p RECOIL $m_e \ll E \ll m_p$

$$\langle |M_{fi}|^2 \rangle = \frac{m_p^2 m_e^2 e^4}{p^4 \sin^4\left(\frac{\theta}{2}\right)} \left[1 + \overset{\sim 1}{\beta_e^2}\, \overset{\gg 1}{\gamma_e^2} \cos^2 \frac{\theta}{2} \right]$$

$K \sim 1 \longrightarrow J^e_{\uparrow\downarrow}$ AND $J^e_{\downarrow\uparrow} \longrightarrow 0$

$$\langle |M_{fi}|^2 \rangle = \frac{m_p^2 m_e^2 e^4}{E^4 \sin^4 \frac{\theta}{2}} \left[\gamma_e^2 \cos^2 \frac{\theta}{2} \right] \qquad E \simeq p$$

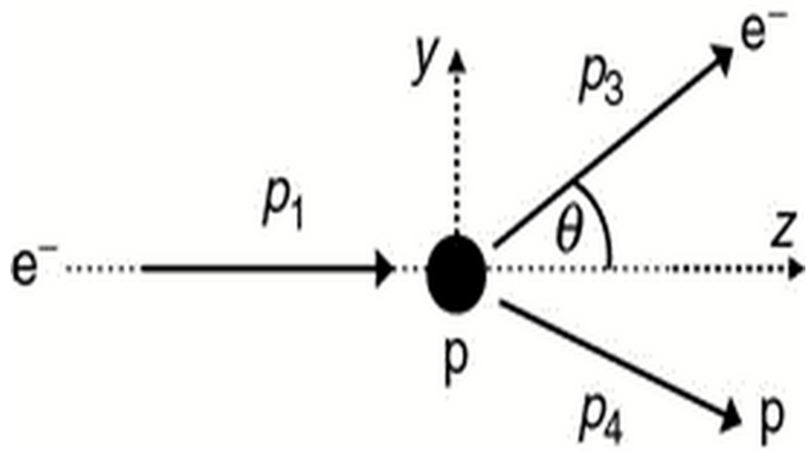
$$= \frac{m_p^2 m_e^2 e^4}{m_e^4 \gamma^4 \sin^4 \frac{\theta}{2}} \cdot \gamma_e^2 \cos^2 \frac{\theta}{2} = \frac{m_p^2 e^4}{\underbrace{m_e^2 \gamma_e^2}_{E^2} \sin^4 \frac{\theta}{2}} \cdot \cos^2 \frac{\theta}{2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{MOTT} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \cos^2 \frac{\theta}{2}$$

AGAIN NO SPIN-SPIN MAGNETIC TERM

RELATIVISTIC ep SCATTERING

IN $ep \to ep$ AT HIGH ENERGIES, PROTON RECOIL AND SPIN-SPIN (MAGNETIC) INTERACTION BETWEEN e AND p CANNOT BE NEGLECTED



$$p_1 = (E_1, 0, 0, E_1)$$

$$p_2 = (m_p, 0, 0, 0)$$

$$p_3 = (E_3, 0, E_3 \sin\theta, E_3 \cos\theta)$$

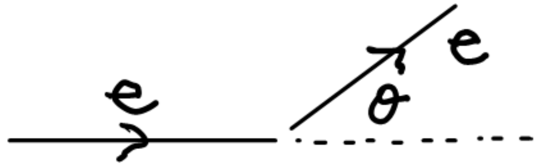
$$p_4 = (E_4, \vec{p}_4)$$

FROM 6.67 IN TEXT

$$\langle |M_{fi}|^2 \rangle = \frac{8e^4}{(p_1 - p_3)^4} \left[(p_1 \cdot p_2)(p_3 \cdot 4) + (p_1 \cdot p_4)(p_2 \cdot p_3) - m_p^2 (p_1 \cdot p_3) \right] \quad (7.22)$$

IN MOST EXPERIMENTS — FINAL p NOT OBSERVED

ELECTRON QUANTITIES — DEFINE KINEMATICS



ELIMINATE p_4 $\quad p_4 = p_1 + p_2 - p_3$

IN MATRIX ELEMENT (7.22) $\quad p_2 \cdot p_3 = E_3 m_p \quad p_1 \cdot p_2 = E_1 m_p$

$$p_1 \cdot p_3 = E_1 E_3 (1 - \cos\theta)$$

$$p_3 \cdot p_4 = p_3 \cdot p_1 + p_3 \cdot p_2 - p_3 \cdot p_3 = E_1 E_3 (1-\cos\theta) + E_3 m_p$$

$$p_1 \cdot p_4 = p_1 \cdot p_1 + p_1 \cdot p_2 - p_1 \cdot p_3 = E_1 m_p - E_1 E_3 (1-\cos\theta)$$

m_e NEGLECTED, $\quad E_3$ AND θ MEASURED

$$\langle |M_{fi}|^2 \rangle = \frac{8e^4}{(p_1 - p_3)^4} m_p E_1 E_3 \left[(E_1 - E_3)(1-\cos\theta) + m_p (1+\cos\theta) \right]$$

$$\langle |M_{fi}|^2 \rangle = \frac{8e^4}{(p_1-p_3)^4} 2m_p E_1 E_3 \left[(E_1-E_3)\sin^2\frac{\theta}{2} + m_p \cos^2\frac{\theta}{2} \right]$$

4-MOMENTUM TRANSFER FROM VIRTUAL γ $\quad$ $\cancel{m_e^2}$

$$q^2 = (p_1-p_3)^2 = p_1^2 + p_3^2 - 2p_1 \cdot p_3 \approx -2E_1E_3(1-\cos\theta)$$

$$q^2 = -4E_1E_3\sin^2\frac{\theta}{2} \rightarrow -q^2 = Q^2 = 4E_1E_3\sin^2\frac{\theta}{2}$$

VIRTUAL γ $\quad q = (\nu, \vec{q}) \qquad \nu = E_4 - E_2 = E_3 - E_1$

$$q^2 = (p_4-p_2)^2 = p_4^2 + p_2^2 - 2p_4 \cdot p_2$$

INVARIANT EVALUATE IN LAB

$$= 2m_p^2 - 2(\nu + m_p, \vec{p}_2)(m_p, 0)$$

$$= 2m_p^2 - 2\nu m_p - 2m_p^2 = -2\nu m_p$$

$$\nu = (E_3 - E_1) = \frac{-q^2}{2m_p} = \frac{Q^2}{2m_p} \rightarrow$$ ENERGY OF VIRTUAL γ

PUTTING INTO $\mathcal{M}_{fi}$ $\quad q^2 = -4E_1E_3\sin^2\frac{\theta}{2}$, $(E_1 - E_3) = \frac{Q^2}{2m_p}$

$$\langle|M_{fi}|^2\rangle = \frac{m_p^2 e^4}{E_1E_3\sin^4\frac{\theta}{2}}\left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right]$$

USING USUAL EXPRESSION FOR $\frac{d\sigma}{d\Omega}$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4\theta/2}\,\frac{E_3}{E_1}\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right)$$

RELATIVISTIC SCATTERING OF ELECTRONS OFF PROTONS AT REST (LIQUID HYDROGEN)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2} \sin^2\frac{\theta}{2} \right)$$

E_3, Q^2, θ → ONLY ONE INDEPENDENT

$$E_3 = \frac{E_1 m_p}{m_p + E_1(1-\cos\theta)}$$

MEASURE ANGLE AND ENERGY OF SCATTERED ELECTRON → CONFIRM ELASTIC SCATTERING

$Q^2 \ll m_p^2$, $E_3 \approx E_1$

REDUCES TO MOTT

ABOVE DIFFERS FROM MOTT BY

$\frac{E_3}{E_1}$ → ENERGY LOSS OF ELECTRON

$\sin^2\frac{\theta}{2}$ → SPIN - SPIN MAGNETIC INTERACTION

SIZE AND SHAPE OF THE PROTONS

- MEASURED IN SAME WAY AS NUCLEI
- ELASTIC $ep \rightarrow ep$ SCATTERING $\rightarrow \frac{d\sigma}{d\Omega}$
- COMPARE $\frac{d\sigma}{d\Omega} \rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{\text{POINT}} \longrightarrow F(q^2)$ FORM FACTOR
- BOTH ELECTRONS & PROTONS HAVE SPIN AND ARE RELATIVISTIC $\rightarrow$ GENERALIZE MOTT CROSS SECTION
- IN ADDITION TO CHARGE DENSITY DISTRIBUTION

RELATIVITY $\longrightarrow$ MAGNETIC MOMENT DISTRIBUTION

NUCLEUS $\longrightarrow$ 1 FORM FACTOR

PROTON $\longrightarrow$ 2 FORM FACTORS

SPIN 1/2 PROTON HAS TWO FORM FACTORS

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{MOTT} \left[\frac{G_E^2 + b\, G_M^2}{1+b} + 2b\, G_M^2 \tan^2\frac{\theta}{2} \right]$$

$-q^2/2m_N^2$

SCATTERING ANGLE

$G_{E,M} \equiv G_{E,M}(q^2)$

ROSENBLUTH CROSS SECTION

ELECTRIC FORM FACTOR

$$G_E(q^2=0) = \frac{Q}{e}$$

$G_E(0)$ PROTON $= 1$

$G_E(0)$ NEUTRON $= 0$

MAGNETIC FORM FACTOR

$$G_M(q^2=0) = \frac{\mu}{\mu_N}$$

NUCLEAR MAGNETON

$G_M(0)_{PROTON} = 2.79$

$G_M(0)_{NEUTRON} = -1.91$

ASIDE ON G_E AND G_M

$F(\bar{q}^2)$ FUNCTION OF 3-MOMENTUM
CAN DO FOURIER TRANSFORM

$G(Q^2)$ FUNCTION OF 4-MOMENTUM
CANNOT DO FOURIER TRANSFORM

CANNOT INTERPRET SIMPLY IN TERMS OF DISTRIBUTION OF CHARGE AND MAGNETIC MOMENT.

$$Q^2 = -q^2 = \bar{q}^2 - (\underbrace{E_1 - E_3}_{})^2 \qquad E_1 - E_3 = \frac{Q^2}{2m_p}$$

$$\bar{q}^2 = Q^2\left(1 + \frac{Q^2}{4m_p^2}\right)$$

FOR $Q^2 \ll 4m_p^2$

$$Q^2 \approx \bar{q}^2$$

$$Q^2 \approx \vec{q}^{\,2} \longrightarrow G_E(Q^2) \approx G_E(\vec{q}^{\,2})$$
$$G_M(Q^2) \approx G_M(\vec{q}^{\,2})$$

$$G_E(Q^2) \approx \int e^{i\vec{q}\cdot\vec{r}} \rho(\vec{r})\, d^3\vec{r}$$

$$G_M(Q^2) \approx \int e^{i\vec{q}\cdot\vec{r}} \mu(\vec{r})\, d^3\vec{r}$$

MAGNETIC MOMENT

POINT DIRAC PARTICLE $\vec{\mu} = \frac{q}{m}\vec{S} \xrightarrow{\text{MEASURE}} \vec{\mu} = 2.79 \frac{e}{m_p} \vec{S}$

PROTON IS EXTENDED OBJECT

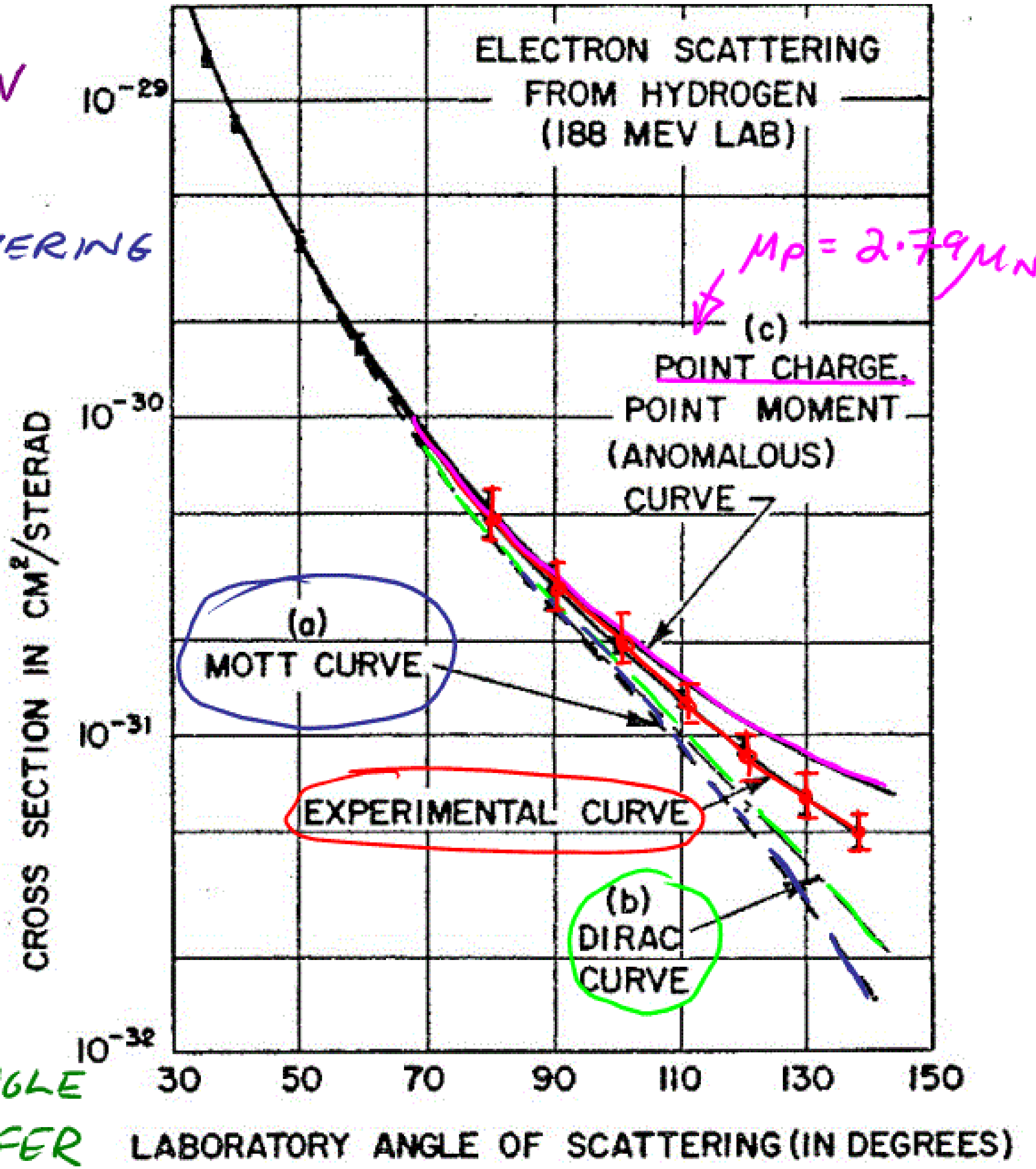
PROTON SHAPE & SIZE HOFSTADTER (AGAIN)

$ep \to ep$ 188 MeV

ELECTRIC & MAGNETIC SCATTERING

BEST FIT GIVES

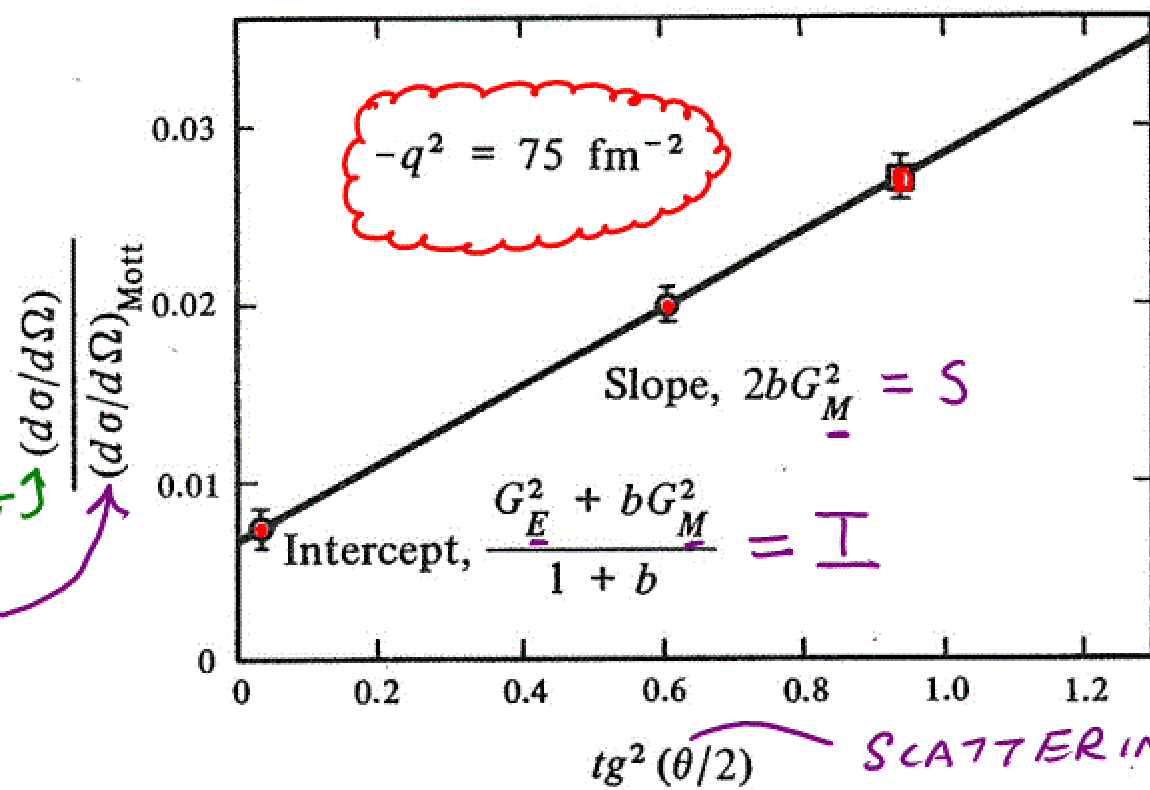
$R_{PROTON} = 7 \times 10^{-14}\ cm$



LARGE SCATTERING ANGLE
LARGE MOMENTUM TRANSFER

EXTRACTING ELECTRIC & MAGNETIC FORM FACTORS

$R =$



EXPERIMENT

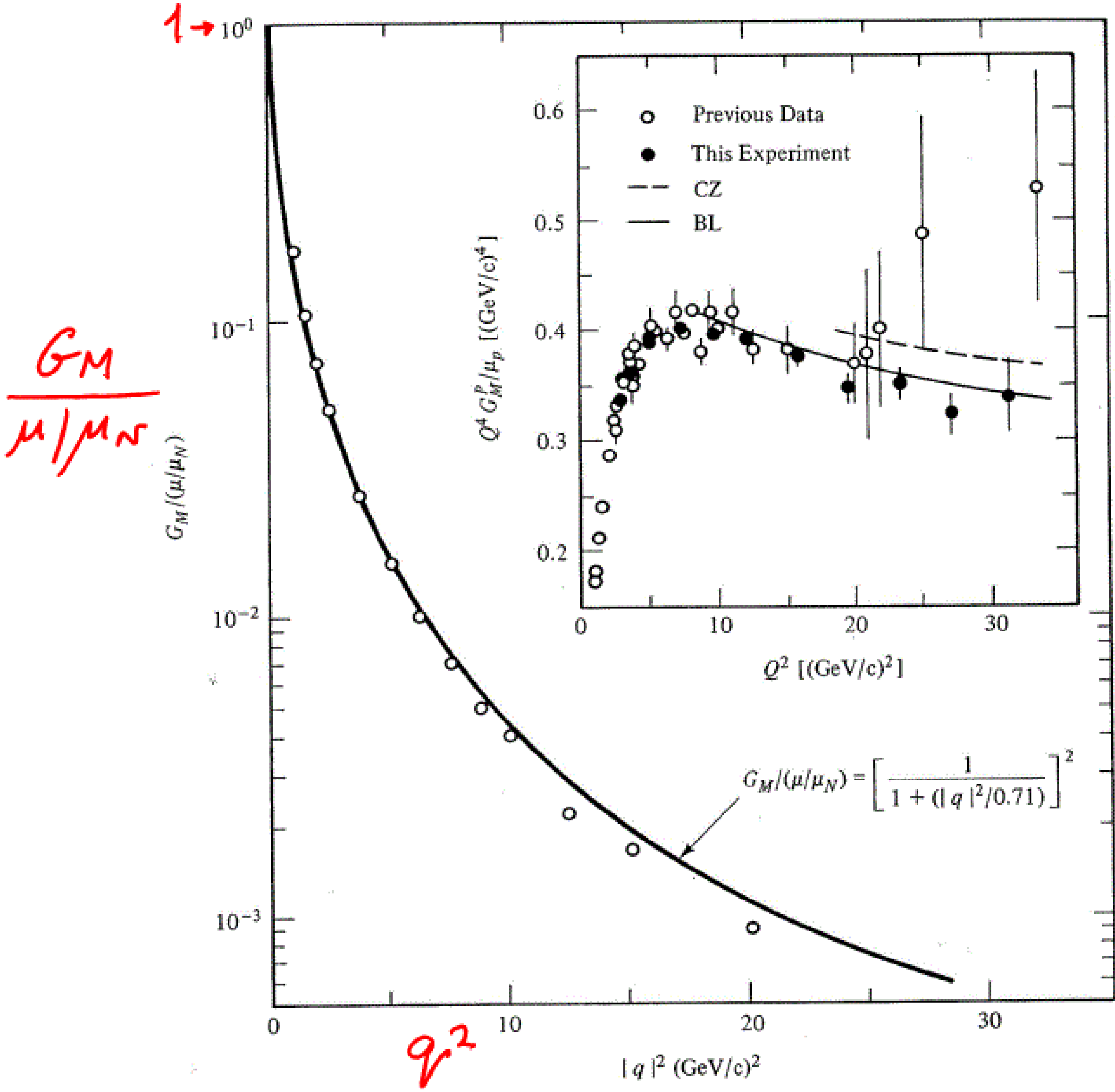
POINT CROSS SECTION

SCATTERING ANLE

AT FIXED q^2 MEASURE $R = I + S \tan^2 \frac{\theta}{2}$

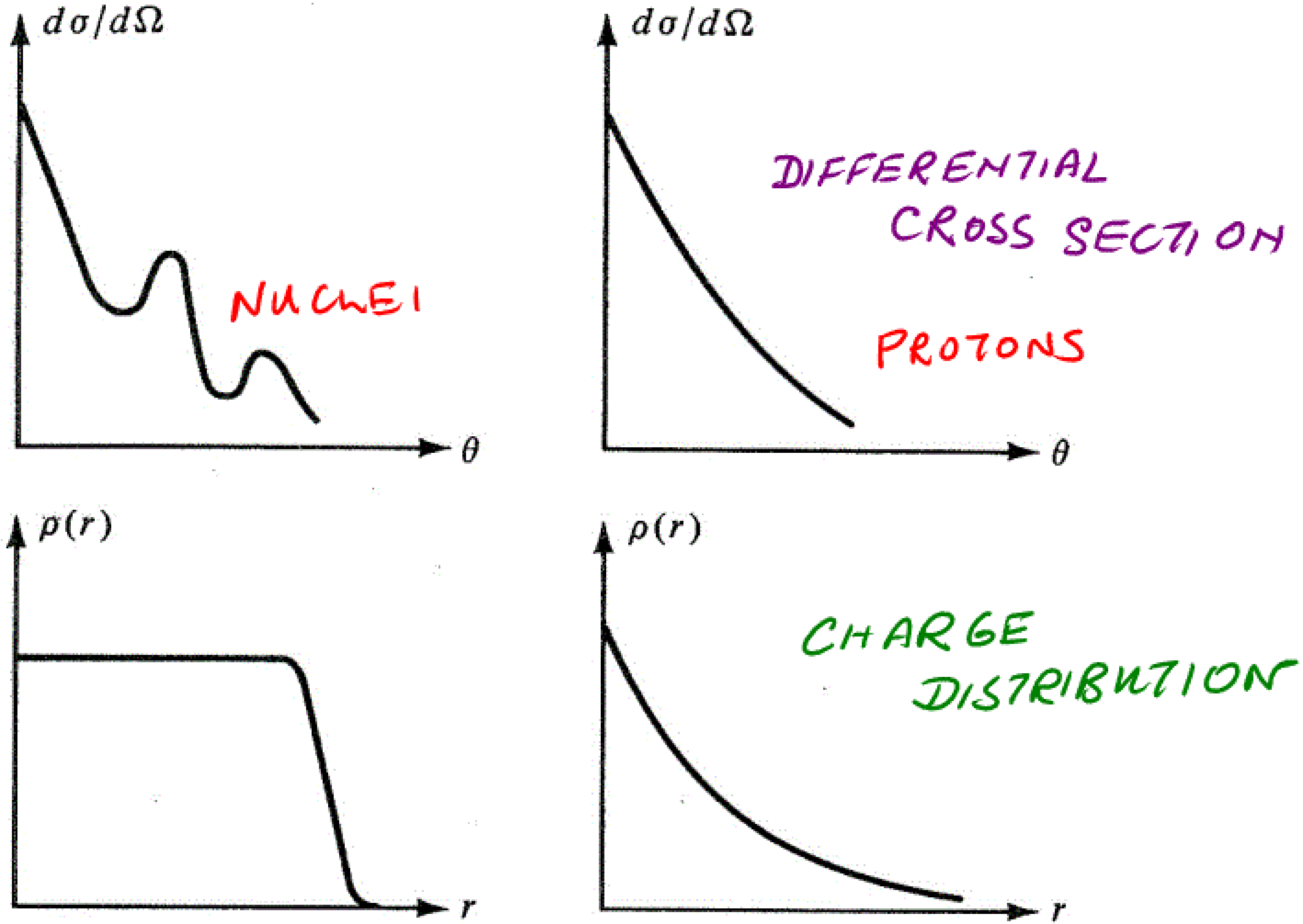
VARY q^2 BY VARYING INCIDENT OR RECOIL ENERGY, AND SCANNING IN ANGLE θ

MAGNETIC DIPOLE MOMENT DISTRIBUTION



BOTH ELECTRIC CHARGE & MAGNETIC MOMENT DISTRIBUTIONS IN PROTON ARE DIFFUSE

COMPARE SHARP EDGE OF NUCLEI



NUCLEI NUCLEI **NUCLEONS** PROTONS

SHARP EDGE

↓

$\frac{d\sigma}{d\Omega}$ → DIFFRACTION PATTERN

CHARGE DISTRIBUTION DIFFUSE

↓

$\frac{d\sigma}{d\Omega}$ → SMOOTH