INELASTIC ELECTRON – PROTON SCATTERING

UP TO NOW CONSIDERED — ELASTIC SCATTERING

ROSENBLUTH CROSS SECTION → ELASTIC SCATTERING

→ THE PROTON REMAINS A PROTON

HAVE ESTABLISHED THAT PROTON HAS STRUCTURE

INELASTIC SCATTERING POSSIBLE

$ep \to ep$ ELASTIC

EXCITE PROTON STRUCTURE → $ep \to e\Delta^+$, $\Delta^+ \to p\pi^0$ } INELASTIC "EXCLUSIVE"

$ep \to en\pi^+$ — "

$ep \to Xe$ — INELASTIC "INCLUSIVE"

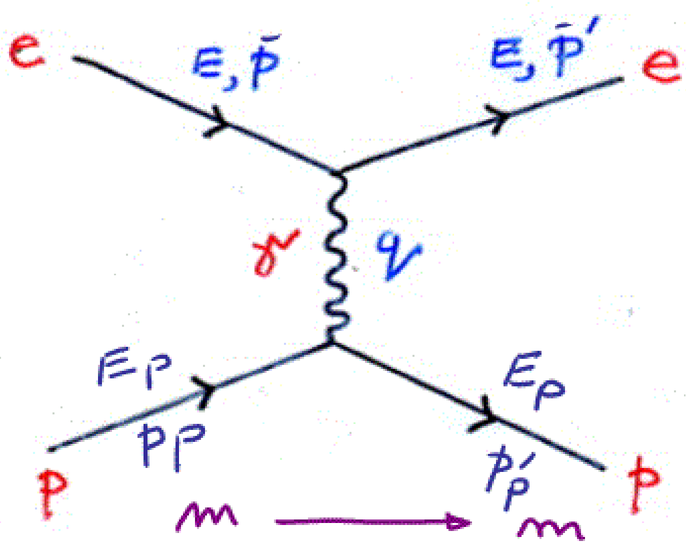
JUST MEASURE ELECTRON

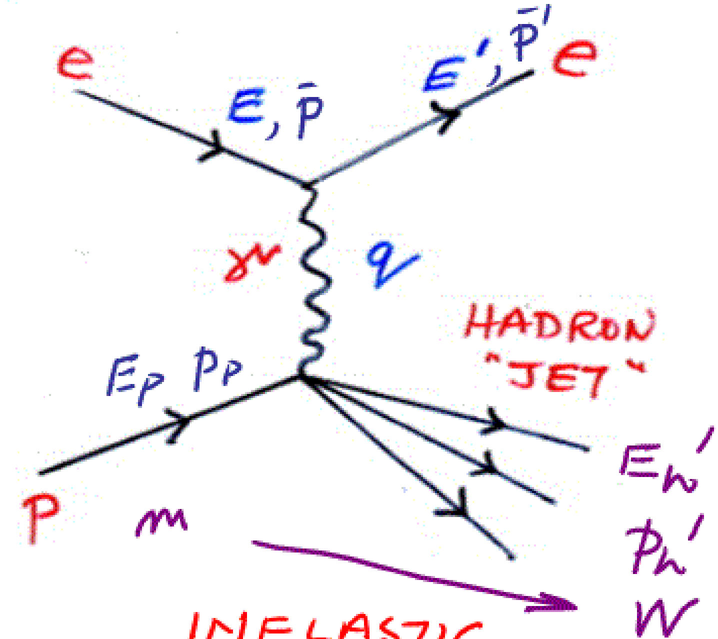
X: ANY HADRONIC SYSTEM FROM BREAK UP OF PROTON

INELASTIC ep SCATTERING

POWERFUL TOOL TO LOOK AT PROTON STRUCTURE

NUCLEUS → NUCLEONS → QUARKS





ELASTIC – TARGET NOT EXCITED / FRAGMENTED

INELASTIC

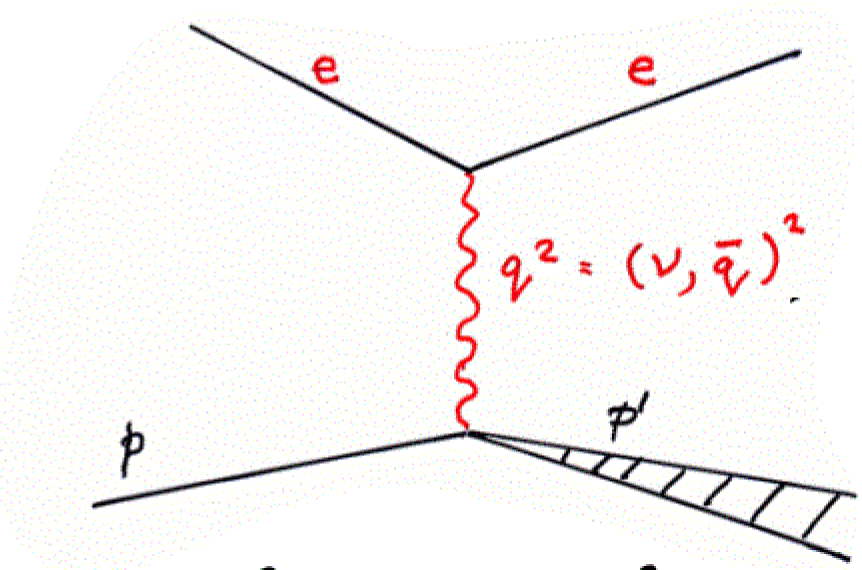
$\nu = E - E'$ ENERGY TRANSFER – INELASTICITY

$q = (\nu, (\bar{p} - \bar{p}'))$

$\nu = \dfrac{|q^2|}{2m_p}$ ← 4-MOMENTUM TRANSFER ← TARGET MASS

$q^2 = \nu^2 - p_h'^2$

ELASTIC & INELASTIC SCATTERING



MASS OF PROTON
ELASTIC SCATTERING

INVARIANT MASS OF
FINAL HADRONIC SYSTEM
INELASTIC SCATTERING

m_p OR W_H

$$q^2 = (p' - p)^2$$

$$q^2 = p'^2 + p^2 - 2p' \cdot p$$

INVARIANT SO EVALUATE
IN LAB FRAME

$$= W_H^2 + m_p^2 - 2(\nu + m_p, \bar{p}') \cdot (m_p, 0)$$

$$= W_H^2 - m_p^2 - 2\nu m_p$$

$$W_H^2 = q^2 + m_p^2 + 2\nu m_p$$

2 INDEPENDENT
VARIABLES FOR
INELASTIC

$$q = (\nu, \bar{q})$$

GENERALLY $W_H^2 = q^2 + m_p^2 + 2\nu m_p$

FOR ELASTIC SCATTERING $W_H^2 = m_p^2$

$$m_p^2 = q^2 + m_p^2 + 2\nu m_p$$

$$q^2 = -2\nu m_p$$

ONLY ONE VARIABLE ν

LOOKING BACK AT DIAGRAM

$$q^2 = (\nu, \bar{q})(\nu, \bar{q}) = -2\nu m_p$$

SO $\nu^2 - \bar{q}^2 = -2\nu m_p \rightarrow \bar{q}^2 = \nu^2 + 2\nu m_p.$

IN LIMIT $\nu \gg m_p \rightarrow \bar{q}^2 \simeq \nu^2$

THEN 4-MOMENTUM XFER $\rightarrow q = (\nu, \bar{q}) \approx (\nu, \nu)$

FOR HIGH ENERGY ELASTIC SCATTERING $q^2 \approx \nu^2 - \nu^2 \approx 0$

REPRISE ON SCATTERING CROSS SECTIONS

RUTHERFORD: NON RELATIVISTIC, SPINLESS POINT TARGET & BEAM

$$\left.\frac{d\sigma}{d\Omega}\right|_R = \frac{4m^2\,(z\,Z\,e^2)}{q^4}$$

z = 1 FOR ELECTRON

q: 3 MOMENTUM

MOTT: RELATIVISTIC SPIN $\frac{1}{2}$ ELECTRON SCATTERING OFF SPINLESS POINT TARGET

$$\left.\frac{d\sigma}{d\Omega}\right|_{MOTT} = \frac{4E^2\,(Ze^2)^2}{q^4}\left(1-\beta^2\sin^2\frac{\theta}{2}\right)$$

q: 4-MOMENTUM

FORM FACTOR: ELECTRON SCATTERING FROM AN EXTENDED TARGET

$$\left.\frac{d\sigma}{d\Omega}\right|_{EXTENDED} = \left|F(q^2)\right|^2\left(\frac{d\sigma}{d\Omega}\right)_{MOTT}$$

MOTT: POINT

ELASTIC SCATTERING OF ELECTRONS & PROTONS:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ro} = \left(\frac{d\sigma}{d\Omega}\right)_{MOTT} \left[\frac{G_E^2 + b\,G_M^2}{1+b} + 2b\,G_M^2 \tan^2\frac{\theta}{2} \right]$$

COULD WRITE $\frac{d\sigma}{dq^2} \rightarrow$ ONE SCATTERING VARIABLE

INELASTIC ELECTRON PROTON SCATTERING

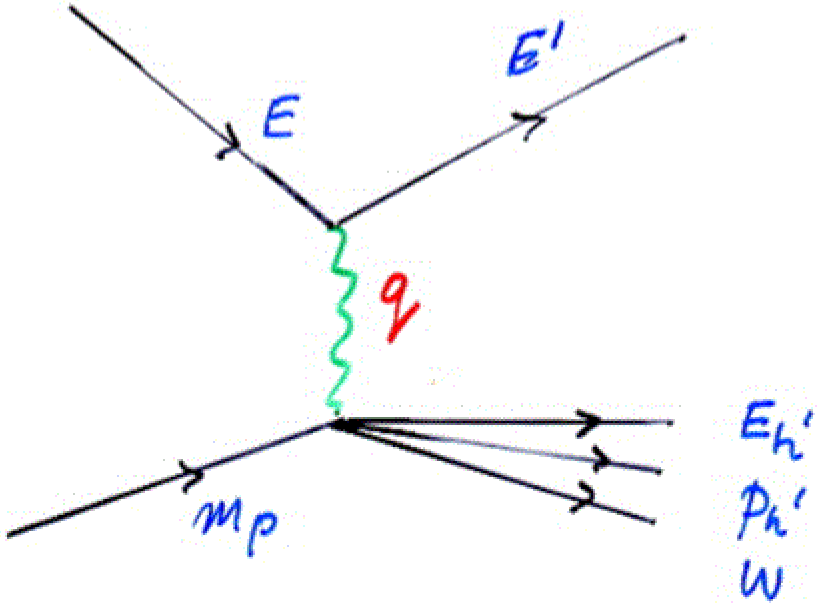
$$\left(\frac{d^2\sigma}{dq^2 d\nu}\right)_{INEL} = \left(\frac{d^2\sigma}{dq^2 d\nu}\right)_{POINT} \left[W_2(q^2,\nu) + 2W_1(q^2,\nu)\, b \tan^2\frac{\theta}{2} \right]$$

TWO SCATTERING VARIABLES

TWO FORM FACTORS
FUNCTIONS OF TWO VARIABLES

STRUCTURE FUNCTIONS

BACK TO INELASTIC SCATTERING!



PROTON HAS ENERGY TRANSFERED TO IT

q^2 HAS 2 COMPONENTS

ν ENERGY

$\bar{q}$ 3-MOMENTUM

$q^2 = \nu^2 - |\bar{q}|^2$ → VARY INDEPENDENTLY

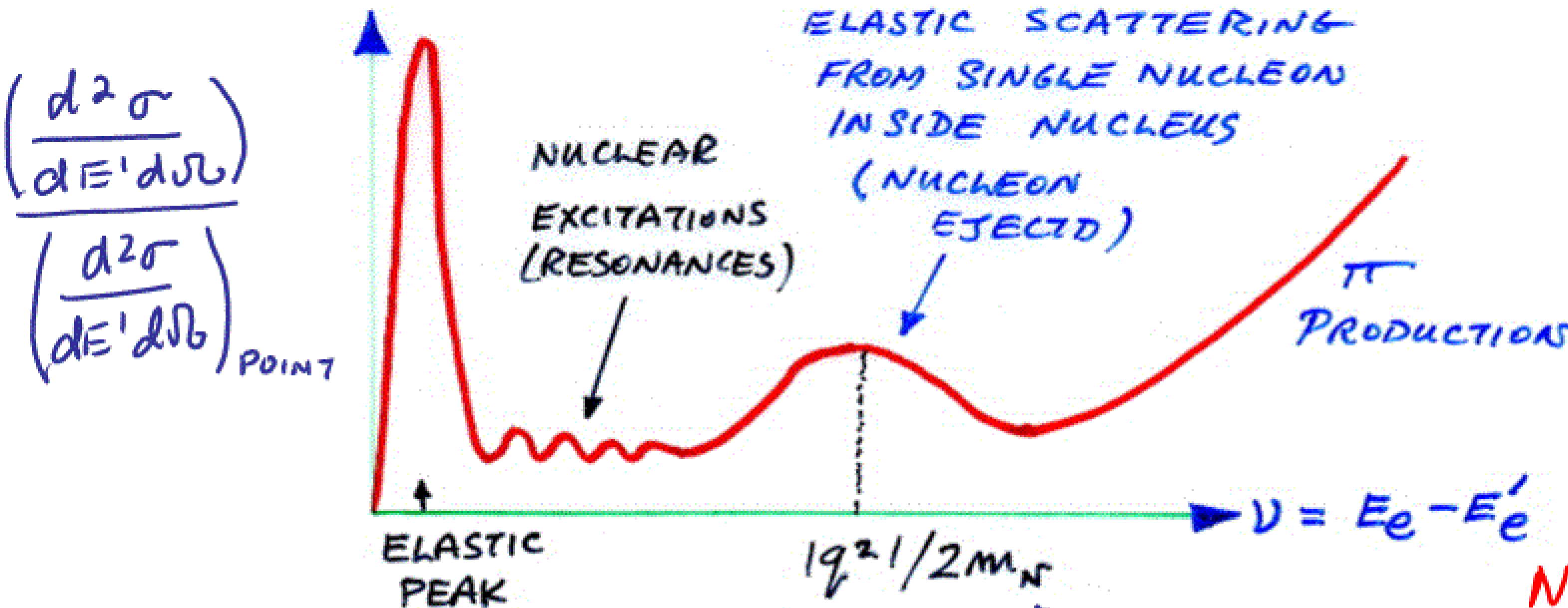
NOW HAVE 2 VARIABLES IN SCATTERING KINEMATICS

FORM FACTORS DEPEND ON ⟶ q^2, ν

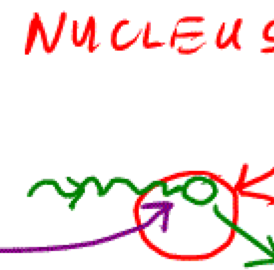
$F(q^2) \longrightarrow W_1(q^2, \nu) \quad W_2(q^2, \nu)$ → STRUCTURE FUNCTIONS

ELASTIC DEEP INELASTIC

REPRISE ON ELECTRON SCATTERING FROM NUCLEUS



QUASI-ELASTIC SCATTERING



- ENERGY TAKEN UP BY SINGLE NUCLEON
- FERMI MOTION OF NUCLEON IN NUCLEUS BROADENS QUASI ELASTIC PEAK

$$R \cdot p_{FERMI} \sim \hbar \rightarrow p_F \sim p_{NUCLEON} \sim \frac{\hbar}{R} \sim 100\ \mathrm{MeV}/c$$

$$\nu = |q^2| / 2m_N$$

- NUCLEON INSIDE NUCLEUS ABSORBS 4-MOMENTUM q^2 FROM VIRTUAL γ^*
- INITIAL 4-MOMENTUM OF NUCLEON p_N

$$(p_N + q)^2 = m_N^2$$

$$\underbrace{p_N^2}_{m_N^2} + q^2 + 2p_N \cdot q = m_N^2$$

$$|q^2| = 2p_N \cdot q = 2(m_N, 0)(\nu, \vec{q})$$

$$= 2m_N \nu$$

$$\nu = \frac{|q^2|}{2m_N}$$

DEEP INSIDE THE NUCLEON

STANFORD LINEAR ACCELERATOR

BASED ON MICROWAVE TECANOLOGY USED BY HOFSTADTER

20 GeV/c

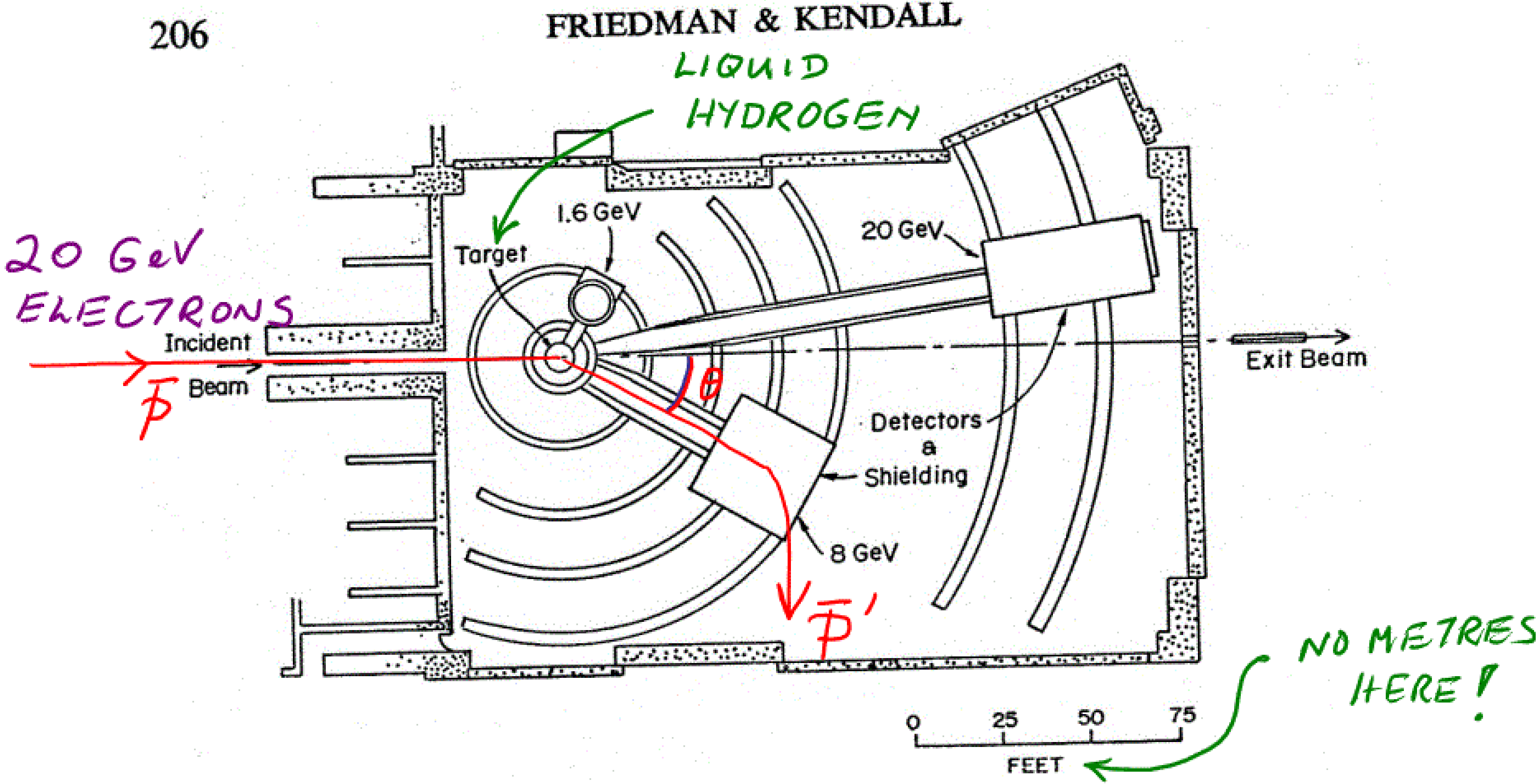
↳ 50 GeV/c

2 MILES LONG

$114 × 10^6$ IN 1965

LHC 10^{10} IN 2005

SLAC SPECTROMETER

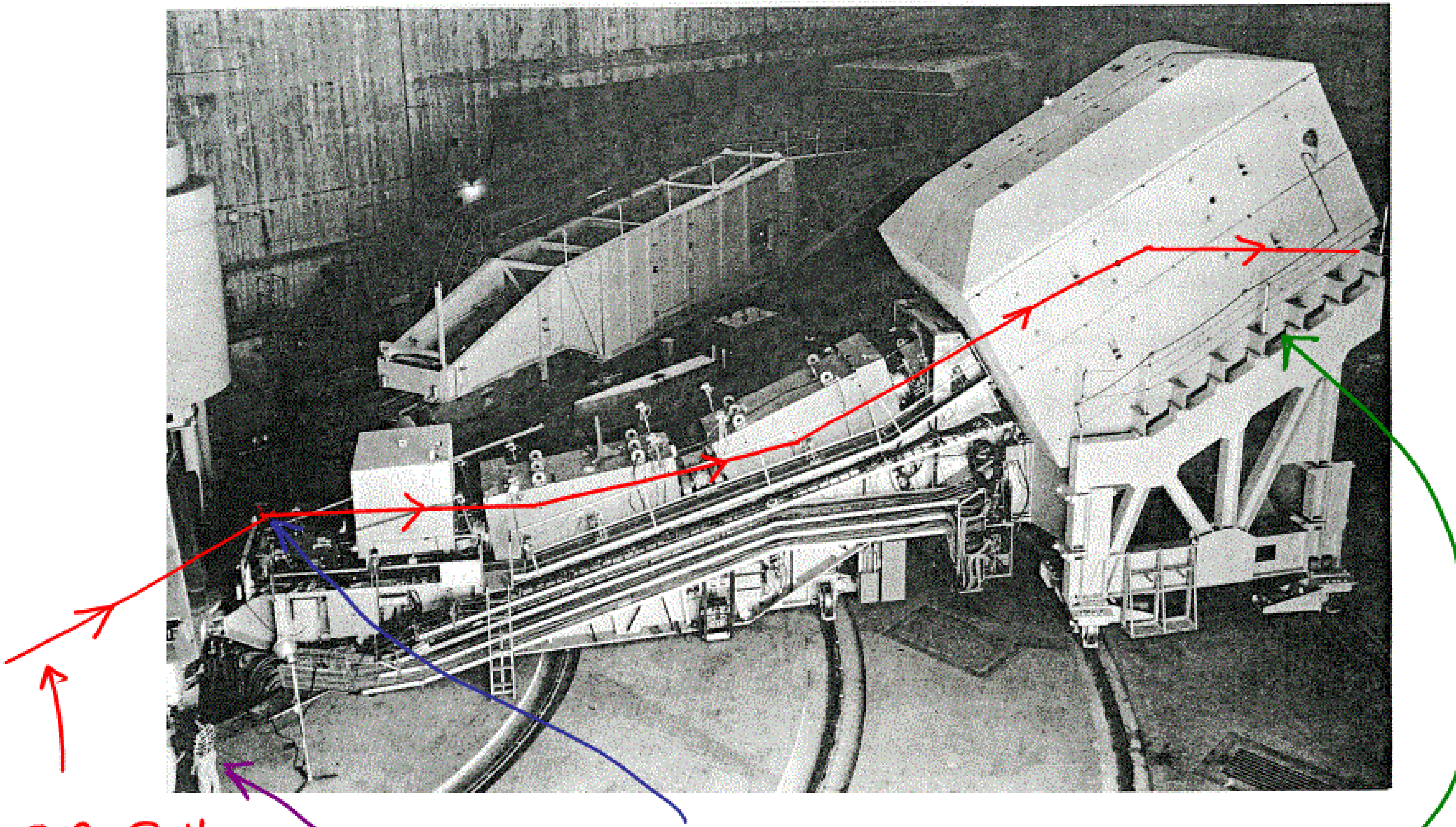


MEASURE SCATTERING ANGLE θ

AND FINAL STATE MOMENTUM $\vec{p}\,'$

OF SCATTERED ELECTRONS

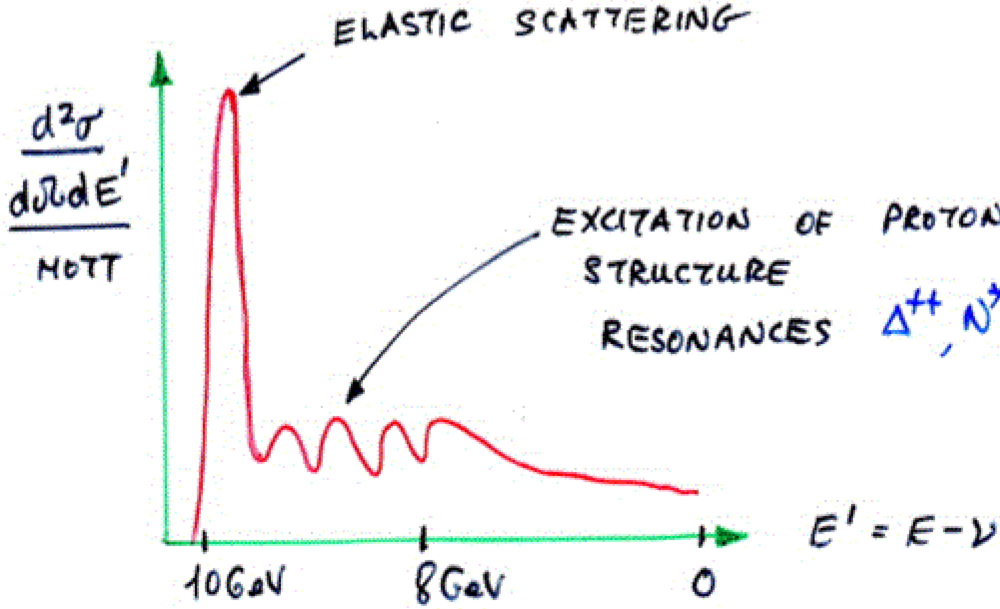
SLAC SPECTROMETER



INELASTIC SCATTERING RESULTS

ELASTIC SCATTERING FROM NUCLEI → NUCLEAR STRUCTURE

INELASTIC SCATTERING FROM PROTONS → PROTON STRUCTURE



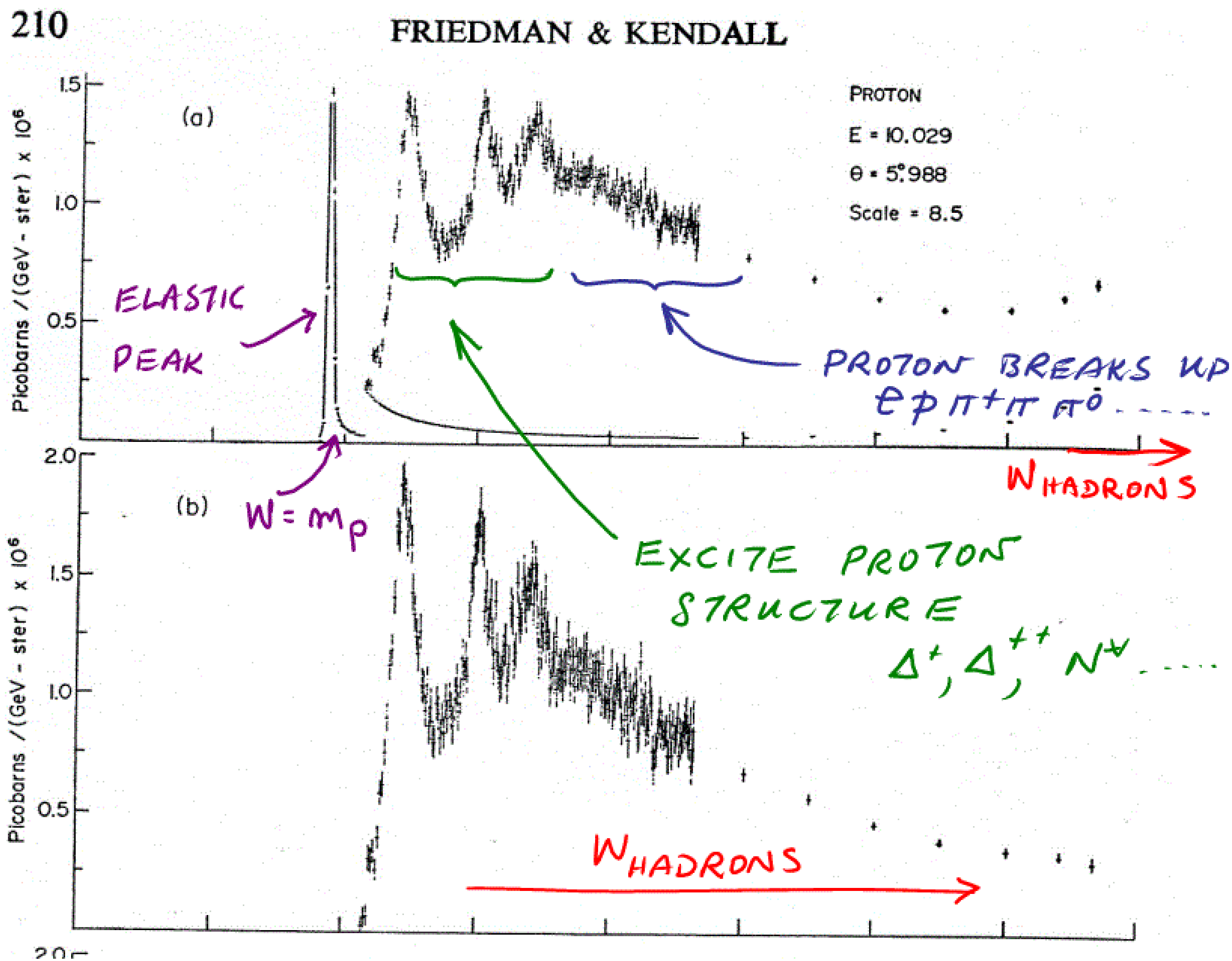
VERY SIMILAR TO SCATTERING FROM NUCLEI

EVIDENCE OF PROTON STRUCTURE

NO QUASI-ELASTIC PEAK. WHATEVER PROTON STRUCTURE IS → CANNOT BE KNOCKED OUT

CONFINED INSIDE PROTON

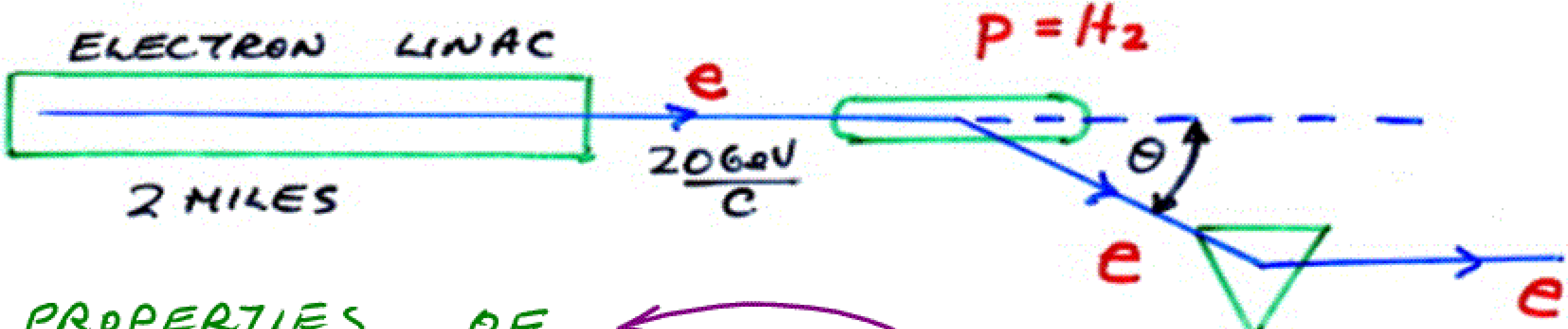
SLAC DATA $ep \to eX$



DEEP INELASTIC SCATTERING & DISCOVERY OF QUARKS

EXPRIMENT — R. E. TAYLOR — BORN MEDICINE HAT - ALBERTA
NOBEL PRIZE 1989

THEORETICAL INTER PRETATIONS — RICHARD FEYNMAN
QUARK - PARTON MODEL



PROPERTIES OF RECOILING HADRONIC SYSTEM

ANALYZE MOMENTUM OF SCATTERED ELECTRON

$$\left.\begin{aligned} E_h &= \nu + m_p \\ \bar{p}_h &= \bar{p}_e - \bar{p}_e' \end{aligned}\right\}$$

$$W_h^2 = q^2 + m_p^2 + 2\nu m_p^2$$

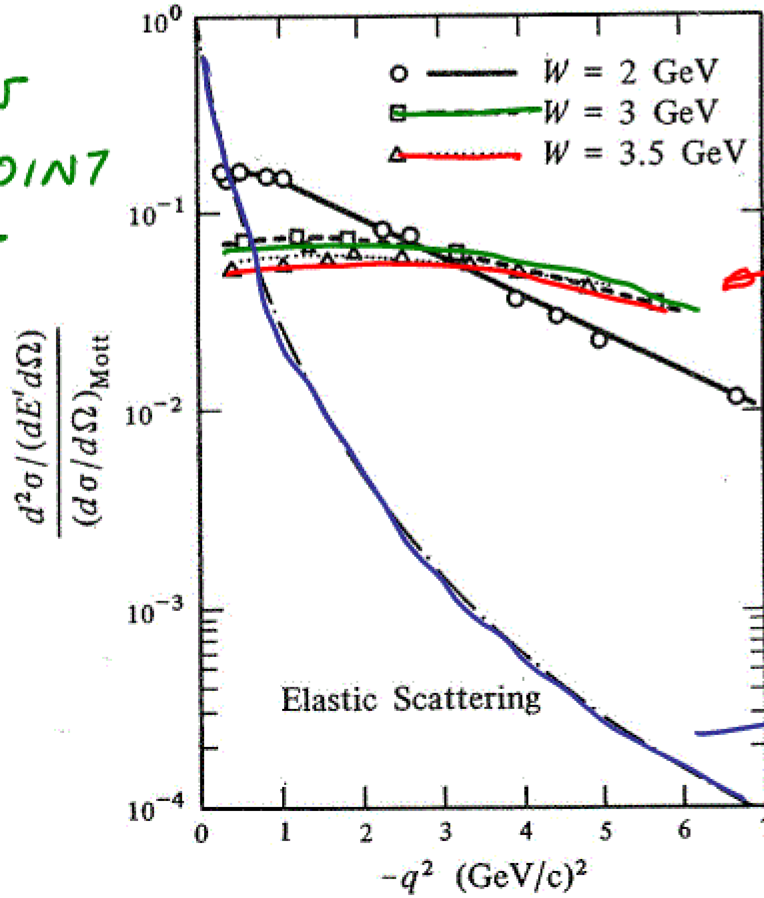
INVARIANT MASS OF RECOILING HADRONIC SYSTEM

DISCOVERY OF PARTONS — SCALING

CROSS SECTION COMPARD TO POINT CROSS SECTIONS

RUN @ FIXED W^2

MEASURE $\frac{d\sigma}{dq^2}$



MASS OF RECOILING HADRONIC SYSTEM

INDEPENDENT OF q^2

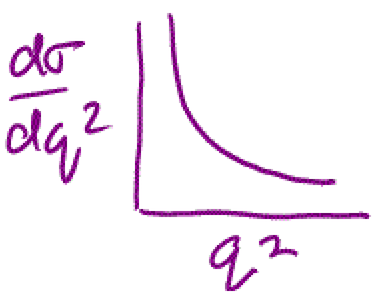
NO FORM FACTOR

POINT TARGET

$W = m_p$

FORM FACTOR

ELASTIC



FORM FACTOR

EXTENDED TARGET

INELASTIC

- NO FORM FACTOR
- POINT TARGET
- RUTHERFORD !

OBSERVED BEHAVIOR OF HIGHLY INELASTIC ELECTRON-PROTON SCATTERING

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DICK TAYLOR

(Received 22 August 1969)

Results of electron-proton inelastic scattering at 6° and 10° are discussed, and values of the structure function W_2 are estimated. If the interaction is dominated by transverse virtual photons, νW_2 can be expressed as a function of $\omega = 2M\nu/q^2$ within experimental errors for $q^2 > 1$ (GeV/c)^2 and $\omega > 4$, where ν is the invariant energy transfer and q^2 is the invariant momentum transfer of the electron. Various theoretical models and sum rules are briefly discussed.

In a previous Letter,[1] we have reported experimental results from a Stanford Linear Accelerator Center–Massachusetts Institute of Technology study of high-energy inelastic electron-proton scattering. Measurements of inelastic spectra, in which only the scattered electrons were detected, were made at scattering angles of 6° and 10° and with incident energies between 7 and 17 GeV. In this communication, we discuss some of the salient features of inelastic spectra in the deep continuum region.

One of the interesting features of the measurements is the weak momentum-transfer dependence of the inelastic cross sections for excitations well beyond the resonance region. This weak dependence is illustrated in Fig. 1. Here we have plotted the differential cross section divided by the Mott cross section, $(d^2\sigma/d\Omega dE')/(d\sigma/d\Omega)_{\rm Mott}$, as a function of the square of the four-momentum transfer, $q^2 = 2EE'(1-\cos\theta)$, for constant values of the invariant mass of the recoiling target system, W, where $W^2 = 2M(E-E') + M^2 - q^2$. E is the energy of the incident electron, E' is the energy of the final electron, and θ is the scattering angle, all defined in the laboratory system; M is the mass of the proton. The cross section is divided by the Mott cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} = \frac{e^4}{4E^2}\frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}$$

in order to remove the major part of the well-known four-momentum transfer dependence arising from the photon propagator. Results from both 6° and 10° are included in the figure for each value of W. As W increases, the q^2 dependence appears to decrease. The striking difference between the behavior of the inelastic and elastic cross sections is also illustrated in Fig. 1, where the elastic cross section, divided by the Mott cross section for $\theta = 10°$, is included. The q^2 dependence of the deep continuum is also consider-

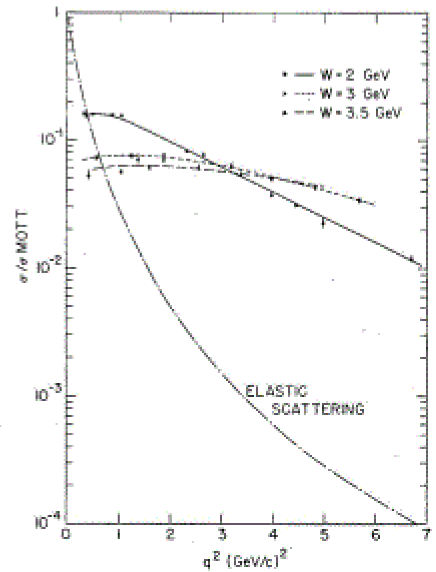
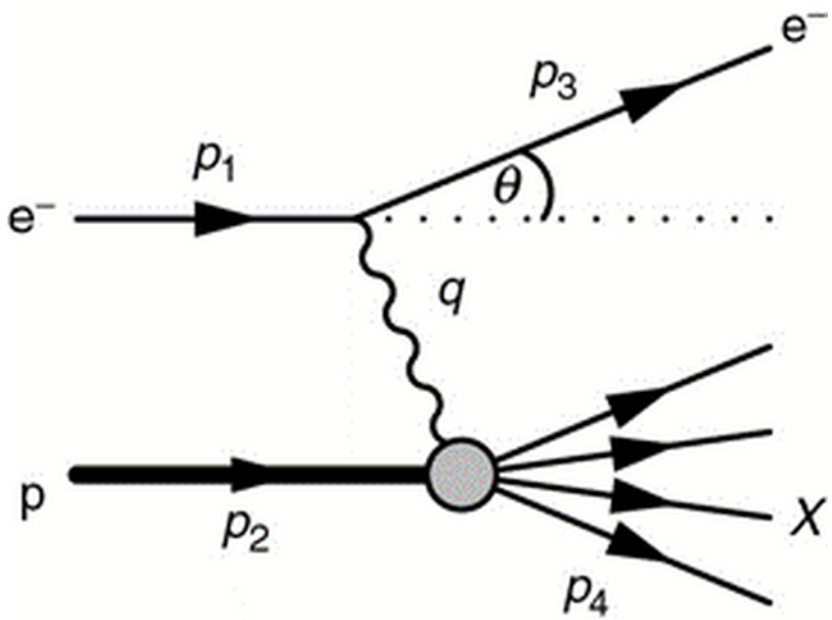


FIG. 1. $(d^2\sigma/d\Omega dE')/\sigma_{\rm Mott}$, in GeV^{-1}, vs q^2 for $W = 2$, 3, and 3.5 GeV. The lines drawn through the data are meant to guide the eye. Also shown is the cross section for elastic e-p scattering divided by $\sigma_{\rm Mott}$, $(d\sigma/d\Omega)/\sigma_{\rm Mott}$, calculated for $\theta = 10°$, using the dipole form factor. The relatively slow variation with q^2 of the inelastic cross section compared with the elastic cross section is clearly shown.

← SEE LAST SLIDE

SOME MORE KINEMATICS



← USE VARIABLE NAMES IN TEXT BOOK

$$Q^2 = -q^2$$

$$Q^2 = -(p_1 - p_3)^2 = -2m_e^2 + 2p_1 \cdot p_3$$

$$= -2m_e^2 + 2E_1E_3 - 2|\bar{p}_1||\bar{p}_3| \cos\theta$$

$$Q^2 \approx 2E_1E_2(1-\cos\theta) = 4E_1E_2 \sin^2\frac{\theta}{2}$$

BJORKEN x

$$x \equiv \frac{Q^2}{2p_2 \cdot q}$$ → LORENTZ INVARIANT

$$W_H^2 \equiv p_4^2 = (p_2+q)^2 = q^2 + 2p_2 \cdot q + p_2^2$$

$$W_H^2 + Q^2 - m_p^2 = 2p_2 \cdot q$$

LOOKS FAMILIAR

$$x = \frac{Q^2}{Q^2 + W_H^2 - m_p^2}$$

$W_H^2 \equiv p_4^2 \geqslant m_p^2$

$Q^2 > 0 \quad W_H^2 \geqslant m_p$

$0 \leqslant x \leqslant 1$

FOR ELASTIC SCATTER

$W_H^2 = m_p^2 \longrightarrow x = 1$

ν AND y

ANOTHER DIMENSIONLESS LORENTZ INVARIANT

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}$$

IN PROTON REST FRAME = LAB $p_2 = (m_p, 0, 0, 0)$

ELECTRONS $p_1 = (E_1, 0, 0, E_1)$, $p_3 = (E_3, E_3 \sin\theta, 0, E_3 \cos\theta)$

$$q = (E_1 - E_3, \bar{p}_1 - \bar{p}_3)$$

$$y = \frac{(m_p, 0, 0, 0)(E_1 - E_3, \bar{p}_1 - \bar{p}_3)}{(m_p, 0, 0, 0)(E_1, 0, 0, E_1)} = \frac{m_p (E_1 - E_3)}{m_p E_1}$$

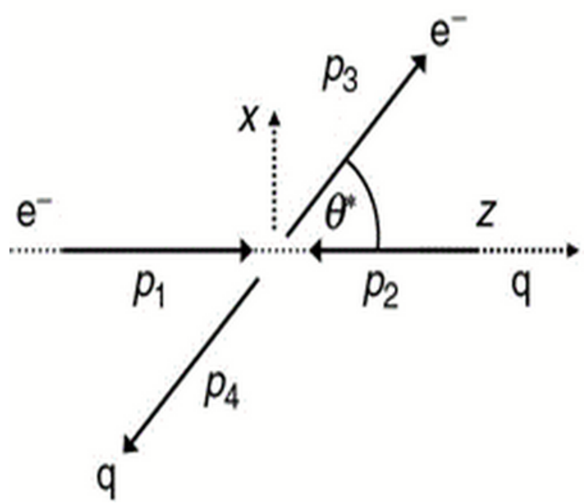
$$y = 1 - \frac{E_3}{E_1} \qquad 0 \leq y \leq 1$$

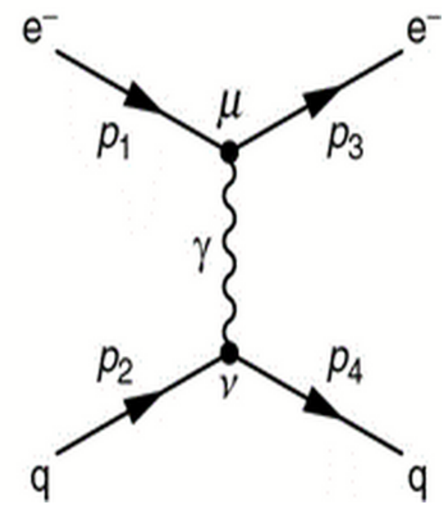
$$\nu \equiv \frac{p_2 \cdot q}{m_p}$$

$$= \frac{m_p (E_1 - E_3)}{m_p}$$

$$= (E_1 - E_3)$$ ← ENERGY LOSS OF ELECTRON

ELECTRON – QUARK SCATTERING





IN THE QUARK MODEL DEEP INELASTIC SCATTERING CAN BE UNDERSTOOD AS ELASTIC e^- QUARK $\rightarrow e^-$ QUARK

THIS IS A QED PROCESS $\rightarrow$ FEYNMAN RULES

ELECTRON CURRENT $\bar{u}(p_3)[ie\gamma^\mu]u(p_1)$

QUARK CURRENT $\bar{u}(p_4)[iQ_q e\gamma^\mu]u(p_2)$

PROPAGATOR $-ig_{\mu\nu}/q^2$

$\hookrightarrow q^2 = p_1 - p_3$

AS USUAL WE PUT THE CURRENTS AND PROPAGATOR TOGETHER TO GET INVARIANT MATRIX ELEMENT.

$$\mathcal{M}_{fi} = \frac{Q_q e^2}{q^2}\left[\bar{u}(p_3)\gamma^\mu u(p_1)\right] g_{\mu\nu} \left[\bar{u}(p_4)\gamma^\nu u(p_2)\right]$$

WE THEN DO SPIN-AVERAGED MATRIX ELEMENT, JUST LIKE WE DID FOR $e^-\mu^- \to e^-\mu^-$

DEEP INELASTIC SCATTERING EXPERIMENTS USE HIGH CMS ENERGY

SLAC $ep \to eX$ @ 20 GeV/c

HERA $ep \to eX$ 30 GeV $\to \leftarrow$ 1000 GeV

NEGLECT m_e, m_q

$$\langle |\mathcal{M}_{fi}|^2 \rangle = 2Q_q^2 e^4 \left(\frac{s^2+u^2}{t^2}\right) = 2Q_q^2 e^4 \frac{(p_1\cdot p_2)^2 + (p_1\cdot p_4)^2}{(p_1\cdot p_3)^2}$$

$s = p_1 + p_2 \quad t = p_1 - p_3 \quad u = p_1 - p_4$

IN CMS FRAME → EXPRESS $\langle |\mathcal{M}_{fi}|^2 \rangle$ IN TERMS OF θ^*

BY DEFINITION ELECTRON ENERGY IN CMS = $E = \sqrt{s}/2$

FOUR MOMENTA OF INITIAL AND FINAL STATE:

e $\quad p_1 = (E, 0, 0, +E) \qquad p_3 = (E, +E\sin\theta^*, 0, +E\cos\theta^*)$

q $\quad p_2 = (E, 0, 0, -E) \qquad p_4 = (E, -E\sin\theta^*, 0, -E\cos\theta^*)$

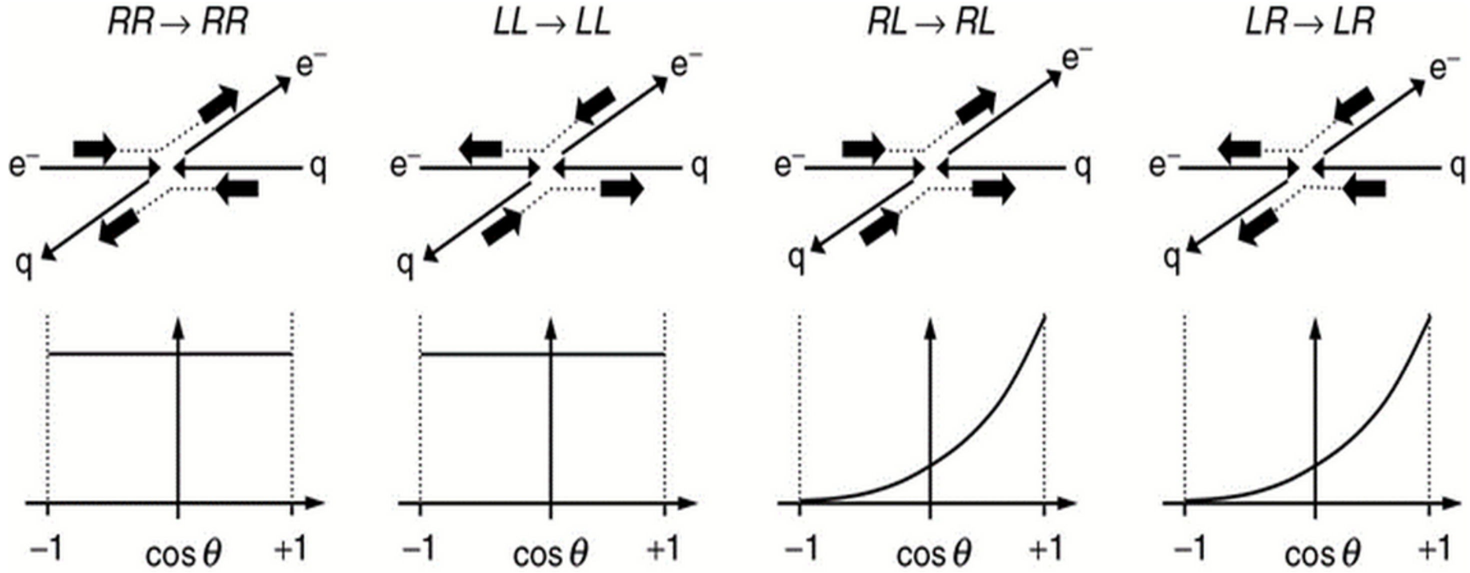
4-VECTOR PRODUCTS in $\langle |\mathcal{M}_{fi}|^2 \rangle$ ARE

$$p_1 \cdot p_2 = 2E^2 \qquad p_1 \cdot p_3 = E^2(1-\cos\theta^*) \qquad p_1 \cdot p_4 = E^2(1+\cos\theta^*)$$

SO SPIN AVERAGED INVARIANT MATRIX ELEMENT

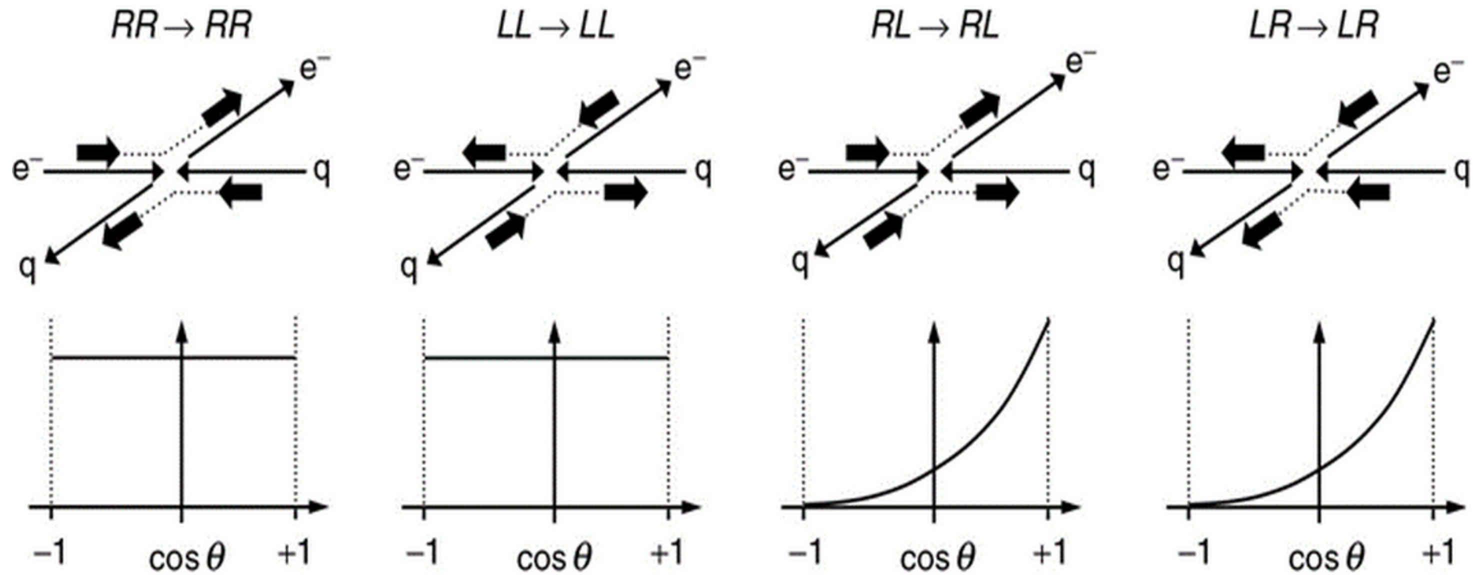
$$\langle |\mathcal{M}_{fi}|^2 \rangle = 2 Q_q e^4 \, \frac{4E^4 + E^4(1+\cos\theta^*)^2}{E^4(1-\cos\theta^*)^2}$$

SUBSTITUTE THIS INTO $\quad \dfrac{d\sigma}{d\Omega^*} = \dfrac{1}{64\pi^2 s} \dfrac{p^*_f}{p^*_i} |\mathcal{M}_{fi}|^2$



$$\frac{d\sigma}{d\Omega^*} = \frac{Q_q^2 e^4}{8\pi^2 s} \frac{\left[1 + \frac{1}{4}(1+\cos\theta^*)^2\right]}{(1-\cos\theta^*)^2} \longrightarrow$$ CHIRAL STRUCTURE OF QED

HELICITY CONSERVED AT HIGH ENERGY → ONLY NON-ZERO MATRIX ELEMENTS → HELICITIES OF e^- & q UNCHANGED DURING INTERACTION → SEE ABOVE



$$1 + \frac{1}{4}(1+\cos\theta^*)^2$$

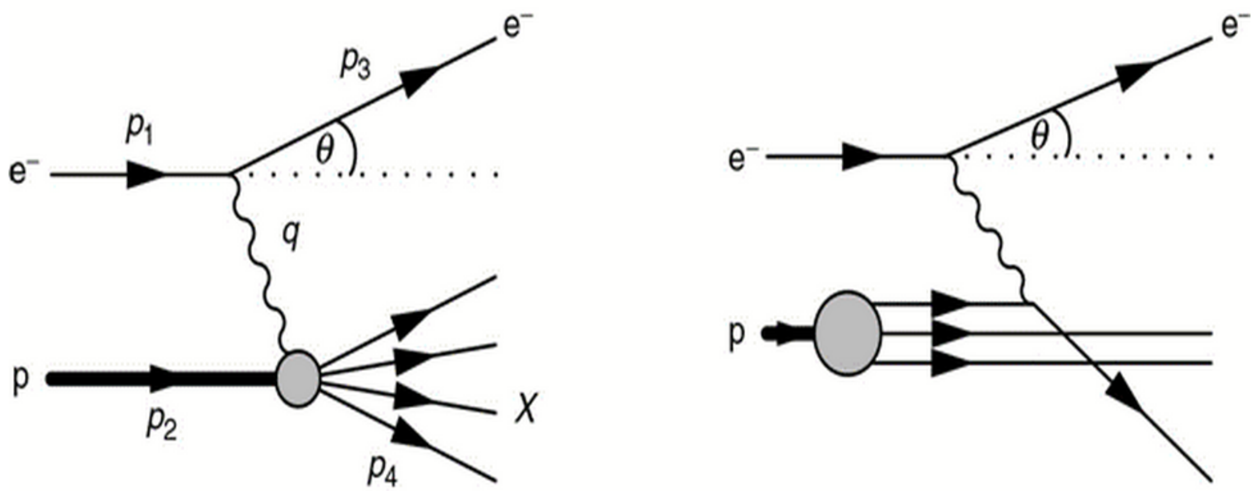
RR → RR, LL → LL $S_z = 0$ NO PREFFERED DIRECTION

RL → RL, LR → LR $S_z = \pm 1$ $\rightarrow \frac{1}{4}(1+\cos\theta^*)^2$

DENOMINATOR $(1-\cos\theta^*)^2 \longrightarrow$ FROM $\frac{1}{q^2}$

$$q^2 = t = (p_1 - p_3) \approx -E^2(1-\cos\theta^*)$$

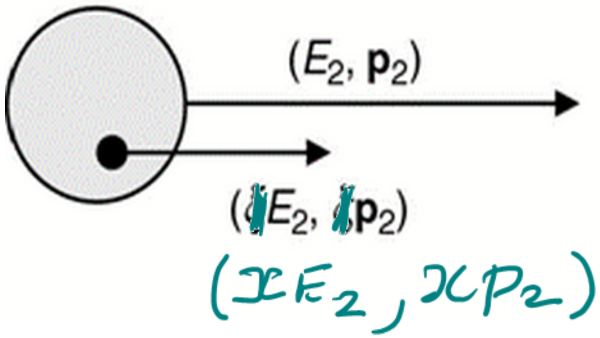
QUARK-PARTON MODEL

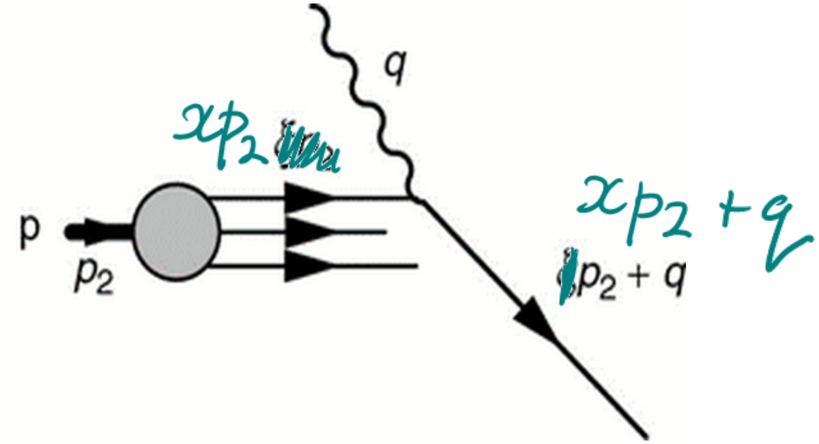


WHEN FEYNMAN SAW THE $ep \to eX$ EXPERIMENTAL RESULTS — HE REALIZED THAT IT WAS JUST $eq \to eq$ ELASTIC SCATTERING $\longrightarrow$ NO FORM FACTOR $\longrightarrow$ RUTHERFORD SCATTERING FROM POINT PARTICLES INSIDE THE PROTON.

WHY "FREE" $\longrightarrow$ SEE QCD — ASYMPTOTIC FREEDOM

$$\alpha_s(q^2) = \frac{1}{b \ln\left(q^2/\lambda^2\right)}$$





QUARK PARTON MODEL, $E \gg m_p \rightarrow \infty$ MOMENTUM FRAME

FOUR MOMENTUM OF PROTON $p_2 = (E_2, 0, 0, E_2)$

NEGLECT TRANSVERSE MOMENTUM OF STRUCK QUARK

QUARK 4-VECTOR $p_q = p_2 = (xE_2, 0, 0, xE_2)$

$x \rightarrow$ FRACTION OF PROTON MOMENTUM CARRIED BY QUARK

4-MOMENTUM OF QUARK BEFORE INTERACTION xp_2

AFTER INTERACTION IT HAS GAINED q^2 FROM THE VIRTUAL γ^* $\rightarrow xp_2 + q$

$$m_q^2 = (x p_2 + q)^2 = \underbrace{x^2 p_2^2}_{} + 2x\, p_2 \cdot q + q^2$$

→ THIS IS ALSO m_q^2 ($m_q^2 = 0$)

$$2x\, p_2 \cdot q + q^2 = 0$$

$$x = \frac{-q^2}{2 p_2 \cdot q} = \frac{Q^2}{2 p_2 \cdot q}$$

IN QUARK PARTON MODEL x → FRACTION OF PROTON MOMENTUM CARRIED BY STRUCK QUARK

x DISTRIBUTION → MOMENTUM DISTRIBUTION OF QUARKS INSIDE THE PROTON

WANT TO RELATE ELECTRON – QUARK KINEMATICS TO ELECTRON – PROTON KINEMATICS ← MEASURE

NEGLECTING m_e m_p IN CMS

$$S = (p_1 + p_2)^2 = \cancel{m_p^2} + \cancel{m_e^2} + 2p_1 \cdot p_2$$

$p_q = x p_2$ → CM ENERGY IN eq SYSTEM

$$S_q = (p_1 + x p_2)^2 \approx 2x\, p_1 \cdot p_2 = x S$$

ELECTRON PROTON

DEFINE $x = \dfrac{Q^2}{2 p_2 \cdot q}$, $y = \dfrac{p_2 \cdot q}{p_2 \cdot p_1}$

$$y_q = \frac{p_q \cdot q}{p_q \cdot p_1} = \frac{x p_2 \cdot q}{x p_2 \cdot p_1} = y$$

FOR ELASTIC SCATTER $x_q = 1$

$$S_q = x S, \quad y_q = y, \quad x_q = 1$$

PUT $s_q = xs$, $y_q = y$, $x_q = 1$ INTO

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s}\right)^2\right]$$

QUARK KINEMATICS

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s_q}\right)\right]$$

CAN RELATE TO ep

HAD $Q^2 = (s - m_p^2)\,xy$ (TEXT 8.8)

$$q^2 = -Q^2 = -(s_q - \cancel{m_q})^2 x_q y_q$$

$$q^2/s_q = -\underset{\underset{1}{\shortparallel}}{x_q}\,\underset{\underset{y}{\shortparallel}}{y_q} = -y$$

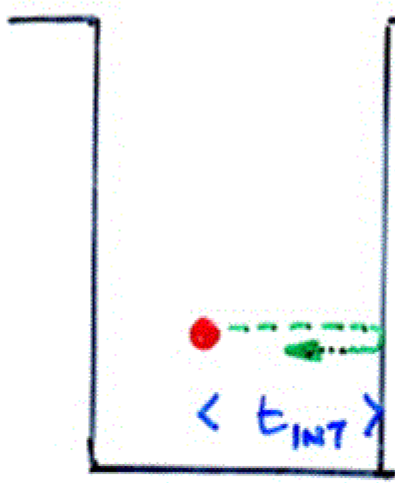
$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + (1-y)^2\right] \rightarrow \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 Q_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]$$

PARTON MODEL OF DEEP INELASTIC SCATTERING

- DATA → SCATTERING FROM FREE POINT CHARGES
- HADRONS → QUARKS NOT FREE → TIGHTLY BOUND

$$\Delta t \sim \frac{\hbar}{\Delta E}$$

$$\Delta x \sim \frac{\hbar}{\Delta p}$$



EXAMINE QUARKS DURING SHORT SPACE-TIME INTERVAL – MOTION FROZEN

FOR TIME << t_{INT} QUARK WILL APPEAR FREE EVEN TH'O' TIGHTLY BOUND

→ IMPULSE APPROXIMATION

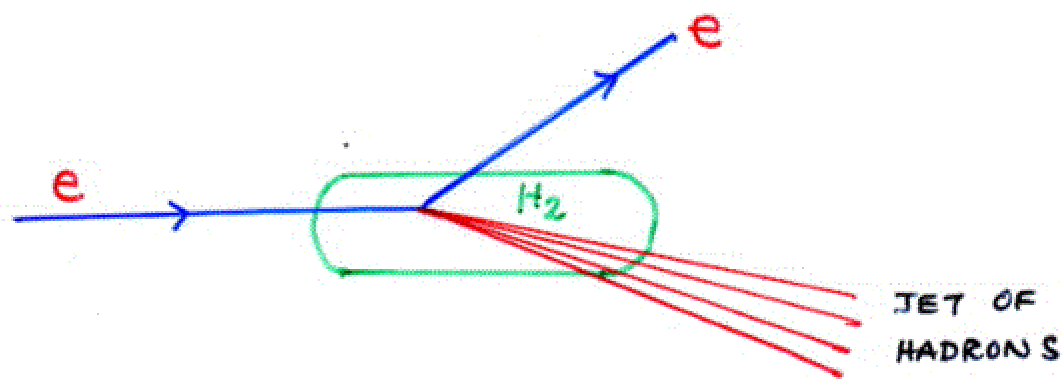
TO INVESTIGATE A SMALL SPACE-TIME INTERVAL $(E, \vec{p})$ BOTH LARGE

SHORT "4-WAVELENGTH"

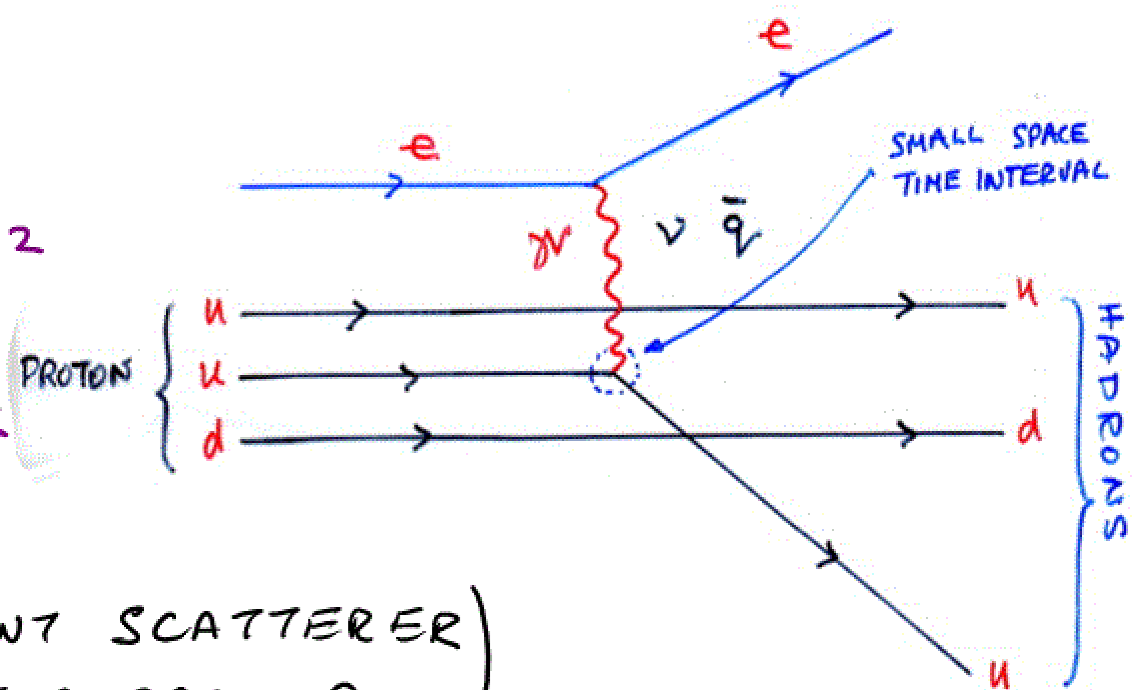
SMALL SPAC-TIME INTERVAL

PHYSICAL INTERPRETATION

EXPERIMENT:



FEYMAN DIAGRAM:



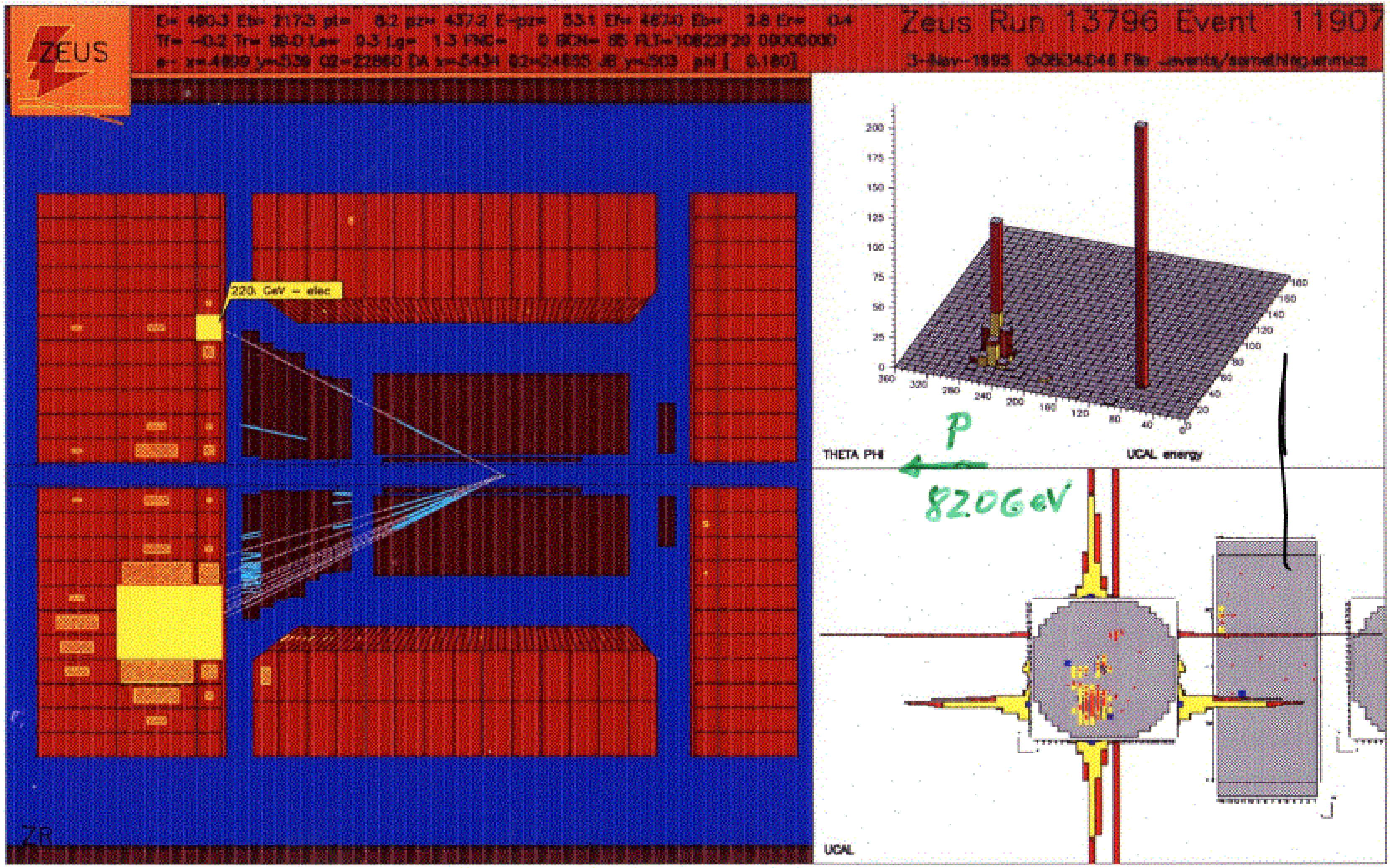
$$\sigma = |A|^2 \sim \left(e \cdot \left\langle \begin{matrix}\text{QUARK}\\ \text{CHARGE}\end{matrix} \right\rangle\right)^2$$

$$\sim \frac{1}{3}\Big[\underset{u}{\left(\tfrac{2}{3}\right)^2} + \underset{u}{\left(\tfrac{2}{3}\right)^2} + \underset{d}{\left(\tfrac{1}{3}\right)^2}\Big] e^2$$

$$\sigma \sim \frac{1}{3} e^2 \sim \frac{1}{3} \left(\begin{matrix}\text{POINT SCATTERER}\\ \text{OF CHARGE } e\end{matrix} \right)$$

ELECTRON - PROTON SCATTERING @ HERA

e
30 GeV



p
820 GeV

RATIOS OF CROSS SECTIONS

IN $\sigma_{DIS} \sim \frac{1}{3} e^2$

KINEMATIC FACTORS DEPENDING ON EXPERIMENT — DROPS OUT IN RATIOS

NEUTRONS ddu

$$\sigma_n \sim \frac{e^2}{3}\left\{ \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 \right\} \sim \frac{2}{9} e^2$$

$$\sigma_{ep} / \sigma_{en} = \frac{e^2}{3} \cdot \frac{2e^2}{9} = 3/2$$

DEUTERIUM → SAME # PROTONS + NEUTRONS

→ SAME # u + d QUARKS (ISOSCALAR TARGET)

$$\sigma_d = e^2 \left\{ \frac{1}{6}\left[3 \cdot \left(\frac{1}{3}\right)^2 + 3\left(\frac{2}{3}\right)^2 \right] \right\} \sim \frac{e^2}{2} \cdot \frac{5}{9}$$

$$\frac{\text{LIQUID DEUTERIUM}}{\text{LIQUID HYDROGEN}} = \frac{\sigma_d}{\sigma_p} = \frac{e^2}{2} \cdot \frac{5}{9} \cdot \frac{3}{e^2} = \frac{5}{6} \checkmark$$

DEEP INELASTIC SCATTERING & SCALING

PROTON 4-MOMENTUM P

EACH PARTON HAS 4-MOMENTUM xP

PARTON ABSORBS q FROM VIRTUAL γ

$$(xP+q)^2 \sim m^2_{PARTON} \sim 0$$ — THEY BEHAVE LIKE THIS

$$x^2P^2 + q^2 + 2xP\cdot q = 0$$

$$x^2P^2 = x^2 M^2_{PROTON} \ll q^2 \Rightarrow q^2 + 2xP\cdot q = 0$$

$$x = \frac{-q^2}{2P\cdot q} \quad 2(M_p, 0)(\nu\, q^2) = 2M_p\nu$$

$$x = \frac{-q^2}{2M_p\nu}$$

→ DIMENSIONLESS

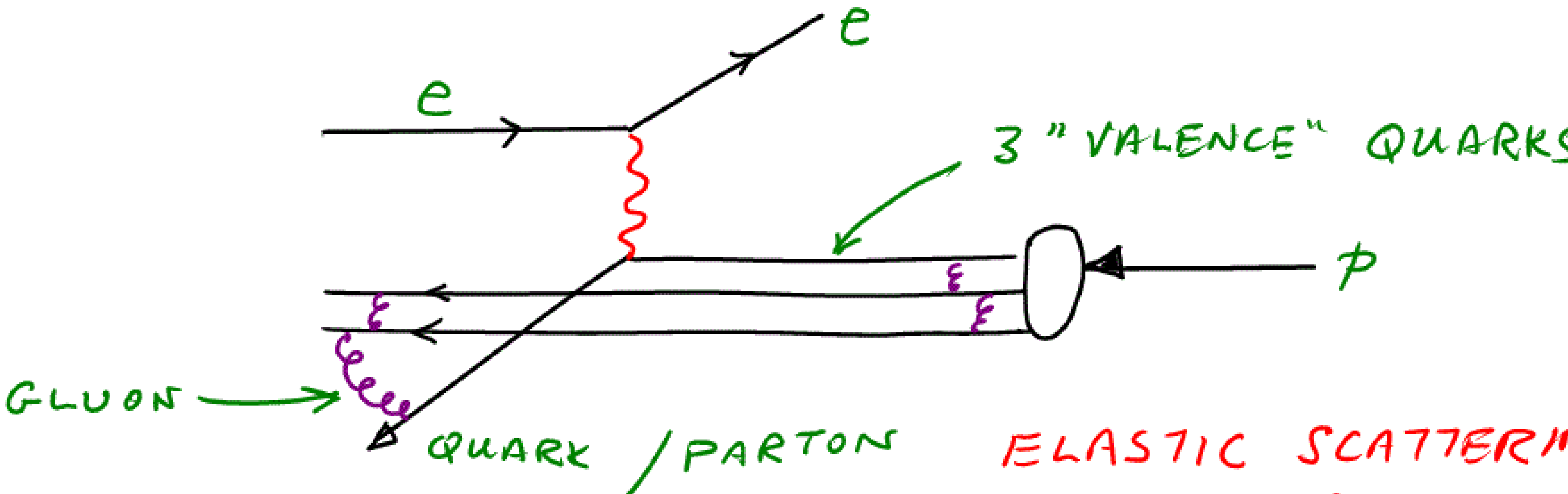
ONLY ONE SCATTERING VARIABLE

→ ELASTIC POINT SCATTERING

SCALING

$$W(q^2,\nu) \to W(q^2/\nu) \to F(x)$$

VISUALIZATION OF SCALING



ELASTIC SCATTERING FROM POINT PARTON

$$x = \frac{\text{QUARK MOMENTUM}}{\text{PROTON MOMENTUM}} \longrightarrow xP = (xE, x\vec{p})$$

$$x = q/P \qquad (0 < x < 1)$$

QUARK/PARTON MOMENTUM DISTRIBUTION INSIDE PROTON

$$\left.\begin{array}{l} W_1(q^2, \nu) \\ W_2(q^2, \nu) \end{array}\right. \longrightarrow \left.\begin{array}{l} F_1(x) \\ F_2(x) \end{array}\right\} \text{SCALE INVARIANCE}$$

ONE VARIABLE

PARTON DISTRIBUTION FUNCTIONS

$$\left(\frac{d\sigma}{dQ^2}\right)_{PROTON} = \left(\frac{d\sigma}{dQ^2}\right)_{QUARK} \times q_i^p(x)$$

FRACTIONAL MOMENTUM DISTRIBUTION OF QUARK i^{TH} FLAVOR

$$\frac{d^2\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right] \times Q_i^2 q_i^p(x)\,\delta x$$

$x \to x + \delta x$

SUM OVER ALL QUARK FLAVORS

$$\frac{d^2\sigma^{ep}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\sum_i Q_i^2 q_i^p(x)$$

$$\frac{d^2\sigma^{ep}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]\sum_i Q_i^2 q_i^p(x)$$

WRITE THIS AS

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2^{ep}(x,Q^2)}{x} + y^2 F_1^{ep}(x,Q^2)\right]$$

$$F_2^{ep}(x,Q^2) = 2x F_1^{ep}(x,Q^2) = x\sum Q_i^2 q_i^p(x)$$

QPM $\rightarrow$ SCALING $\rightarrow$ POINT NATURE

CALLAN-GROSS $2xF_1 = F_2$

$$F_2^{ep} = x \sum_i Q_i^2 q_i^p(x) \approx x\left(\frac{4}{9}u^p(x) + \frac{1}{9}d^p(x) + \frac{4}{9}\bar{u}^p(x) + \frac{1}{9}\bar{d}^p(x)\right)$$

(QUARK CHARGE)^2

ISOSPIN SYMMETRY $d^n(x) = u^p(x)$

NEUTRON PROTON

CONVENTIONALLY FOR PROTON WRITE IN TERMS

$$u(x),\ d(x),\ \bar{u}(x),\ \bar{d}(x)$$

FOR NEUTRON

$$d^n(x) = u^p(x) \equiv u(x)$$

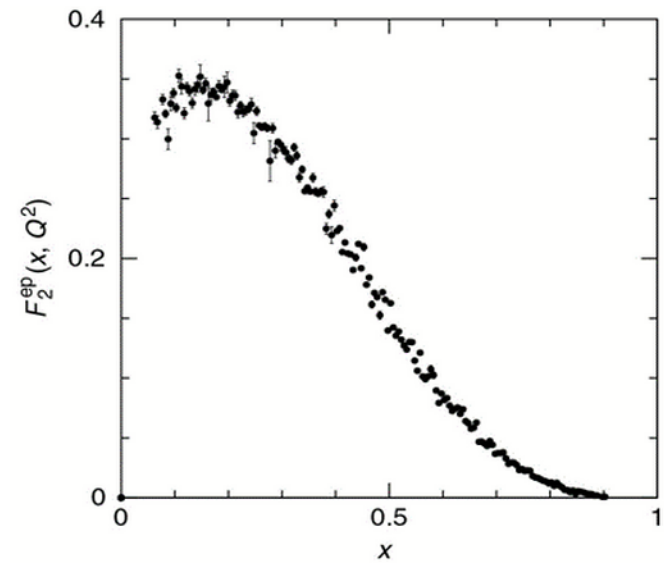
$$u^n(x) = d^p(x) \equiv d(x)$$

$$F_2^{ep}(x) = 2x F_1^{ep}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x)\right)$$

$$F_2^{en}(x) = 2x F_1^{en}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}\bar{u}(x)\right)$$

$$\int_0^1 F_2^{ep}(x)\,dx = \frac{4}{9} f_u + \frac{1}{9} f_d \ , \quad \int_0^1 F_2^{en}(x)\,dx = \frac{4}{9} f_d + \frac{1}{9} f_u$$

$$f_u = \int_0^1 [x u(x) + x\bar{u}(x)]\,dx \qquad f_d = \int_0^1 [x d(x) + x\bar{d}(x)]\,dx$$



EXPERIMENTALLY

$2\,\mathrm{GeV}^2 < Q^2 < 30\,\mathrm{GeV}^2$ SLAC

$$\int F_2^{ep}(x)\,dx \simeq 0.18$$

$$\int F_2^{en}(x)\,dx \simeq 0.12$$

$f_u \simeq 0.36$ (2u)

$f_d \simeq 0.18$ (1d)

$\Downarrow$

PROTON

50% OF PROTON MOMENTUM

$u + d + \bar{u} + \bar{d}$

$\Longrightarrow$ REST GLUONS

CAN SPLIT PDF INTO VALENCE + SEA

$$u(x) = u_v(x) + u_s(x) \quad , \quad d(x) = d_v(x) + u_v(x)$$

ANTIQUARKS ARE ALL SEA $\quad \bar{u}(x) \equiv \bar{u}_s(x)$

$\bar{d}(x) \equiv \bar{d}_s(x)$

PROTON $\rightarrow$ uud

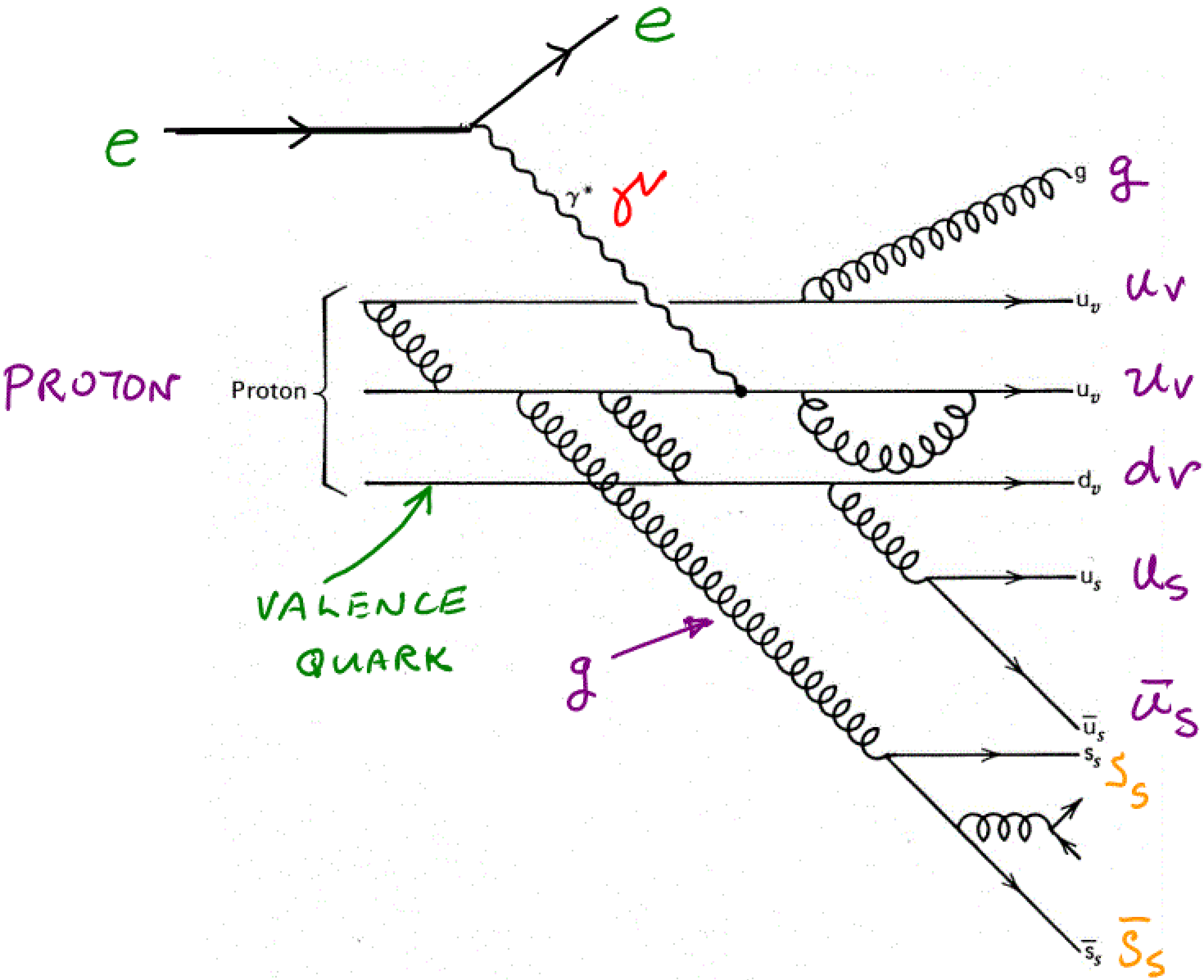
EXPECT $\int_0^1 u_v(x)\,dx = 2, \quad \int_0^1 d_v(x)\,dx = 1$

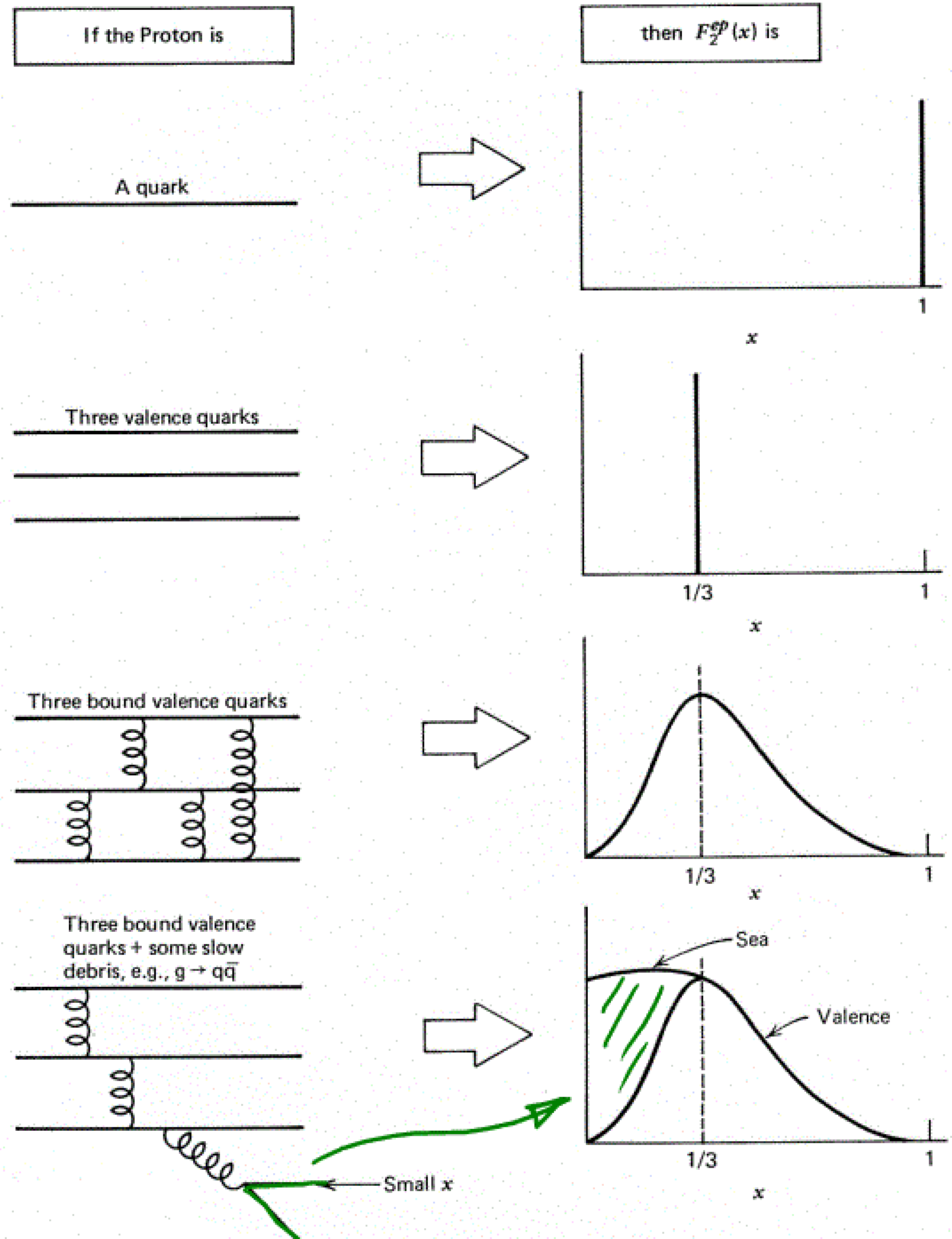
$$u_s(x) = \bar{u}_s(x) \approx d_s(x) = \bar{d}_s(x) \simeq S(x)$$

$$F_2^{ep}(x) = x\left(\frac{4}{9}u_v(x) + \frac{1}{9}d_v(x) + \frac{10}{9}S(x)\right)$$

$$F_2^{en}(x) = x\left(\frac{4}{9}d_v(x) + \frac{1}{9}u_v(x) + \frac{10}{9}S(x)\right)$$

WHERE DO THE ANTIQUARKS COME FROM ?





If the Proton is
then $F_2^{ep}(x)$ is
A quark
1
x
Three valence quarks
1/3
1
x
Three bound valence quarks
1/3
1
x
Three bound valence quarks + some slow debris, e.g., g → q$\bar{q}$
Sea
Valence
Small x
1/3
1
x

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{4d_v(x) + u_v(x) + 10S(x)}{4u_v(x) + d_v(x) + 10S(x)}$$

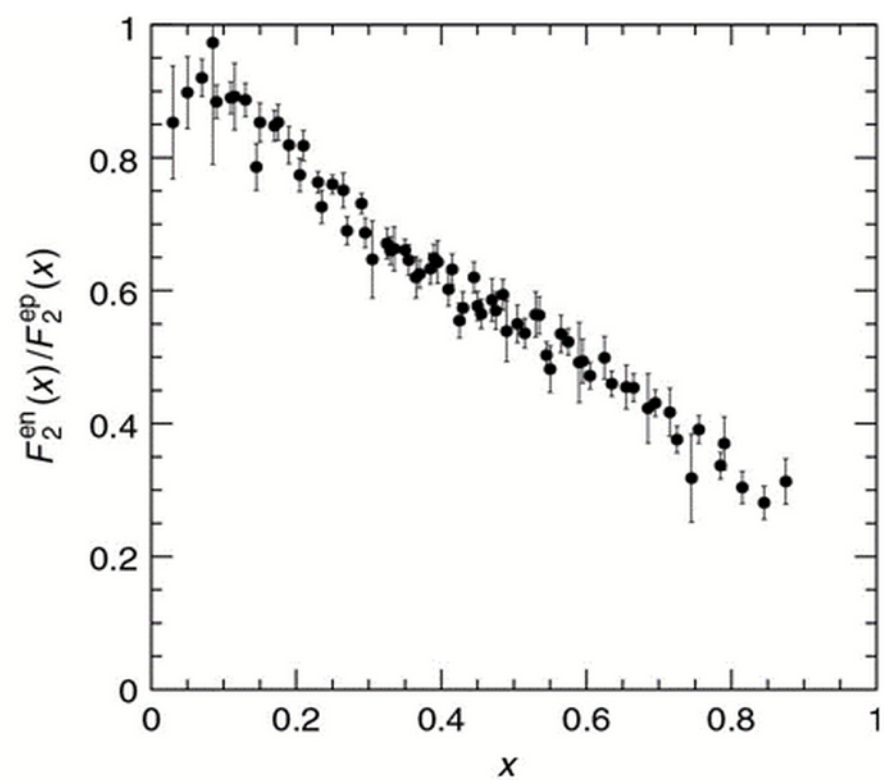
EXAMPLE PREDICTION OF QPM

SEA QUARKS CONCENTRATED AT LOW x

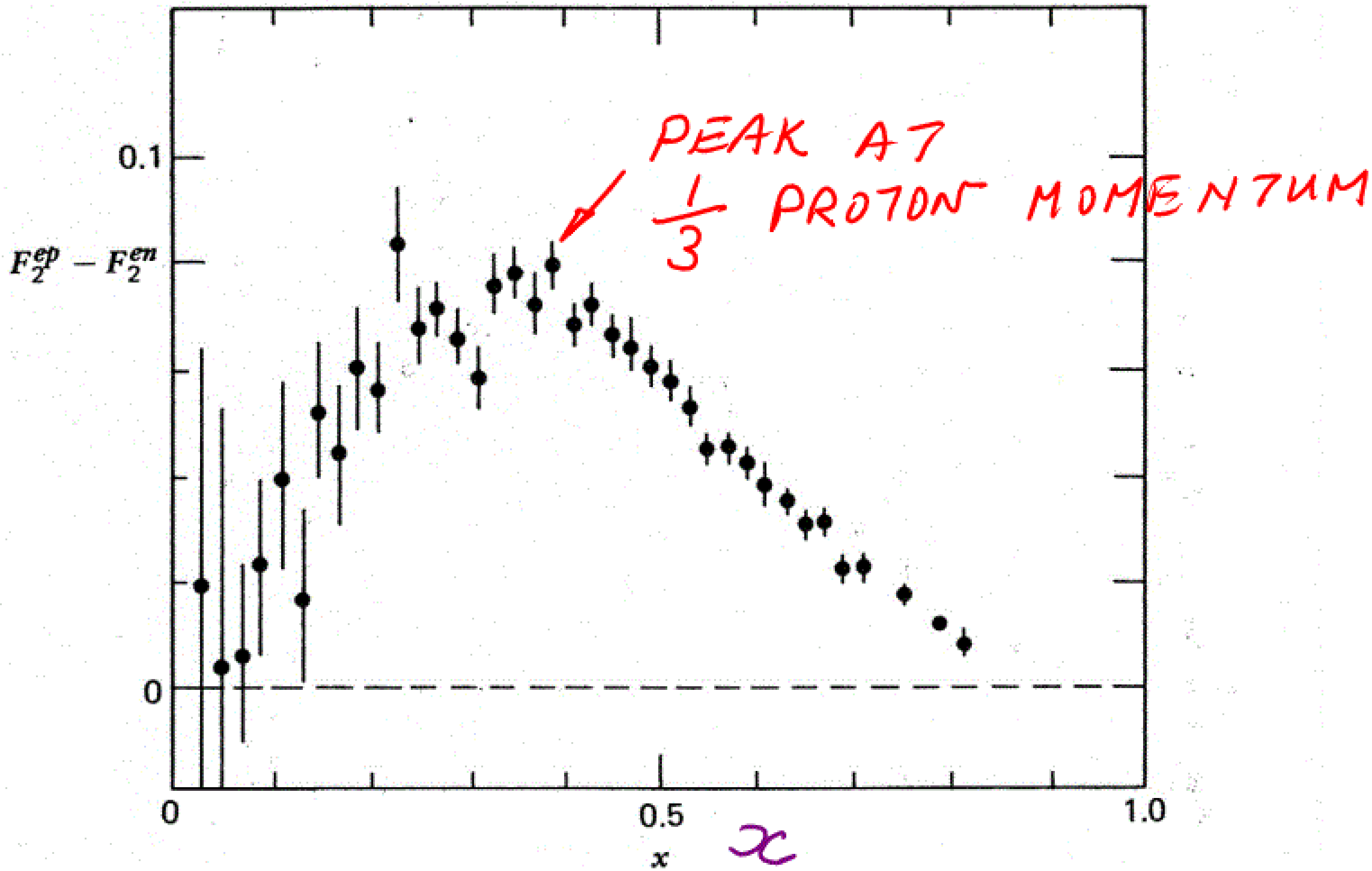
→ VALENCE CARRY MOST MOMENTUM

PROTON & NEUTRON HAVE SAME SEA

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \to 1, \text{ AS } x \to 0$$



MOMENTUM DISTRIBUTION OF VALENCE QUARKS.



NEUTRON + PROTON HAVE SAME SEA

DISTBIBUTION → DIFFERENCE → VALENCE

SUMMARY

- FOR DEEP INELASTIC SCATTERING IN GENERAL
  - TWO KINEMATIC VARIABLES $q \equiv q(\nu, \vec{p})$
  - TWO STRUCTURE FUNCTIONS

$$W_1(\nu, q^2) \qquad W_2(\underbrace{\nu, q^2}_{\text{2 VARIABLES}})$$

- FOR LARGE q^2 OR W_H STRUCTURE FUNCTIONS DEPEND ON ONE VARIABLE $x = q^2/2M_p\nu$

STRUCTURE FUNCTIONS $\rightarrow F_1(x) \quad F_2(x)$

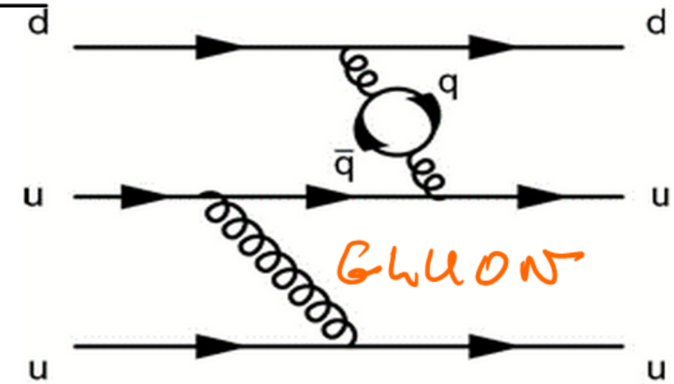
- ARISES FROM ELASTIC SCATTERING FROM

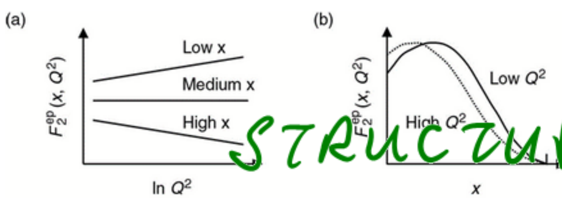
POINT PARTONS (QUARKS) $\longrightarrow$ NO LENGTH SCALE

SCALE INVARIANCE

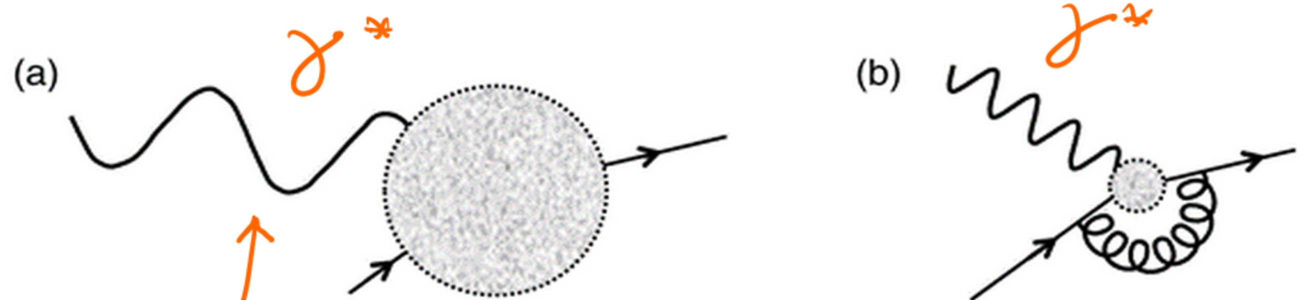
SCALING VIOLATIONS

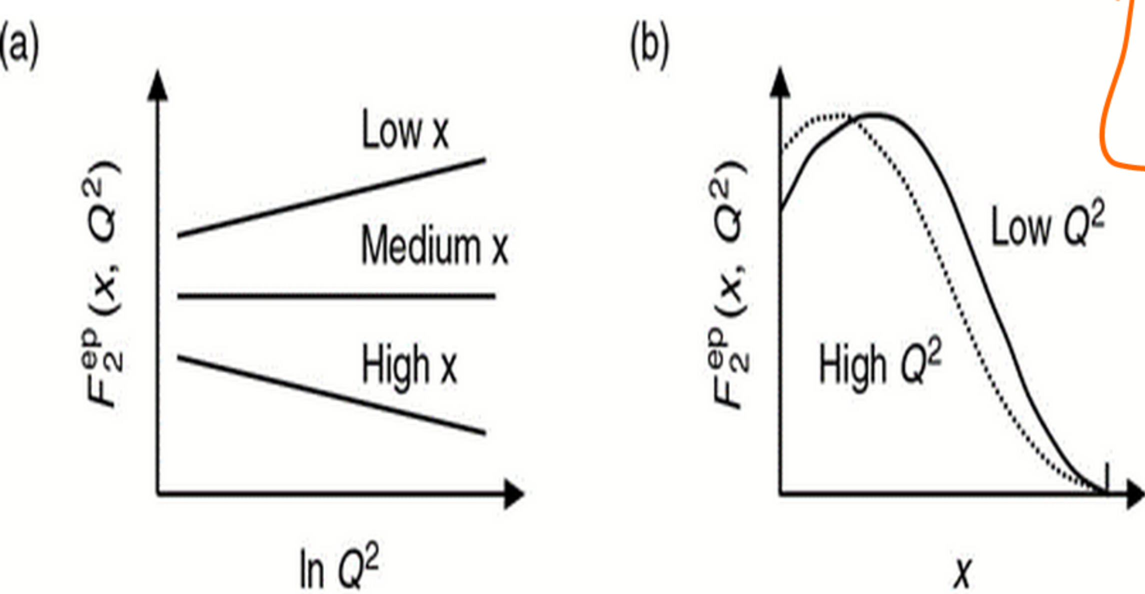
QUARKS ARE NOT FREE





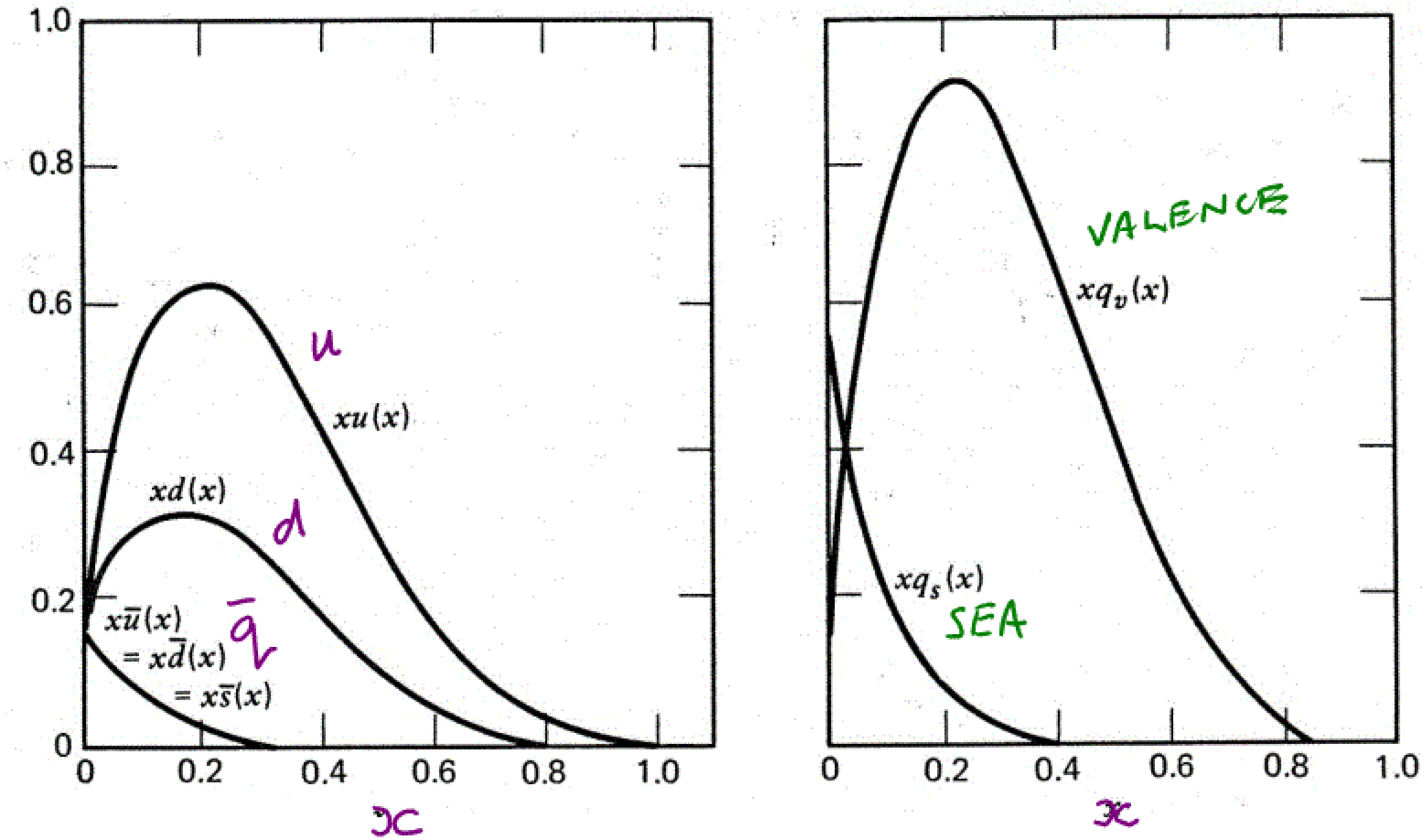
STRUCTURE SEE INSIDE PROTON DEPENDS ON DISTANCE SCALE RESOLVED





"WAVE LENGTH" $\sim \frac{1}{Q^2}$

STRUCTURE FUNCTIONS



THESE ARE THE MOMENTUM DISTRIBUTIONS OF VARIOUS QUARK SPECIES INSIDE PROTON

PARTON DISTRIBUTION FUNCTIONS