THE WEAK INTERACTION

I'VE GONE ON TO THE WEAK INTERACTION

- NOT BECAUSE QCD IS NOT IMPORTANT → THE WORLD WE SEE AROUND US IS MAINLY THE QCD WORLD
- UNIFICATION OF WEAK AND ELECTROMAGNETIC IS ONE OF THE TRIUMPHS OF MODERN SCIENCE
- THE WEAK INTERACTION HAS SOME RESEMBLANCE TO QED — NOT SURPRISING
- MEDIATED BY MASSIVE BOSONS cf $m_\gamma = 0$ TO MAKE IT A GAUGE THEORY — HAD TO INVENT HIGGS MECHANISM
- VIOLATES PARITY, CHARGE CONJUGATION, AND TIME REVERSAL SYMMETRY.

SYMMETRY OPERATION

SYMMETRY OPERATOR → TRANSFORMATION → LEAVES HAMILTONIAN UNCHANGED

$$\psi'(\bar{x},t) = \mathcal{U}\,\psi(\bar{x},t)$$

NEW STATE   ORIGINAL STATE

$$i\hbar \frac{d}{dt}\psi = H\psi \quad \text{AND} \quad i\hbar \frac{d}{dt}\mathcal{U}\psi = H\mathcal{U}\psi$$

IF $\mathcal{U}\mathcal{U}^{+} = 1$ → UNITARY

$$\rightarrow \quad H = \mathcal{U}^{+} H \mathcal{U} \quad \rightarrow \quad [H, \mathcal{U}] = 0$$

$[H, \mathcal{U}]$

SYMMETRY OPERATOR ONLY NEEDS TO BE UNITARY

$$\mathcal{U}^{+}\mathcal{U} = 1$$

ONLY HERMITIAN OPERATORS CORRESPOND TO OBSERVABLES $A = A^{+}$

GENERALLY SYMMETRY TRANSFORMATIONS DO NOT CORRESPOND TO OBSERVABLES

→ BUT A SYMMETRY TRANSFORMATION OPERATOR IS ALWAYS RELATED TO SOME OTHER OPERATOR WHICH IS AN OBSERVABLE

TWO DISTINCT KINDS OF TRANSFORMATION

- CONTINUOUS → DEPEND ON SOME CONTINUOUS PARAMETER. CAN DIFFER FROM UNITY (= DO NOTHING) BY AN ARBITRARILY SMALL AMOUNT

  EXAMPLE → ROTATION IN SPACE

- DISCRETE → EITHER HAPPEN OR NOT

  REFLECTION, TIME REVERSAL, PARTICLE → ANTIPARTICLE

SOME DISCRETE TRANSFORMATIONS → OBSERVABLES

SPATIAL REFLECTION → PARITY = $\mathcal{U}_P$

$$\psi(\bar{x}) \to \psi(-\bar{x}) \quad ; \quad \psi(-\bar{x}) = \mathcal{U}_P\, \psi(\bar{x})$$

$\mathcal{U}_P \mathcal{U}_P = 1$ → HERMITIAN UNITARY → MUST CORRESPOND TO AN OBSERVABLE

DISCRETE SYMMETRIES

- SOME OF THE MOST SURPRISING RESULTS IN 20TH CENTURY PHYSICS

PARITY VIOLATION RIGHT-LEFT ASYMMETRY IN UNIVERSE AT MOST FUNDAMENTAL LEVEL
→ COMPLETE SURPRISE

TIME REVERSAL ASYMMETRY MAY BE ORIGIN OF ASYMMETRY BETWEEN MATTER & ANTIMATTER IN THE UNIVERSE
ALSO MAY BE CONNECTED TO THE OBSERVED 3 GENERATIONS OF QUARKS & LEPTONS

DISCRETE SYMMETRIES — PROPERTIES OF UNIVERSE

PARITY

- NOT VERY INTERESTING AT FIRST SIGHT

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \xrightarrow{P} \begin{pmatrix} ct \\ -x \\ -y \\ -z \end{pmatrix}$$

P REVERSES 3 SPATIAL COORDINATES

— CANNOT BUILD FROM INFINITESIMALS

— NOT A ROTATION IN SPACE

EXPECT THAT

$$[P, H] = 0$$

AND SINCE $PP\psi(\bar{x}) = \psi(\bar{x})$

PARITY OF $\psi(\bar{x})$ WILL BE A CONSERVED OBSERVABLE

→ EXPERIMENTAL QUESTION

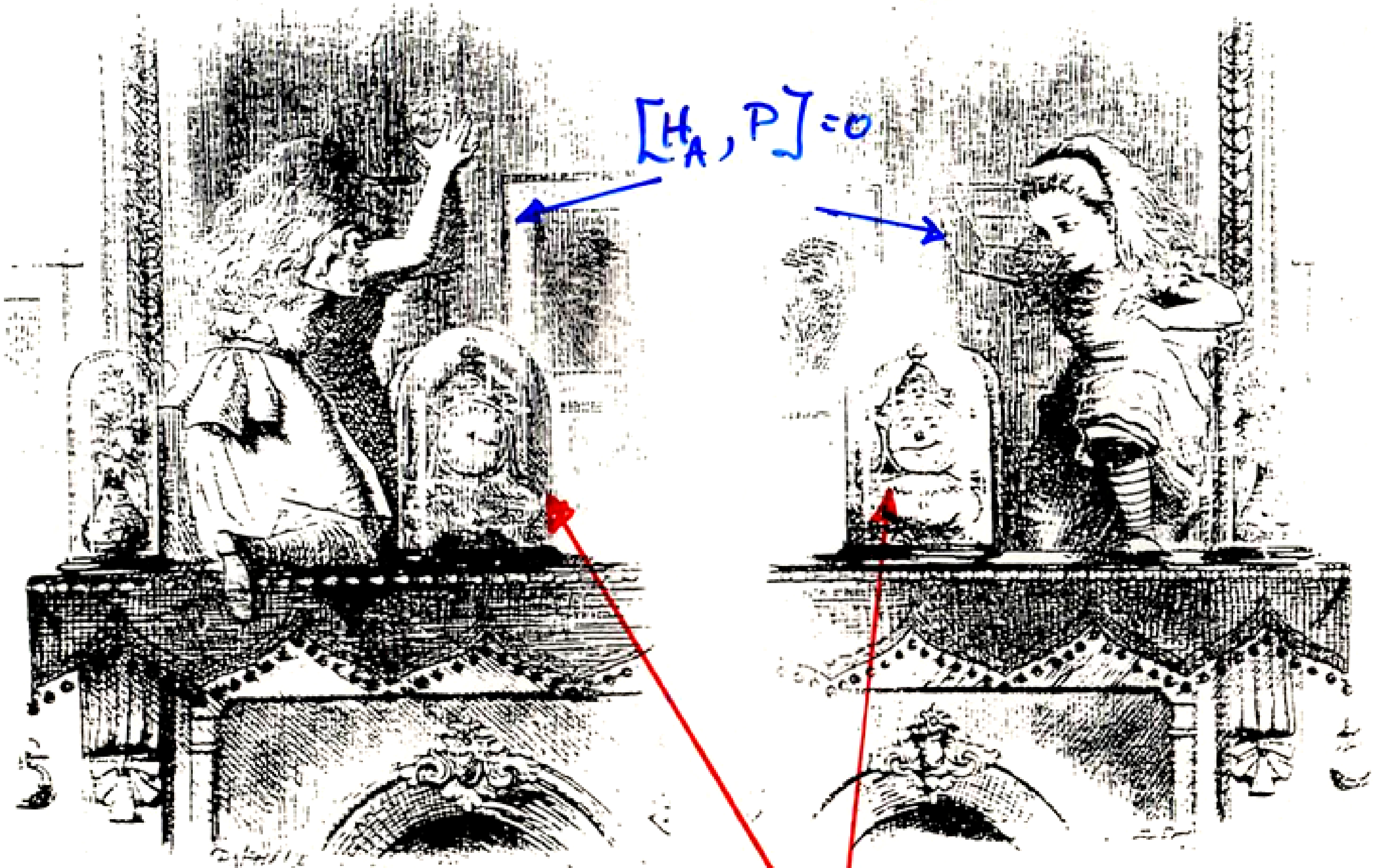
- OBSERVATIONS OF STRONG & ELECTROMAGNETIC DECAYS OF NUCLEI SHOWED ONLY TRANSITIONS BETWEEN STATES OF SAME PARITY

OK → PARITY CONSERVED AS EXPECTED

THROUGH THE LOOKING GLASS

PARITY

$[H_A, P] = 0$



OH, OH!

$[H_{\text{clock}}, P] \neq 0$

PARITY OPERATOR

$$\bar{r} \rightarrow -\bar{r} \qquad \text{VECTOR}$$

$$\bar{p} = m\dot{\bar{r}} \rightarrow -m\dot{\bar{r}} = -\bar{p} \qquad \text{VECTOR}$$

$$|\bar{r}| = (\bar{r}\cdot\bar{r})^{\frac{1}{2}} \rightarrow [(-\bar{r})\cdot(-\bar{r})]^{\frac{1}{2}} = |\bar{r}| \qquad \text{SCALAR}$$

$$|\bar{p}| = (\bar{p}\cdot\bar{p})^{\frac{1}{2}} \rightarrow [(-\bar{p})\cdot(-\bar{p})]^{\frac{1}{2}} = |\bar{p}| \qquad \text{SCALAR}$$

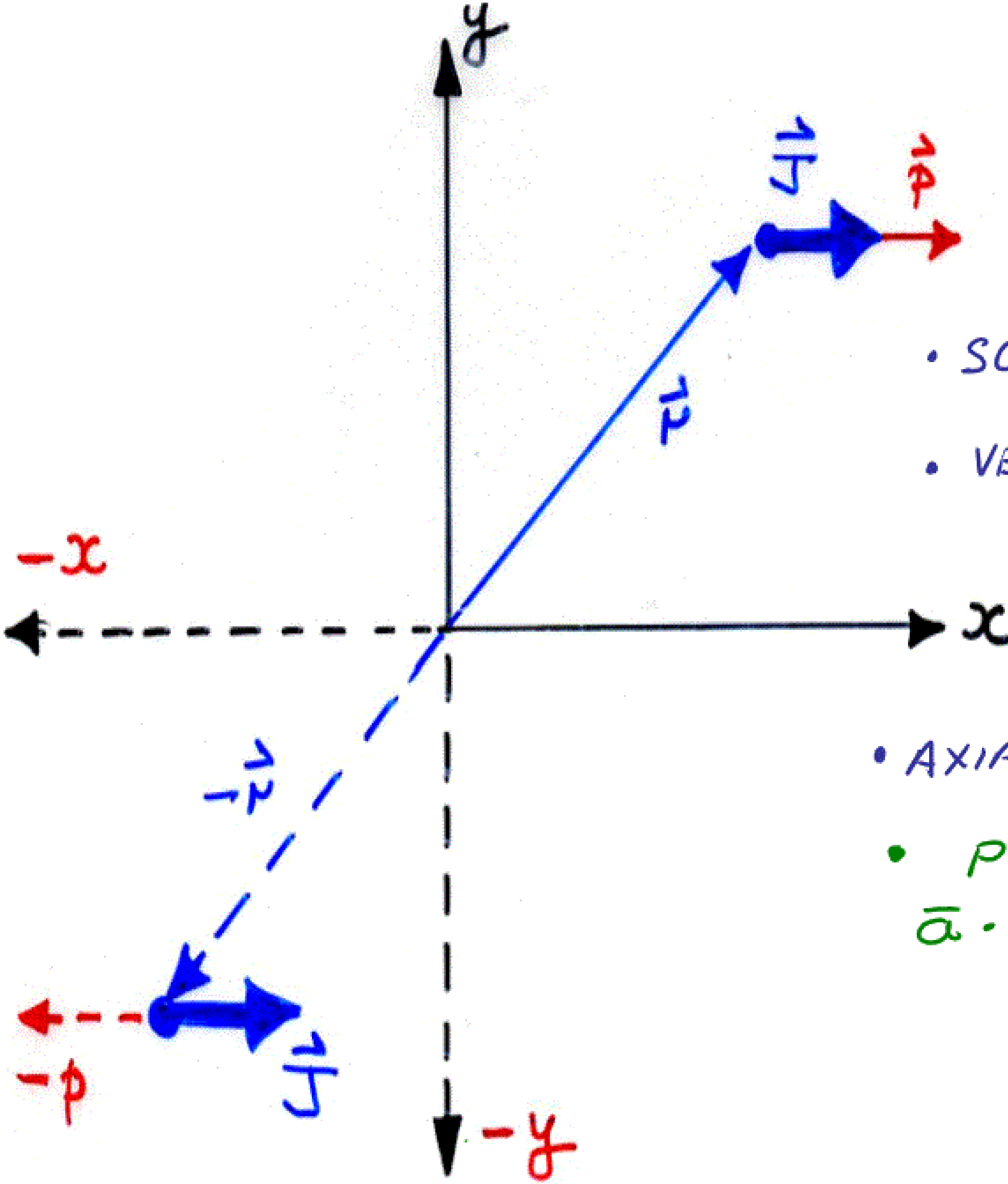
THIS DEFINES TRANSFORMATION OF VECTORS & SCALARS

ANGULAR MOMENTUM ?

$$\bar{L} = \bar{r}\times\bar{p} \rightarrow (-\bar{r})\times(-\bar{p}) = \bar{r}\times\bar{p} = \bar{L}$$

DOES NOT TRANSFORM AS TRUE VECTOR

$$\left.\begin{array}{l}\text{AXIAL VECTOR}\\ \text{PSEUDOVECTOR}\end{array}\right\}$$



- SCALARS UNCHANGED
- VECTORS CHANGE SIGN

- AXIAL VECTORS UNCHANGED
- PSEUDSCALARS

$\bar{a}\cdot(\bar{b}\times\bar{c}) \rightarrow -\bar{a}(\bar{b}\times\bar{c})$

PARITY $\longrightarrow$ UNITARY $\rightarrow$ SYMMETRY OF H

$$P|\psi(\bar{x})\rangle \rightarrow |\psi(-\bar{x})\rangle$$

SECOND PARITY OPERATION RETURNS TO ORIGINAL

$$P.P|\psi(\bar{x})\rangle \propto P|\psi(-\bar{x})\rangle \propto |\psi(\bar{x})\rangle$$

$P^2 = I \longrightarrow$ HERMITIAN, OBSERVABLE

$$P^2|\psi(\bar{x})\rangle = \pi^2|\psi(\bar{x})\rangle$$

PARITY OPERATOR

PARITY EIGENVALUE

$\pi = +1$ EVEN

$ -1$ ODD

CONSERVATION OF PARITY

CONSERVATION OF PARITY IS A MULTIPLICATIVE CONSERVATION LAW

$$a + b \rightarrow c + d$$

$$|\text{INITIAL}\rangle = |a\rangle |b\rangle |\text{RELATIVE MOTION}\rangle$$

$$P|i\rangle = P|a\rangle \, P|b\rangle \, P|\text{RELATIVE}\rangle$$

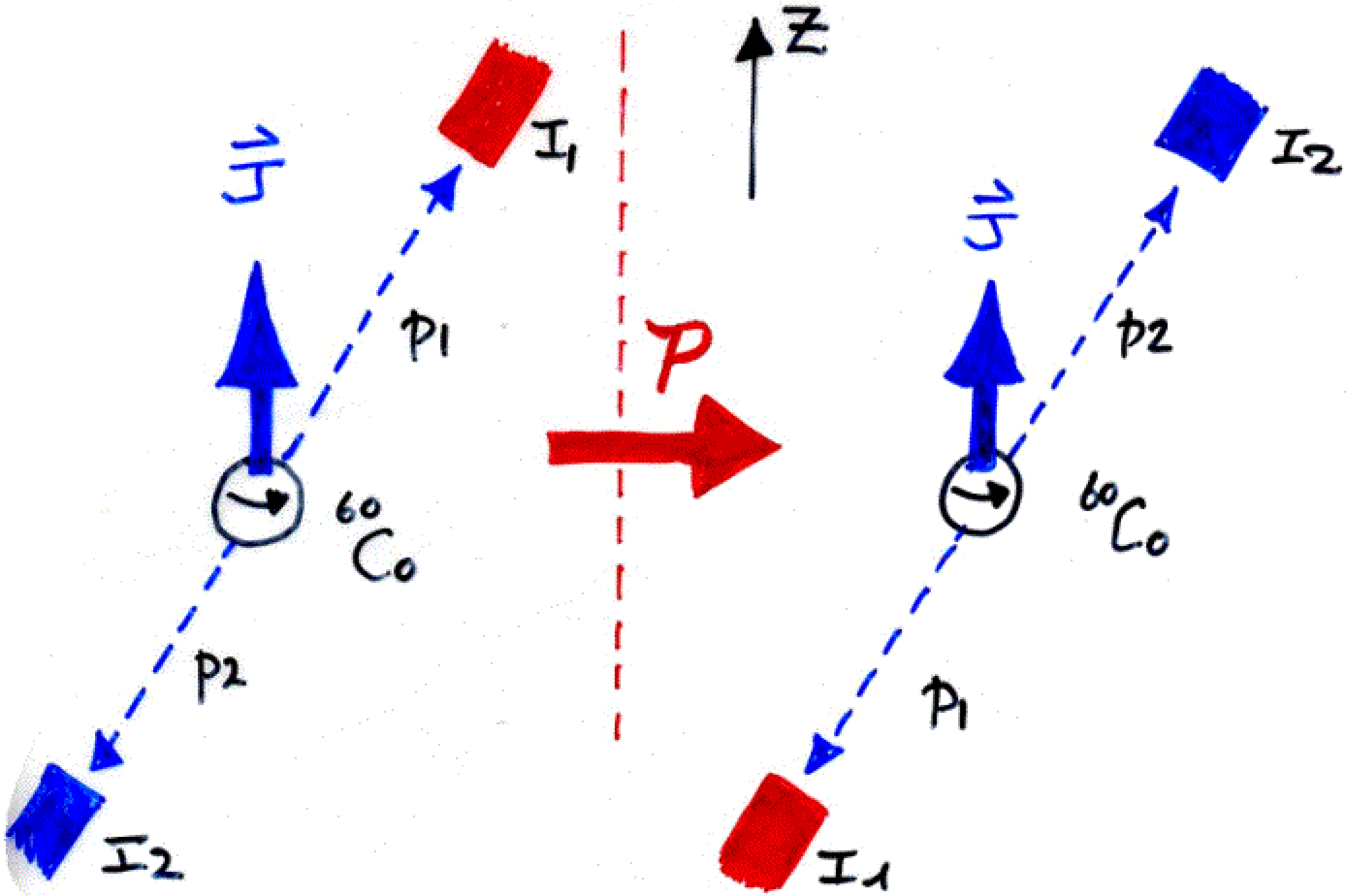
RADIAL $|\vec{r}| \rightarrow |\vec{r}| \rightarrow +1$

ANGULAR (ORBITAL) $\rightarrow (-1)^{\ell}$

$$P|a\rangle = \pi_a |a\rangle$$

INTRINSIC PARITY $\rightarrow$ AN ATTRIBUTE OF THE PARTICLE $\rightarrow$ cf SPIN, FLAVOR

LEE & YANG → WEAK INTERACTION VIOLATES PARITY CONSERVATION

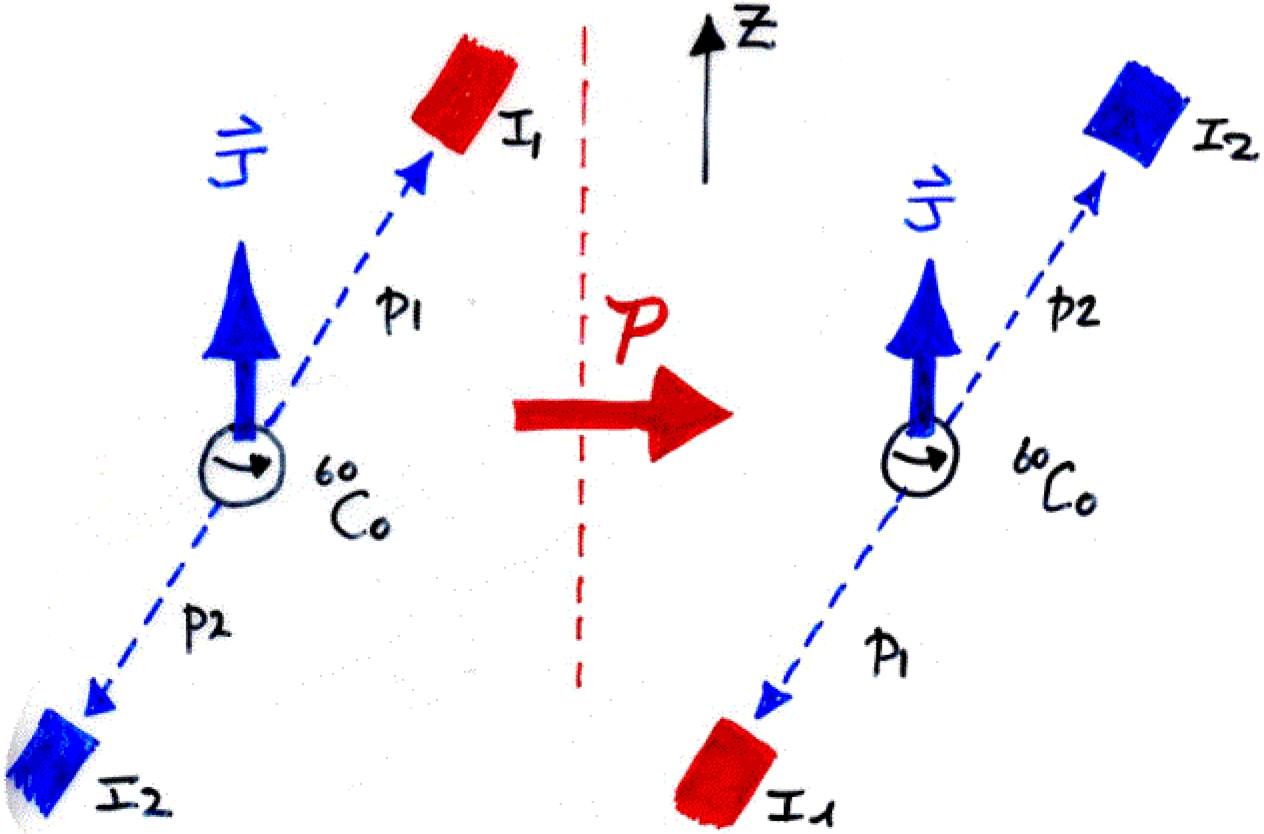


WU ET AL, ←

$^{60}Co \rightarrow {}^{60}Ni \; e^- \; \bar{\nu}_e$

↑ POLARIZED, $\vec{J}$ ALONG Z-AXIS

IS CONFIGURATION DRAWN ON RIGHT POSSIBLE IN NATURE?

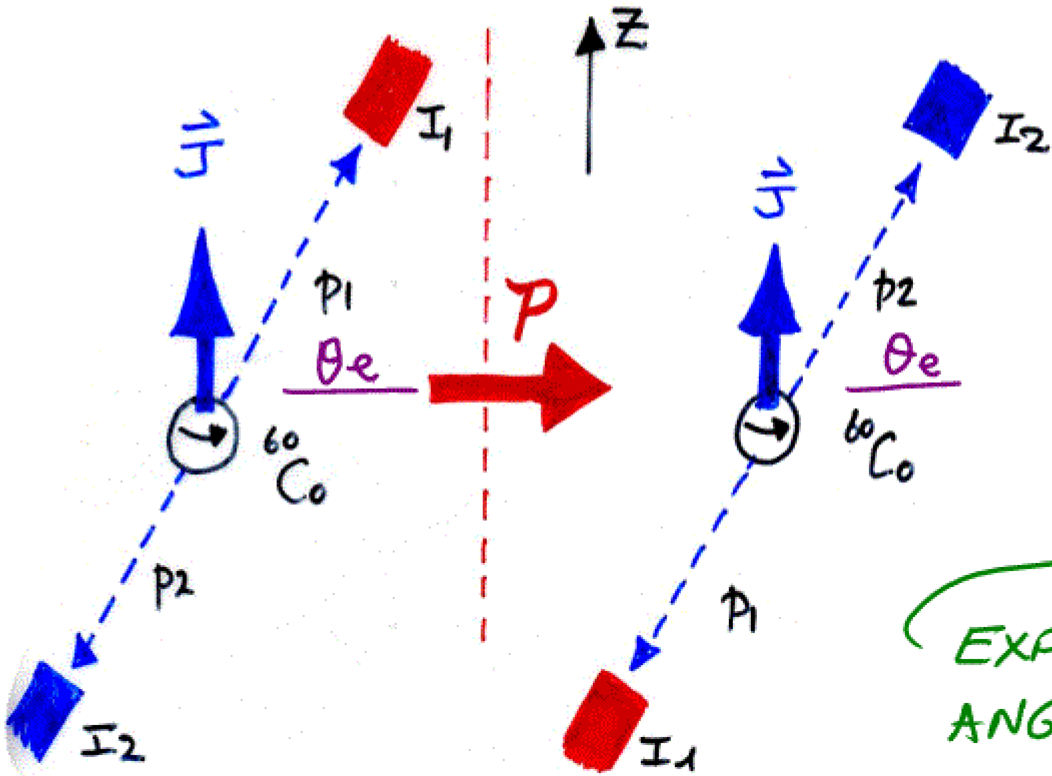


PARITY OPERATION $\longrightarrow$

$$\bar{J} \to \bar{J}$$
$$\bar{p}_1 \to -\bar{p}_1$$
$$\bar{p}_2 \to -\bar{p}_2$$

INVARIANCE UNDER PARITY OPERATION

$$I_1 = I_2$$



INVARIANCE UNDER PARITY

$I_1 = I_2$

ELECTRONS HAVE ANGULAR DISTRIBUTION θ_e

EXPERIMENT MEASURES THIS ANGULAR DISTRIBUTION

EXPECTATION VALUE OF $\cos\theta_e$

$$\langle \cos\theta_e \rangle = \left\langle \frac{\vec{J}\cdot\vec{p}}{|\vec{J}||\vec{p}|} \right\rangle$$

PSEUDO SCALAR - CHANGES SIGN UNDER PARITY

SCALAR DOES NOT CHANGE SIGN

PARITY INVARIANCE $\rightarrow I_1 = I_2 \rightarrow \langle \cos\theta_e \rangle$ UNCHANGED

$$\langle \cos\theta_e \rangle = \left\langle \frac{\bar{J}\cdot\bar{p}}{|\bar{J}||\bar{p}|} \right\rangle$$

BUT $\bar{J}\cdot\bar{p} \xrightarrow{P} -\bar{J}\cdot\bar{p}$

ONLY SOLUTION FOR PARITY INVARIANCE

$\bar{J}\cdot\bar{p} = 0 \rightarrow \langle \cos\theta_e \rangle = 0 \rightarrow$ NO UP-DOWN ASYMMETRY W.R.T Z-AXIS

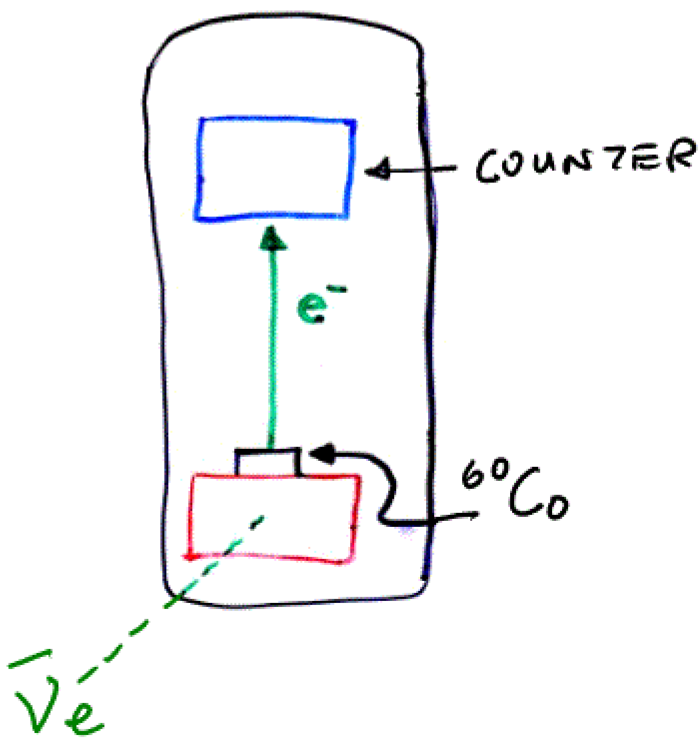
- EXPERIMENT OBSERVED LARGE UP-DOWN ASYMMETRY
- ELECTRONS PREFERENTIALLY EMITTED OPPOSITE TO NUCLEAR SPIN

PARITY INVARIANCE VIOLATED

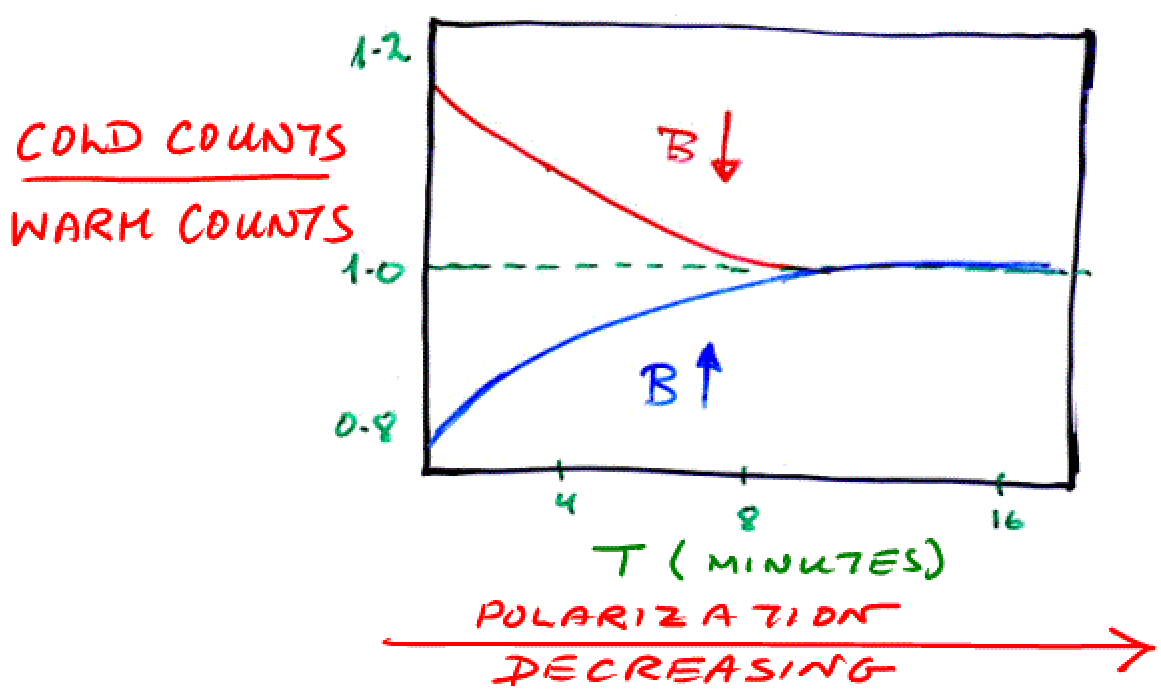
$$\vec{J}\cdot\vec{p} \xrightarrow{P} -\vec{J}\cdot\vec{p}$$

EXPERIMENTALLY INVERT $\vec{J}$ BY REVERSING THE MAGNETIC FIELD POLARIZING ^{60}Co

EXPERIMENTAL TOUR DE FORCE → TEXT BOOK P. 250
FEYNMAN LECTURES
ADIABATIC DEMAGNETIZATION



β ASYMMETRY



Mme Wu APPARATUS

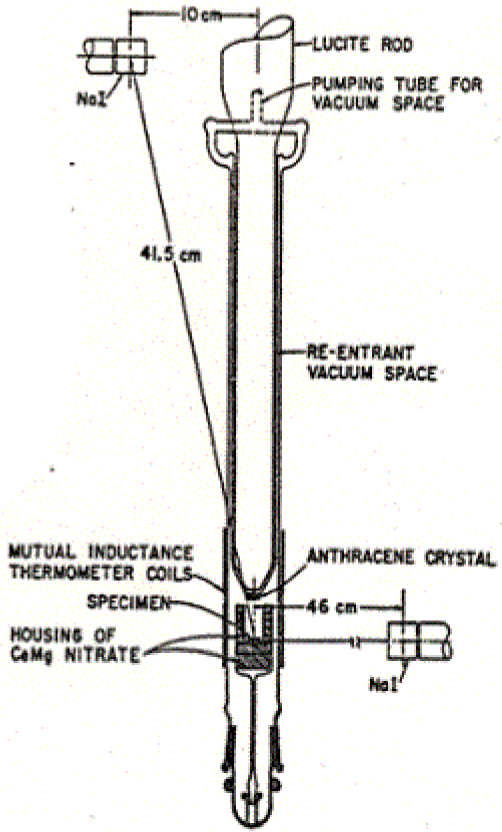


FIG. 1. Schematic drawing of the lower part of the cryostat.

EXPERIMENTAL RESULTS

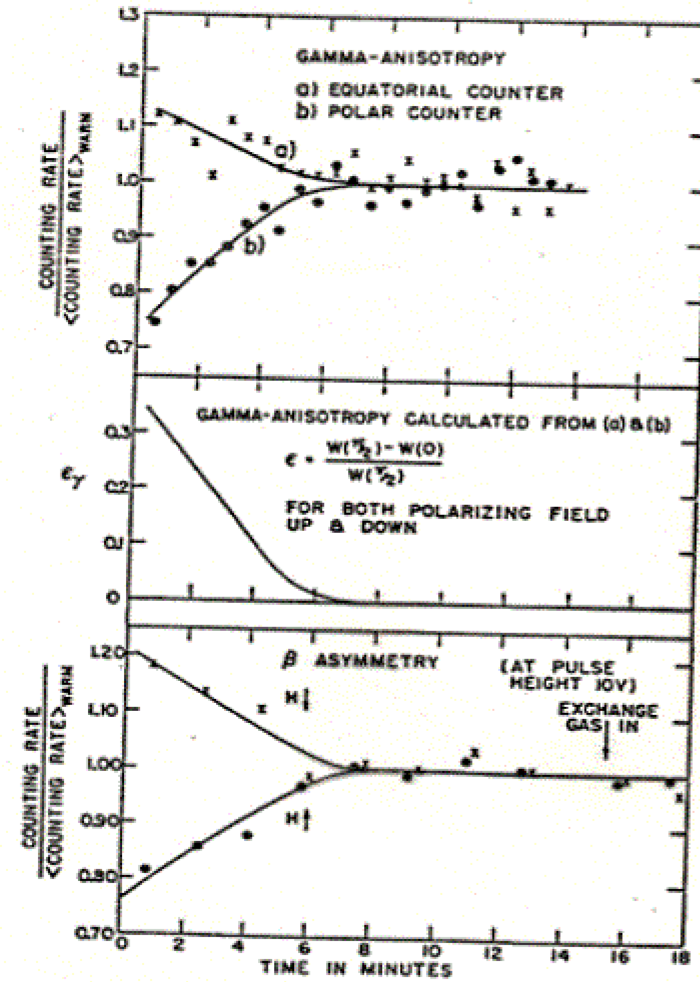
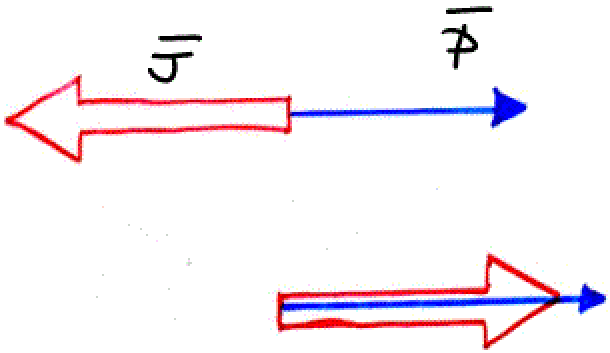


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

NEUTRINOS $\rightarrow$ MANIFESTLY VIOLATE PARITY INVARIANCE



ν HELICITY $= 2\dfrac{\bar{J}\cdot\hat{p}}{\hbar} = \begin{matrix} +1 & \nu \\ -1 & \bar{\nu} \end{matrix}$

$\bar{\nu}$

$\hat{p}$ IS A UNIT VECTOR IN THE DIRECTION OF THE MOMENTUM

MASSLESS ν PROPAGATE AT VELOCITY $= c$

PARITY TRANSFORM $\nu \rightarrow \bar{\nu}$ CANNOT BE PARITY EIGEN STATE

$$A \rightarrow B + C + \nu$$

$$\downarrow P$$

$$A \rightarrow B + C + \bar{\nu}$$

NOT SAME PHYSICAL PROCESS

PARITY CONSERVATION IN QED

PARITY CONSERVATION IN QED ARISES FROM FORM OF THE INTERACTION

$$\mathcal{M} = \frac{Q_q e^2}{q^2} j_e \cdot j_q \qquad e^- q \to e^- q$$

CURRENTS ARE

$$j_e^\mu = \bar{u}(p_3)\gamma^\mu u(p_1)$$

$$j_q^\nu = \bar{u}(p_4)\gamma^\nu u(p_2)$$

WE'LL SEE THAT THIS IS WHAT IS IMPORTANT

APPLY PARITY TO THIS, IN QED $\hat{P} = \gamma^0 \qquad u \xrightarrow{\hat{P}} \hat{P}u = \gamma^0 u$

$$\bar{u} = u^\dagger \gamma^0 \xrightarrow{\hat{P}} (\hat{P}u)^\dagger \gamma^0 = u^\dagger \gamma^{0\dagger} \gamma^0 = u^\dagger \gamma^0 \gamma^0 = \bar{u}\gamma^0$$

SO $j_e^\mu = \bar{u}(p_3)\gamma^\mu u(p_1) \xrightarrow{\hat{P}} \bar{u}(p_3)\gamma^0\gamma^\mu\gamma^0 u(p_1)$

$$\hat{P}\, j_e^{\mu} = \bar{u}(p_3)\gamma^0\gamma^{\mu}\gamma^0 u(p_1)$$

$$\gamma^0\gamma^0 = I \quad \text{so} \quad j_e^0 \xrightarrow{\hat{P}} \bar{u}\gamma^0\gamma^0\gamma^0 u = \bar{u}\gamma^0 u = j_e^0$$

$$j_e^k \xrightarrow{\hat{P}} \bar{u}\gamma^0\gamma^k\gamma^0 u = -\bar{u}\gamma^k\gamma^0\gamma^0 u = -\bar{u}\gamma^k u = -j_e^k$$

PARITY CHANGES SIGN OF SPACELIKE (p_x, p_y, p_z)
LEAVES TIMELIKE (ENERGY) UNCHANGED

$$j_e \cdot j_q = j_e^0 j_q^0 - j_e^k j_q^k \xrightarrow{\hat{P}} j_e^0 j_q^0 - (-j_e^k)(-j_q^k) = j_e \cdot j_q$$

$\mathcal{M} \xrightarrow{\hat{P}} \mathcal{M} \rightarrow$ INVARIANT UNDER PARITY OPERATION

PARITY IS CONSERVED IN QED

QCD IS ALSO $\bar{u}\gamma^{\mu}u \rightarrow$ PARITY CONSERVED IN STRONG INTERACTION

LORENTZ STRUCTURE OF WEAK INTERACTION

QED AND QCD ARE VECTOR INTERACTIONS

CURRENT $\longrightarrow$ $j^\mu = \bar{u}(p')\gamma^\mu u(p)$

TRANSFORMS AS A 4-VECTOR

WEAK INTERACTION VIOLATES PARITY

$\longrightarrow$ CANNOT BE A VECTOR INTERACTION

$\longrightarrow$ WHAT IS THE FORM OF THE CURRENT??

LORENTZ INVARIANCE IS A SEVERE RESTRICTION

GENERAL BILINEAR COMBINATION OF 2 SPINORS

$$\bar{u}(p')\,\Gamma\,u(p)$$

$\hookrightarrow$ 4×4 MATRIX MADE UP OF COMBINATIONS O DIRAC γ^μ MATRICES

ALLOWED BY LORENTZ INVARIANCE

BILINEAR COVARIANTS $\rightarrow \bar{u}(p')\,\Gamma\, u(p)$

TYPE	FORM	RANK	BOSON SPIN
SCALAR	$\bar{\psi}\phi$	1	0
PSEUDOSCALAR	$\bar{\psi}\gamma^5\phi$	1	0
VECTOR	$\bar{\psi}\gamma^\mu\phi$	4	1
AXIAL VECTOR	$\bar{\psi}\gamma^\mu\gamma^5\phi$	4	1
TENSOR	$\bar{\psi}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)\phi$	6	2

QED $g^{\mu\nu}$ → COMES FROM SUM OVER $(2J+1)+1$ POLARIZATION STATES OF $J^P = 1^-$ OF THE VIRTUAL γ^ν

4 POLARIZATION STATES → $j^\mu = \bar{\psi}\gamma^\mu\phi$

$\mu = 0, 1, 2, 3$

SINGLE COMPONENT SCALAR AND PSEUDOSCALAR

↳ SPIN 0 BOSON EXCHANGE $J=0$

6 COMPONENTS OF TENSOR — SPIN 2 EXCHANGE

$J=2$ → $(2J+1)+1 = 6$ POLARIZATION STATES

MOST GENERAL LORENTZ FORM

MOST GENERAL FORM OF INTERACTION BETWEEN BOSON AND FERMION IS A LINEAR COMBINATION OF BILINEAR COVARIANTS

IF RESTRICT TO EXCHANGE OF SPIN-1 BOSON MOST GENERAL IS LINEAR COMBINATION OF

VECTOR + AXIAL VECTOR

$$J^\mu \propto \bar{u}(p')\left(g_V \gamma^\mu + g_A \gamma^\mu \gamma^5\right) u(p) = g_V j_V^\mu + g_A J_A^\mu$$

$$j_V^\mu = \bar{u}(p') \gamma^\mu u(p) \quad , \quad j_A = \bar{u}(p') \gamma^\mu \gamma^5 u(p)$$

WE LOOKED AT PARITY TRANSFORMATION OF j^{μ}_{V}

→ i.e. QED VECTOR INTERACTION

IN THE SAME WAY:

$$j^{\mu}_{A} = \bar{u}\gamma^{\mu}\gamma^{5}u \xrightarrow{P} \bar{u}\gamma^{0}\gamma^{\mu}\gamma^{5}\gamma^{0}u = -\bar{u}\gamma^{0}\gamma^{\mu}\gamma^{0}\gamma^{5}u$$

$$\rightarrow \gamma^{5}\gamma^{0} = -\gamma^{0}\gamma^{5}$$

THE TIME-LIKE COMPONENT OF j^{μ}_{A} TRANSFORMS:-

$$j^{0}_{A} \xrightarrow{P} -\bar{u}\gamma^{0}\gamma^{0}\gamma^{0}\gamma^{5}u = -\bar{u}\gamma^{0}\gamma^{5}u = -j^{0}_{A}$$

$$j^{k}_{A} \xrightarrow{P} -\bar{u}\gamma^{0}\gamma^{k}\gamma^{0}\gamma^{5}u = +\bar{u}\gamma^{k}\gamma^{5}u = +j^{k}_{A}$$

SCALAR PRODUCT OF TWO j_{A} → INVARIANT UNDER PARITY

$$j_1 \cdot j_2 = j^{0}_{1}j^{0}_{2} - j^{k}_{1}j^{k}_{2} \xrightarrow{P} (-j_1^{0})(-j_2^{0}) - j_1^{k}j_2^{k} = j_1 \cdot j_2$$

MATRIX ELEMENT IS A SCALAR $\rightarrow j_1 \cdot j_2$

PRODUCT OF 2 SCALARS OR 2 VECTORS $\rightarrow$ PARITY INVARIANT

$$j_V^0 \xrightarrow{P} +j_V^0, \quad j_V^k \xrightarrow{P} -j_V^k, \quad j_A^0 \xrightarrow{P} -j_A^0, \quad j_A^k \xrightarrow{P} +j_A^k$$

WEAK INTERACTION DOES NOT CONSERVE PARITY

$\rightarrow$ EXPERIMENTAL OBSERVATION !

$$j_V \cdot j_A \xrightarrow{P} -j_V \cdot j_A$$ FEYNMAN

NUCLEAR β-DECAY WAS WHERE MME. WU FIRST OBSERVED PARITY VIOLATION

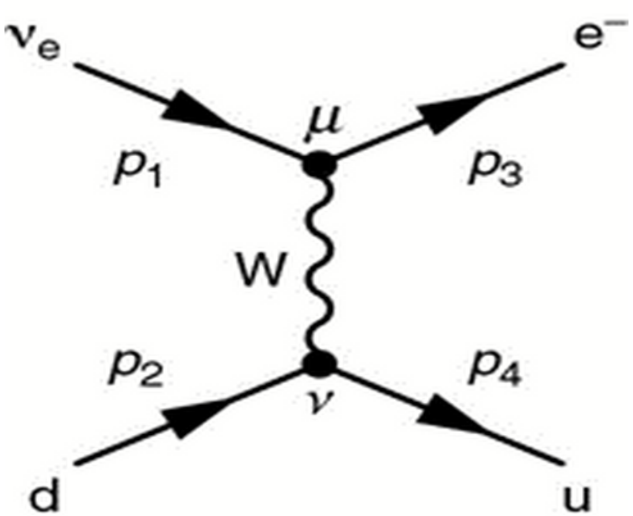
$\rightarrow$ USE OUR FORMALISM TO LOOK AT IT.

WEAK CHARGED CURRENT

NUCLEAR β-DECAY $\quad n \to p\, \bar{\nu}_e\, e^-$ $\qquad n = ddu$

$\quad d \to u\, \bar{\nu}_e\, e^-$

BY CROSSING $\quad \nu_e\, d \to e^-\, u$



$$j^{\mu}_{\nu e} = \bar{u}(p_3)(g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) u(p_1)$$
$$= g_V\, j^V_{\nu e} + g_A\, j^A_{\nu e}$$

$$j^{\nu}_{du} = \bar{u}(p_4)(g_V \gamma^\nu + g_A \gamma^\nu \gamma^5) u(p_2)$$
$$= g_V\, j^V_{du} + g_A\, j^A_{du}$$

$$\mathcal{M} \to j_{\nu e} \cdot j_{du}$$

$$\mathcal{M}_{fi} \propto j_{\nu e} \cdot j_{du} = g_V^2\, j^V_{\nu e} \cdot j^V_{du} + g_A^2\, j^A_{\nu e} \cdot j^A_{du}$$
$$+ g_V g_A \left(j^V_{\nu e} \cdot j^A_{du} + j^A_{\nu e} \cdot j^V_{du} \right)$$

$j^V_{\nu e} \cdot j^V_{\nu e}$, $j^A_{\nu e} \cdot j^A_{\nu e}$ CONSERVE PARITY

$j^V_{\nu e} \cdot j^A_{du}$ AND $j^A_{\nu e} \cdot j^V_{du}$ VIOLATE PARITY

$$j_{\nu e} \cdot j_{du} \xrightarrow{P} g_V^2\, j^V_{\nu e} \cdot j^V_{du} + g_A^2\, j^A_{\nu e} \cdot j^A_{du}$$
$$- g_V g_A \left(j^V_{\nu e} \cdot j^A_{du} + j^A_{\nu e} \cdot j^V_{du} \right)$$

RELATIVE STRENGTH OF PARITY VIOLATING CURRENT AND PARITY CONSERVING CURRENT

$$\frac{g_V g_A}{g_V^2 + g_A^2}$$

$$\frac{g_V\, g_A}{g_V^2 + g_A^2}$$

IF EITHER g_V OR $g_A = 0 \rightarrow$ PARITY CONSERVATION

MAXIMAL PARITY VIOLATION $|g_A| = |g_V|$

↳ PURE (V−A) INTERACTION → FEYNMAN

WEAK CHARGED CURRENT $W^\pm$ EXCHANGE

W VERTEX FACTOR → $-\frac{i g_W}{\sqrt{2}} \frac{1}{2} \gamma^\mu (1-\gamma^5)$

CURRENT $j^\mu = \frac{g_W}{\sqrt{2}} \bar{u}(p') \frac{1}{2} \gamma^\mu (1-\gamma^5) u(p)$

HISTORICAL