DISCRETE SYMMETRIES

- SOME OF THE MOST SURPRISING RESULTS IN 20TH CENTURY PHYSICS

PARITY VIOLATION RIGHT-LEFT ASYMMETRY IN UNIVERSE AT MOST FUNDAMENTAL LEVEL

→ COMPLETE SURPRISE

TIME REVERSAL ASYMMETRY MAY BE ORIGIN OF ASYMMETRY BETWEEN MATTER & ANTIMATTER IN THE UNIVERSE

ALSO MAY BE CONNECTED TO THE OBSERVED 3 GENERATIONS OF QUARKS & LEPTONS

DISCRETE SYMMETRIES — PROPERTIES OF UNIVERSE

TIME REVERSAL

IN QUANTUM FIELD THEORY, ALL SENSIBLE HAMILTONIANS HAVE THE SYMMETRY!

$$[CPT, H]$$

CHARGE CONJUGATION

PARTICLES → ANTIPARTICLES

PARITY

TIME REVERSAL

WEAK INTERACTION → VIOLATES P ∴ → ALSO SOME COMBINATION OF CT

$$t \xrightarrow{T} -t$$

$$\bar{r} \longrightarrow \bar{r}$$

$$\bar{p} = m\dot{\bar{r}} \longrightarrow -m\dot{\bar{r}} = -\bar{p}$$

$$\bar{L} = \bar{r} \times \bar{p} \longrightarrow \bar{r} \times (-\bar{p}) = -\bar{L}$$

CLASSICALLY

$$m \frac{d^2 \bar{r}}{dt^2} = \bar{F}$$

MANIFESTLY INVARIANT UNDER $t \to -t$

2ND ORDER DIFFERNTIAL → ALL MICROSCOPIC CLASSICAL SYSTEMS ARE T INVARIANT

USUALLY "ARROW OF TIME" ASSERTED TO COME FROM ENTROPY → READ R. PENROSE.

IN QUANTUM MECHANICS SCHRÖDINGER SHOWS HOW TIME EVOLUTION DEPENDS ON HAMILTONIAN

$$i\hbar \frac{\partial \psi(\bar{r},t)}{\partial t} = H \psi(\bar{r},t)$$

1ST ORDER IN t →

∴ NOT T INVARIANT →

$$\psi(\bar{r},t) \xrightarrow{T} \psi(\bar{r},-t)$$

IF T IS A SYMMETRY OF THE HAMILTONIAN

$$[H, T] = 0 \quad \text{AND IF} \quad \psi(-t) = T\psi(t)$$

THEN

$$T\left(i\hbar \frac{\partial \psi}{\partial t} = H\psi\right) \rightarrow i\hbar \frac{\partial [TH]}{\partial(-t)} = HT\psi$$

SINCE

$$T\psi(t) \longrightarrow \psi(-t)$$

$$-i\hbar \frac{\partial \psi(-t)}{\partial t} = H\psi(-t)$$

THIS IS NOT SAME AS $i\hbar \dfrac{\partial \psi(t)}{\partial t} = H\psi(t)$

$\psi(-t)$ AND $\psi(t)$ OBEY DIFFERENT DYNAMICS

CAN MAINTAIN TIME REVERSAL SYMMETRY

IF DEFINE! $\psi(\vec{r},t) \xrightarrow{T} \psi^*(\vec{r},-t)$

TIME REVERSED SCHRÖDINGER

$$i\hbar \frac{\partial[T\psi]}{\partial(-t)} = HT\psi$$

IS $$-i\hbar \frac{\partial \psi^*(-t)}{\partial t} = H\psi^*(-t)$$

TAKE COMPLEX CONJUGATE

$$i\hbar \frac{\partial \psi(-t)}{\partial t} = H\psi(-t)$$

THIS IS SAME SCHRÖDINGER, IF

$$\psi(t) \xrightarrow{T} \psi^*(-t)$$

NOTE THAT $$\psi(t)\psi^*(t) = \psi^*(-t)\psi(-t)$$

TRANSFORMATION $\psi(t) \xrightarrow{T} \psi^*(-t)$

IS OK, SINCE PHYSICS $\rightarrow \psi\psi^*$

FOR PARITY, CAN HAVE EIGENVALUE EQUATION

$$P|\psi_P\rangle = \pi\,|\psi_P\rangle$$

CANNOT HAVE EIGENVALUES OF T

$$T|\psi\rangle \rightarrow \langle\psi(-t)|$$

T IS A SYMMETRY OF THE HAMILTONIAN

DOES NOT CORRESPOND TO AN OBSERVABLE

T NOT UNITARY, HERMITIAN

ANTIUNITARY, ANTILINEAR

TIME REVERSAL IS MOTION REVERSAL

FREELY PROPAGATING PLANE WAVE STATE

$$\psi(\bar{r}, t) = e^{i(\bar{p}\cdot\bar{r} - Et)/\hbar}$$

$$T\,\psi(\bar{r}, t) = \psi^*(\bar{r}, -t)$$

$$= e^{-i(\bar{p}\cdot\bar{r} + E\cdot t)/\hbar}$$

$$= e^{i(-\bar{p}\cdot\bar{r} - Et)}$$

cf

THIS DEFINITION OF TIME REVERSAL IS SAME AS REVERSING MOMENTUM

$\rightarrow$ EXACTLY SAME AS CLASSICAL IDEA OF "RUNNING THE MOVIE BACKWARDS"

$$T\,|\bar{p}, \bar{J}\rangle \longrightarrow |-\bar{p}, -\bar{J}\rangle$$

CHARGE CONJUGATION SYMMETRY

P & T → SPACE-TIME SYMMETRIES

C → CHANGE THE ELECTRIC CHARGE OF ALL PARTICLES

IN QUANTUM FIELD THEORY THIS MEANS INTERCHANGING PARTICLES ↔ ANTIPARTICLES

ELECTROMAGNETISM

$$Q \xrightarrow{C} -Q$$

$$\vec{E} \to -\vec{E}$$

$$\vec{B} \to -\vec{B}$$

INTUITIVELY ELECTROMAGNETISM IS INVARIANT UNDER SUCH A TRANSFORMATION

QUANTUM MECHANICALLY

$$|\psi(Q, \vec{r}, t)\rangle \xrightarrow{C} |\psi(-Q, \vec{r}, t)\rangle$$

A PARTICLE CAN BE AN EIGENSTATE OF C

$$C|\pi^0\rangle \rightarrow |\pi^0\rangle$$

FOR COMPOSITE PARTICLES, INTERNAL QUANTUM NUMBERS MAY CHANGE → EVEN FOR NEUTRAL PARTICLES

$$C|n\rangle = C|ddu\rangle \rightarrow |\bar{d}\bar{d}\bar{u}\rangle = |\bar{n}\rangle$$

$$C|K^0\rangle = C|d\bar{s}\rangle \rightarrow |\bar{d}s\rangle = |\bar{K}^0\rangle$$

JUST AS WITH PARITY, 2 SUCCESSIVE C OPERATIONS LEAVE A STATE UNCHANGED

C HAS EIGENVALUES ± 1

$C|\vec{E}\rangle \to -|\vec{E}\rangle$

$C|\vec{B}\rangle \to -|\vec{B}\rangle$

$\to \quad C|\gamma\rangle = -|\gamma\rangle$

γ HAS -VE "C-PARITY"

IF $[C, H] = 0$ TOTAL C-PARITY IS CONSERVED IN INTERACTIONS

$$\pi^0 \to \gamma\,\gamma$$

$$\eta_c(\pi^0) = \eta_c(\gamma)\cdot\eta_c(\gamma)$$

$$= (-1)\cdot(-1) = +1$$

OBSERVABLE CONSEQUENCE

$$\pi^0 \not\to 3\gamma \qquad C = -1$$

$$5\gamma$$

$$\vdots$$

CHARGE CONJUGATION IN QED

IS CHARGE CONJUGATION A "GOOD SYMMETRY" FOR QED

CLASSICALLY, TO GET MOTION OF CHARGED PARTICLE IN AN ELECTROMAGNETIC FIELD $A^\mu = (\phi, \bar{A})$

$$E \to E - q\phi \qquad \bar{p} \to \bar{p} - q\bar{A}$$

ENERGY

CHARGE ON PARTICLE

$$p_\mu \to p_\mu - qA_\mu$$

FOR QUANTUM MECHANICAL VERSION

$$\bar{p} \to -i\bar{\nabla} \qquad E = i\partial/\partial t$$

$$i\partial_\mu \to i\partial_\mu - qA_\mu$$

FREE DIRAC $\quad (i\gamma^\mu \partial_\mu - m)\psi = 0$

INTERACTING $\quad \gamma^\mu(\partial_\mu - ieA_\mu)\psi + im\psi = 0$

COMPLEX CONJ $\times\ -i\gamma^2$

$$-i\gamma^2(\gamma^\mu)^*(\partial_\mu + ieA_\mu)\psi^* - m\gamma^2\psi^* = 0$$

$(\gamma^0)^* = \gamma^0$, $(\gamma^1)^* = \gamma^1$, $(\gamma^2)^* = -\gamma^2$, $(\gamma^3)^* = +\gamma^3$, $\quad \gamma^2\gamma^\mu = -\gamma^\mu\gamma^2 \quad \mu \neq k$

$$(\gamma^\mu)^* = \left[(\gamma^0)^*, (\gamma^1)^*, (\gamma^2)^*, (\gamma^3)^*\right]$$

$$\begin{array}{lcccc}
 & \downarrow & \downarrow & \downarrow & \downarrow \\
 & \gamma^0 & \gamma^1 & -\gamma^2 & \gamma^3 \\
\times\ \gamma^2 & \gamma^2\gamma^0 & \gamma^2\gamma^1 & -\gamma^2\gamma^2 & \gamma^2\gamma^3 \\
 & -\gamma^0\gamma_2 & -\gamma^1\gamma^2 & -\gamma^2\gamma^2 & -\gamma^3\gamma^2
\end{array}$$

$$\gamma^2(\gamma^\mu)^* = -\gamma^\mu\gamma^2$$

$$i\gamma^\mu\gamma^2(\partial_\mu + ieA_\mu)\psi^* - m\gamma^2\psi = 0$$

$-m = iim = imi$

$$\gamma^\mu(\partial_\mu + ieA_\mu)\, i\gamma^2\psi^* + imi\psi = 0$$

$\psi' = i\gamma^2\psi^*$

$$\gamma^\mu(\partial_\mu + ieA_\mu)\psi' + im\psi' = 0$$

TERM $-ieA_\mu \rightarrow +ieA_\mu$

THIS EQUATION → SAME MASS, OPPOSITE CHARGE

PARTICLE → ANTIPARTICLE

$$\psi' = C\psi = i\gamma^2\psi^*$$

CHARGE CONJUGATION OPERATOR FOR QED

LOOK AT EFFECT OF C ON

$$\psi = u_1 e^{i(\bar{p}\cdot\bar{x} - Et)}$$

$$\psi' = C\psi = i\gamma^2\psi^* = i\gamma^2 u_1^* e^{-i(\bar{p}\cdot\bar{x} - Et)}$$

$$i\gamma^2 u_1^* = i\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}\sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix} = \sqrt{E+m}\begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}$$

ANTIPARTICLE SPINOR

$$\psi = u_1 e^{i(\bar{p}\cdot\bar{x} - Et)} \xrightarrow{C} \psi' = v_1 e^{-i(\bar{p}\cdot\bar{x} - Et)}$$

$$\psi = u_2 e^{i(\bar{p}\cdot\bar{x} - Et)} \xrightarrow{C} \psi' = v_2 e^{-i(\bar{p}\cdot x - Et)}$$

CP SYMMETRY

CPT → GOOD SYMMETRY FOR ALL H

EXPECT T SYMMETRY GOOD

∴ CP SHOULD ALSO BE A GOOD SYMMETRY

WE KNOW WEAK INTERACTION VIOLATES P
SO IT MUST VIOLATE C FOR CP GOOD

$$|\nu_L\rangle \xrightarrow{C} |\bar{\nu}_L\rangle$$

DOESN'T EXIST
∴ NO C SYMMETRY

$$|\nu_L\rangle \xrightarrow{C} |\bar{\nu}_L\rangle \xrightarrow{P} |\bar{\nu}_R\rangle$$

DOES EXIST

∴ LOOKS LIKE CP IS A GOOD SYMMETRY
OF THE WEAK INTERACTION

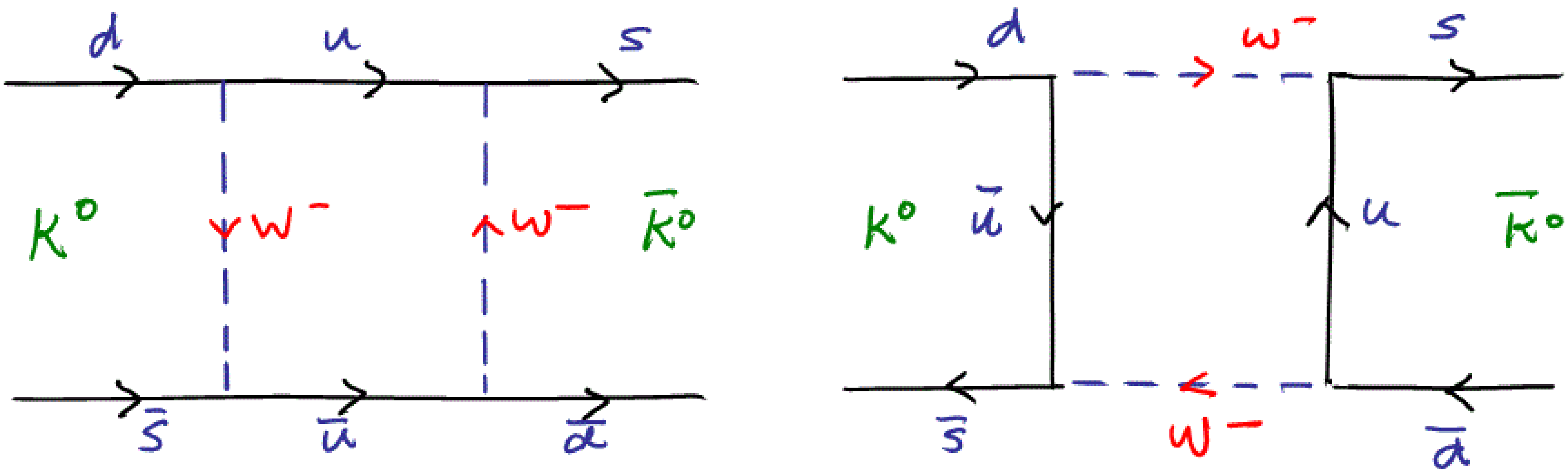
CP CONSERVED IN THE WEAK INTERACTION ?

SINCE $|\nu_L\rangle \xrightarrow{CP} |\bar{\nu}_R\rangle$

CP COULD BE A GOOD SYMMETRY OF WEAK FORCE

WEAK FORCE CAN INDUCE

$$|K^0\rangle \xrightarrow{wk} |\bar{K}^0\rangle$$



SECOND ORDER WEAK TRANSITIONS

$|K^0\rangle$ AND $|\bar{K}^0\rangle$ ARE EIGESTATES OF

STRONG INTERACTION

$$H_{ST}|K^0\rangle = m_{K^0}|K^0\rangle$$
$$H_{ST}|\bar{K}^0\rangle = m_{\bar{K}^0}|\bar{K}^0\rangle$$

DEFINITE MASSES

MASSES EQUAL → CPT

$$S|K^0\rangle = +1\,|K^0\rangle$$
$$S|\bar{K}^0\rangle = -1\,|K^0\rangle$$

DEFINITE FLAVOUR

⇒ STRANGENESS

STRANGENESS OPERATOR

THEY ARE PRODUCED BY STRONG INTERACTION

→ STRONG EIGENSTATES

$$\pi^- p \to \bar{K}^0 \Lambda$$

CAN ONLY DECAY THROUGH WEAK INTERACTION

SINCE

$$|K^0\rangle \xrightarrow{WK} |\bar{K}^0\rangle$$

THESE STRONG EIGENSTATES CANNOT BE EIGENSTATES OF THE WEAK INTERACTION

HYPOTHESIZE THAT WEAK FORCE CONSERVES

CP → T INVARIANCE

$$C|K^0\rangle = |\bar{K}^0\rangle$$

$$C|\bar{K}^0\rangle = |K^0\rangle$$

AND

$$P|K^0\rangle = -|K^0\rangle$$

$$P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$$

$$\rightarrow \quad \underbrace{\begin{aligned} CP|K^0\rangle &= -|\bar{K}^0\rangle \\ CP|\bar{K}^0\rangle &= -|K^0\rangle \end{aligned}}_{\text{NOT CP EIGENSTATES}}$$

CP CONSERVED IN WEAK INTERACTIONS

$$CP|K^0\rangle = -|\bar{K}^0\rangle$$

$$CP|\bar{K}^0\rangle = -|K^0\rangle$$

NOT WEAK EIGENSTATES

BUT, THEY ARE A COMPLETE SET OF STRONG EIGENSTATES → CAN CONSTRUCT ANY OTHER STATES OF K^0 $\bar{K}^0$ BY LINEAR SUPERPOSITION OF $|K^0\rangle, |\bar{K}^0\rangle$ INCLUDING WEAK EIGENSTATES

TRY!

$$|K^0_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

$$|K^0_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$CP|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(CP|K^0\rangle - CP|\bar{K}^0\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(-|\bar{K}^0\rangle + |K^0\rangle\right)$$

$$= +1\ |K_1^0\rangle \rightarrow \text{CP EIGENSTATE}$$

$$CP|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(CP|K^0\rangle + CP|\bar{K}^0\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(-|\bar{K}^0\rangle - |K^0\rangle\right)$$

$$= -1\ |K_2^0\rangle \rightarrow \text{CP EIGENSTATE}$$

$|K_1^0\rangle$ $|K_2^0\rangle$ $\rightarrow$ DO NOT HAVE DEFINITE STRANGENESS

↳ OK, NOT CONSERVED BY WEAK

- NOT EIGENSTATES OF S, C, P

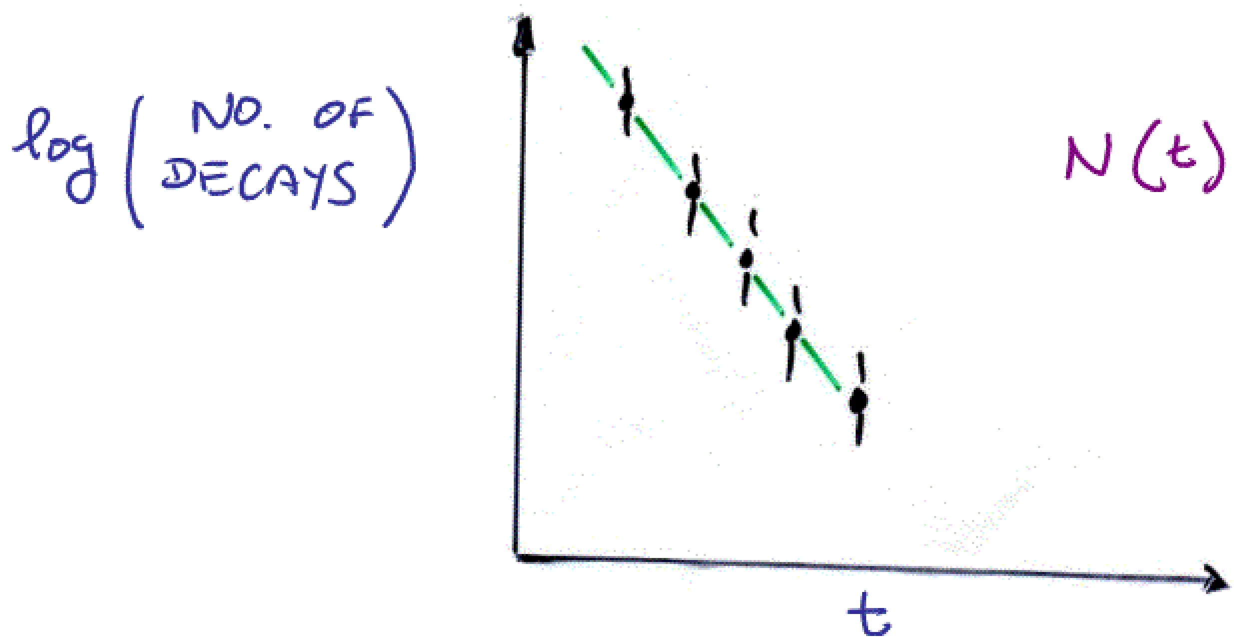
$|K^0\rangle$, $|\bar{K}^0\rangle$ PRODUCED BY STRONG FORCE

$|K_1^0\rangle$, $|K_2^0\rangle$ STATES WE SEE DECAYING BY WEAK FORCE

→ DIFFERENT LIFETIMES & DECAY MODES

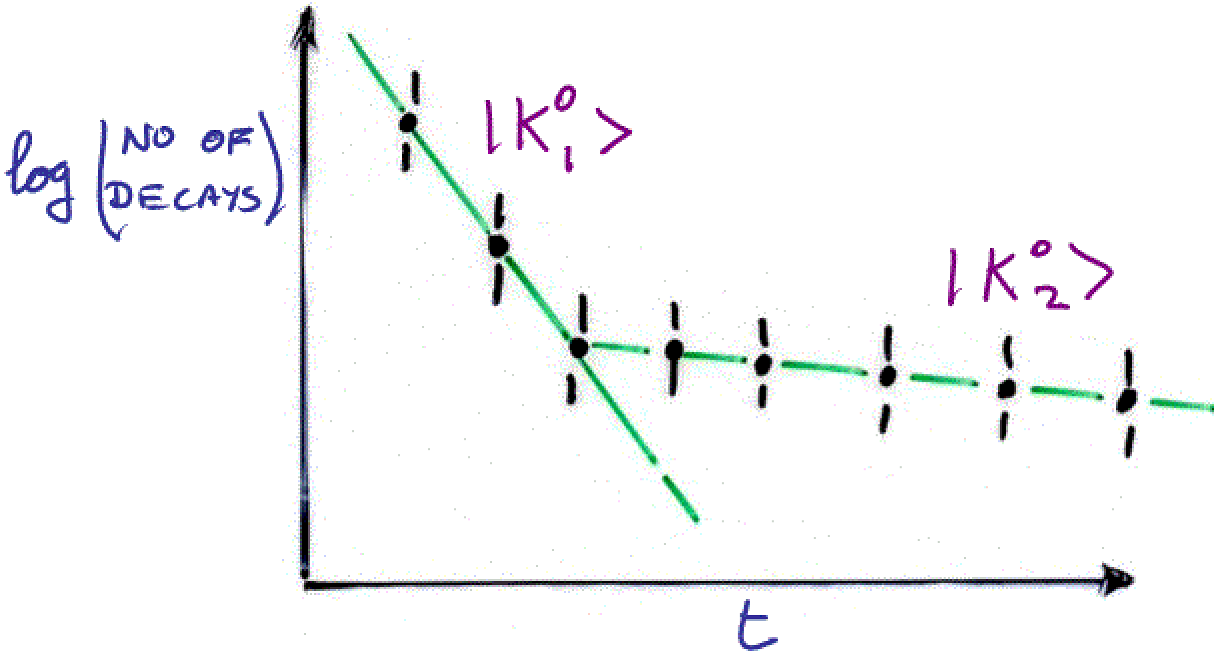
PRODUCE A BEAM OF $|K^0\rangle$ BY STRONG FORCE

IF K^0 DECAYED WITH A SINGLE LIFETIME



$$N(t) = N(0)\, e^{-t/\tau}$$

WHAT ONE SEES EXPERIMENTALLY:



K_1^0 K_2^0 HAVE DIFFERENT LIFETIMES → DECAY AT DIFFERENT RATES

STRONG PRODUCTION GIVES $|K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + |K_2^0\rangle\right)$

AFTER SOME TIME $|K_1^0\rangle$ COMPONENT OF BEAM DECAYS → NO LONGER PURE $|K^0\rangle$

→ WEAK INTERACTION SLOWLY CHANGES STRANGENESS OF THE BEAM

AT $t=0$

$$|BEAM\rangle = |K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + |K_2^0\rangle\right)$$

CP MIXTURE

$t \gg \tau_{K_1^0}$ $|K_1^0\rangle$ DECAYED

AT $t \gg \tau_{K_1^0}$

$$|BEAM\rangle = \text{PURE } K_2^0 = |K_2^0\rangle$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

WEAK INTERACTION HAS CHANGED

MATTER $\rightarrow$ MATTER + ANTIMATTER

FAR FROM PRODUCTION OF $|K^0\rangle$ BEAM

HAVE PURE $|K_2^0\rangle \rightarrow CP = -1$ EIGENSTATE

IF CP IS CONSERVED BY WEAK INTERACTION

$$|K_2^0\rangle \rightarrow |CP=-1\rangle \qquad J_\pi^{PC} = 0^{-+}$$

MOSTLY $$|K_2^0\rangle \rightarrow \pi^+\pi^-\pi^0 \quad CP = -1 \;\checkmark$$

SURPRISE, SURPRISE

$$|K_2^0\rangle \rightarrow \pi^+\pi^- \quad \text{OBSERVED}$$

$$CP = +1$$

$$\frac{\text{No.}\,(K_2^0 \rightarrow \pi^+\pi^-)}{\text{No.}\,(K_2^0 \rightarrow \text{ANYTHING})} = 2\times 10^{-3}$$

CP NOT CONSERVED BY WEAK INTERACTION

CPT $\Rightarrow$ TIME REVERSAL NON-INVARIANCE

CONSIDER 2π STATE $K^0_2 \rightarrow \pi^+\pi^-$

K^0_2, π^+, π^- ALL SPIN 0 $\rightarrow$ $\ell_{\pi\pi} = 0$ $(-1)^{\ell} = 0$

$$P|\pi^+\pi^-\rangle = + |\pi^+\pi^-\rangle$$

$$C|\pi^+\pi^-\rangle = + |\pi^-\pi^+\rangle$$

$$CP|\pi^+\pi^-\rangle = +1 \ |\pi^+\pi^-\rangle$$

$$|K^0_2\rangle \rightarrow |\pi^+\pi^-\rangle$$

$CP = -1$ $CP = +1$

BOOM !

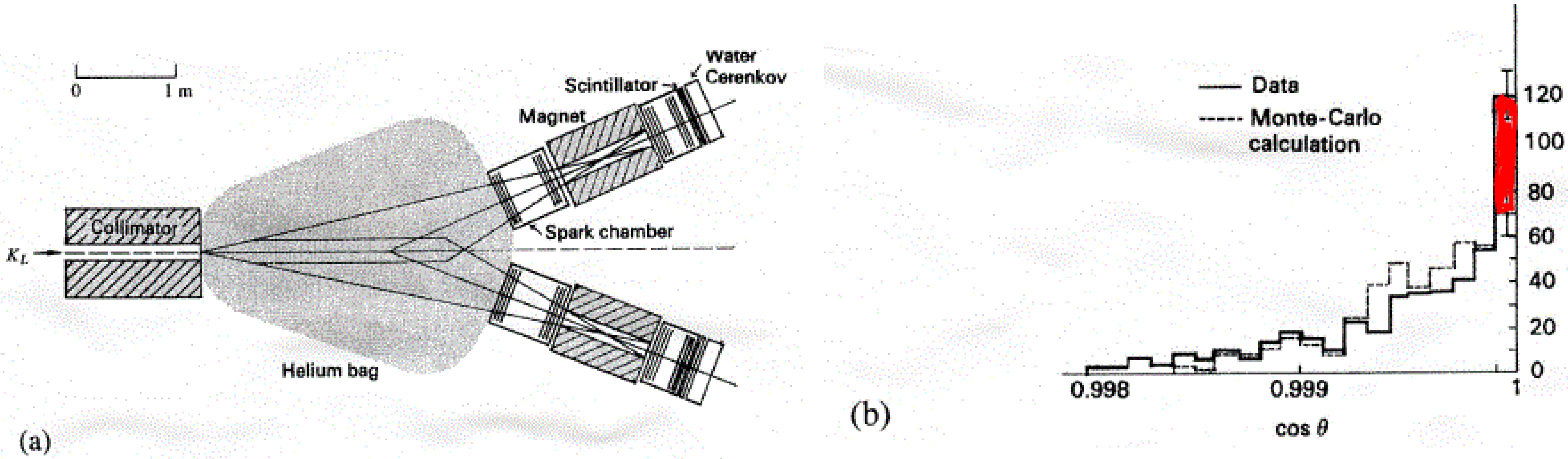


Fig. 7.22. (a) Arrangement of Christenson *et al.* (1964) demonstrating the CP-violating decay $K_L \to \pi^+\pi^-$. K_L decays are observed in a helium bag, the charged products being analysed by two magnet spectrometers instrumented with spark chambers and scintillators. (b) Rare two-pion decays are distinguished from the common three-pion decays by the invariant mass of the pair (490 MeV $< M_{\pi\pi} <$ 510 MeV) and the direction, θ, of the resultant momentum vector. The $\cos\theta$ distribution is that expected from three-body decays, plus 50 events (shaded) collinear with the beam and attributed to the two-pion decay mode.

STRANGENESS OSCILLATIONS

- WEAK INTERACTION EIGENSTATES ARE RELEVANT FOR K^0 $\bar{K}^0$ PROPAGATION
- WE DEFINED K^0_1 & K^0_2 $\rightarrow$ CP EIGENSTATES
- SHORT & LONG LIVED COMPONENTS OF K^0, $\bar{K}^0$ BEAM ARE NOT CP EIGENSTATES $\longrightarrow$ $\cancel{CP}$
- DEFINE

$$K^0_L = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(K^0_2 + \varepsilon K^0_1\right)$$

$$K^0_S = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(K^0_1 - \varepsilon K^0_2\right)$$

LEVEL OF CP VIOLATION

AS $\varepsilon \rightarrow 0$

$$K^0_L \rightarrow K^0_2$$

$$K^0_S \rightarrow K^0_1$$

• PROB AMPLITUDE OF K_S^0 IN REST FRAME

$$A_S(t) = A_S(0)\, e^{-\left(\Gamma_S/2 + i m_S\right)\cdot t}$$

$m_S \rightarrow$ REST MASS OF K_S^0

$\Gamma_S \rightarrow \hbar/\tau_S \rightarrow$ WIDTH OF K_S^0

SIMILARLY $$A_L(t) = A_L(0)\, e^{-\left(\Gamma_L/2 + i m_L\right)t}$$

• START WITH PURE K^0 BEAM FROM STRONG INTERACTION

• FORGET $\cancel{CP} \rightarrow \varepsilon \rightarrow 0$

$$\left.\begin{aligned} K_S^0 &= \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \\ K_L^0 &= \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \end{aligned}\right\} \quad |K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_S^0\rangle + |K_L^0\rangle\right)$$

PURE $|K^0\rangle$

$$\therefore A_S(0) = A_L(0) = \frac{1}{\sqrt{2}}$$

- K^0_L & K^0_S EVOLVE DIFFERENTLY IN TIME
- AFTER TIME t

$$I(K^0) = \frac{1}{2}\left[A_S(t) + A_L(t)\right]\left[A_S^*(t) + A_L^*(t)\right]$$

$$= \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m \cdot t\right]$$

$\Delta m = m_L - m_S$

OSCILLATION DEPENDING ON THE MASS DIFFERENCE

SIMILARLY

$$I_0(\bar{K}_0) = \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-(\Gamma_S+\Gamma_L)t/2}\cos\Delta m \cdot t\right]$$

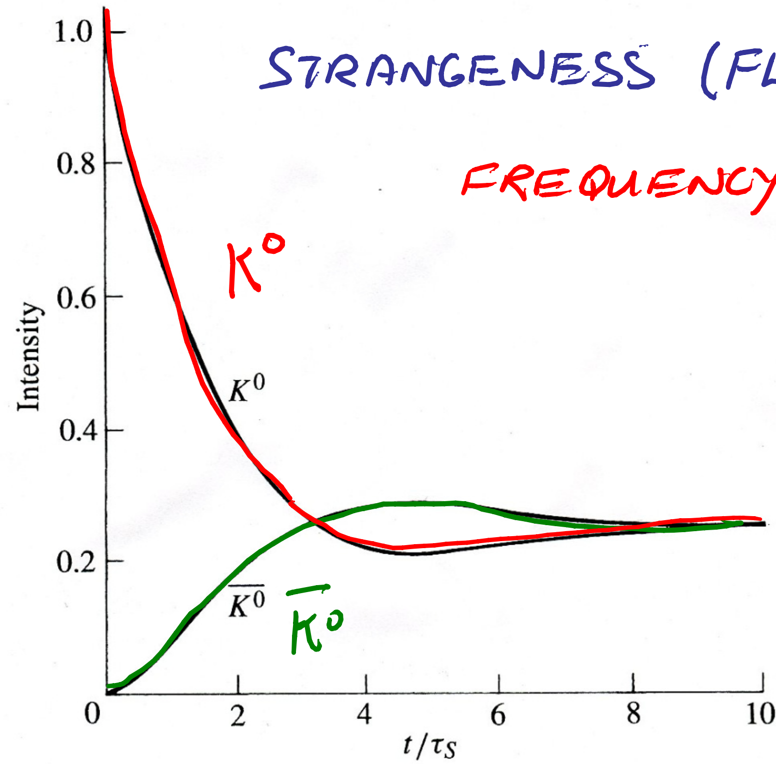
FLAVOUR OSCILLATIONS

$\Delta m \rightarrow$ $K^0 \leftrightarrow \bar{K}^0$ OSCILLATION FREQUENCY

STRANGENESS (FLAVOUR) OSCILLATIONS

FREQUENCY $\sim \Delta m$

I



t/τ_S

$$\Delta m = \frac{0.5}{\tau_S}$$

- FOR $K^0\ \overline{K^0}$ SYSTEM $\quad \Delta m = (3.491 \pm 0.009) \times 10^{-12}$ MeV

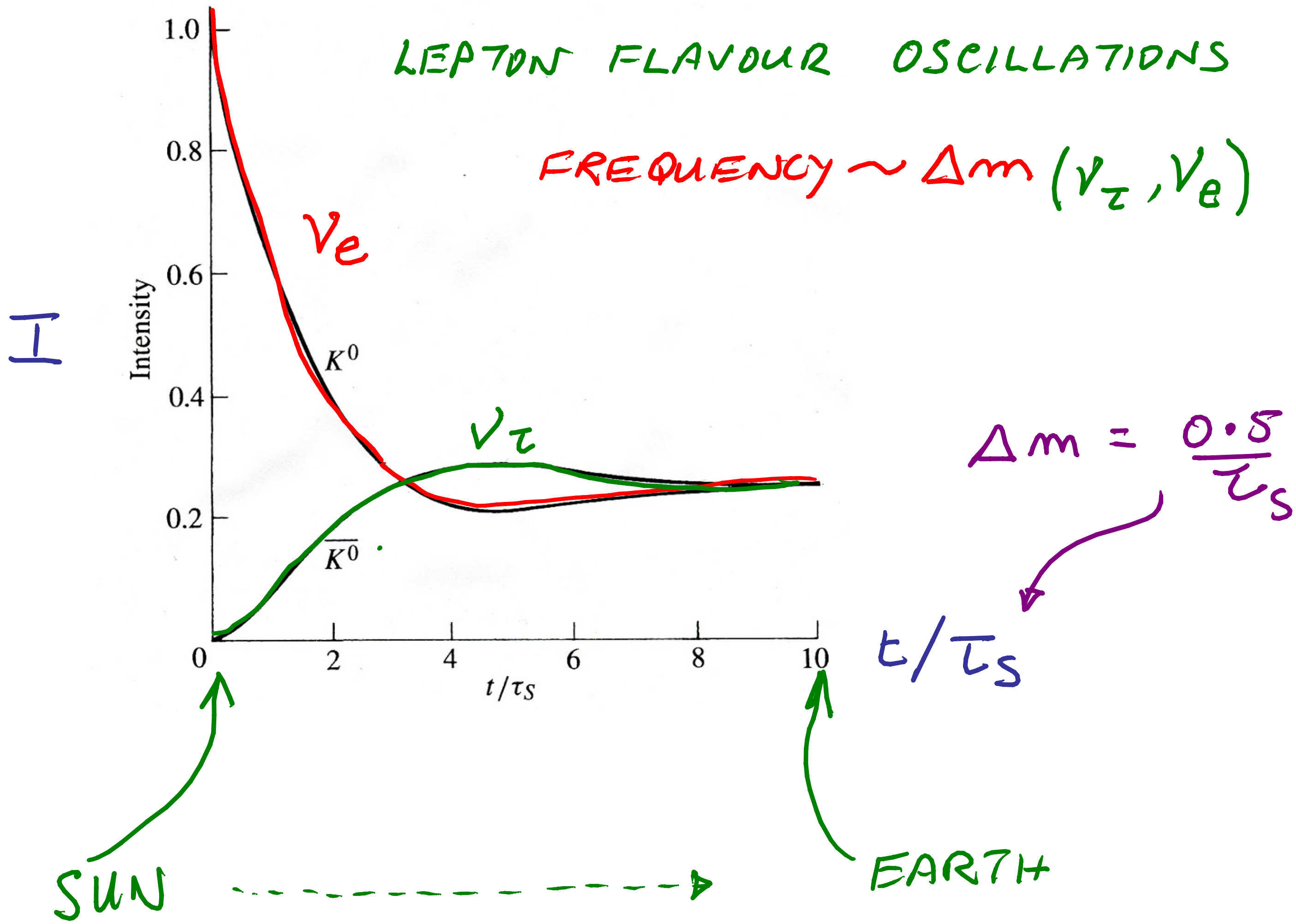
$$\frac{\Delta m}{m_{K^0}} = 7 \times 10^{-15}$$

SENSITIVE TO SMALL Δm

DO LEPTON FLAVOUR OSCILLATIONS EXIST?

$\rightarrow$ YOU BET!

TWO ν "MODEL" OF SOLAR ν PROBLEM



CABIBBO – KOBAYASHI – MASKAWA MATRIX

FOR 3 GENERATIONS OF QUARK – GENERALIZE THE CABIBBO FLAVOUR MIXING MATRIX.

WEAK EIGENSTATES

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

COLOUR EIGENSTATES

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$V_{ab} \rightarrow$ STRENGTH OF TRANSITION $a \rightarrow b$

$V_{ud} \sim \cos\theta_c$

$V_{us} \sim \sin\theta_c$

CKM - MATRIX

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

eg $c_{12} = \cos\theta_{12}$, $s_{23} \equiv \sin\theta_{23}$ $\qquad \theta_{ij} \rightarrow$ SMALL

EXPERIMENTALLY $V \approx \begin{pmatrix} 1 & s_{12} & s_{13}e^{i\delta} \\ -s_{12} & 1 & s_{23} \\ -s_{13}e^{i\delta} & -s_{23} & 1 \end{pmatrix}$

θ_{12} = CABIBBO ANGLE

RED >> BLUE >> GREEN

IN THE LARGE SCALE UNIVERSE

MATTER $\neq$ ANTIMATTER

IN ELEMENTARY PARTICLES

$K^0 \rightarrow \begin{matrix} 2\pi \\ 3\pi \end{matrix} \rightarrow \cancel{CP} \rightarrow$ MATTER $\neq$ ANTIMATTER

NEED 3 GENERATIONS FOR $\cancel{CP}$

NEED 3 GENERATIONS FOR MATTER $\neq$ ANTIMATTER

$\cancel{CP}$ IN QUARK SECTOR NOT ENOUGH TO EXPLAIN LARGE SCALE MATTER - ANTIMATTER ASYMMETRY

BUT WE HAVE $m_\nu \neq 0$ & 3 GENERATIONS OF LEPTONS $\rightarrow$ HOW MUCH $\cancel{CP}$ $\rightarrow$ WE'LL SEE.

WHY IS $m_\nu \neq 0$ SO IMPORTANT?

REMEMBER $\begin{pmatrix} u \\ d \end{pmatrix}$ ARE MASS EIGENSTATES

WE CAN LABLE THEM $\begin{pmatrix} m_u \\ m_d \end{pmatrix}$

FOR LEPTONS "CABIBBO" MIXING WOULD BE

$$\begin{pmatrix} m_e \\ m_{\nu_e}\cos\theta + m_{\nu_\mu}\sin\theta \end{pmatrix} \quad \begin{pmatrix} m_\mu \\ m_{\nu_e}\sin\theta - m_{\nu_\mu}\cos\theta \end{pmatrix}$$

FOR $m_\nu = 0 \rightarrow \begin{pmatrix} m_e \\ 0 \end{pmatrix} \begin{pmatrix} m_\mu \\ 0 \end{pmatrix}$

$\rightarrow$ NO MIXING $\rightarrow$ NO $\cancel{CP}$ IN LEPTON SECTOR

WHAT IS MASS "FOR" ?

UMM ----> HOLDS ME ON THE EARTH ?

NO !

HOLDS GALAXIES TOGETHER ?

NO !

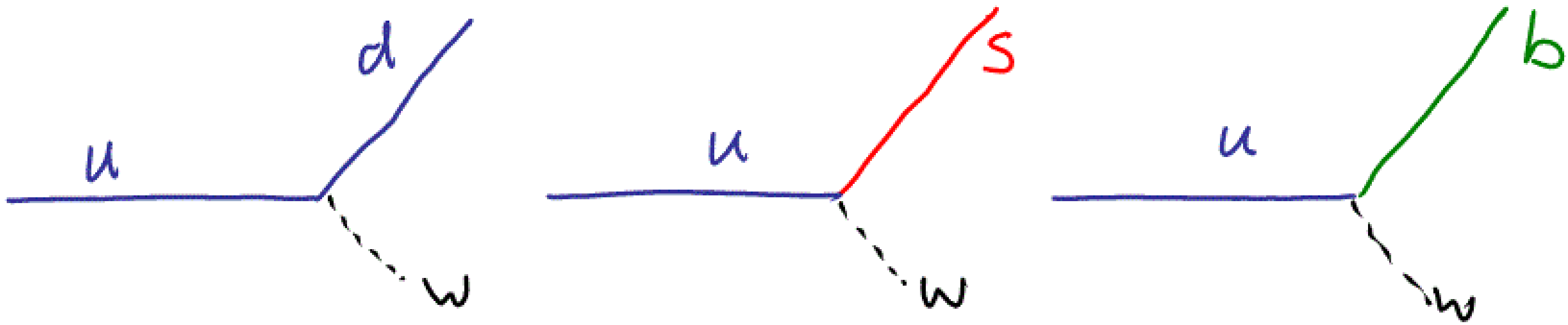
GOVERNS EXPANSION OF UNIVERSE ?

WELL ......

LABELS THE 3 GENERATIONS

↓

$\cancel{CP}$



HOW DO I DISTINGUISH d s b?

- SAME ELECTRIC CHARGE
- SAME COLOUR CHARGE
- SAME WEAK CHARGE
- ud cs tb COUPLINGS SAME

- DIFFERENT ub, us, ud COUPLINGS
- DIFFERENT MASS
- IF $m_d = m_s = m_b \rightarrow \pi^+(u\bar{d}) = K^+(u\bar{s}) = B^+(u\bar{b})$

$B^+ \rightarrow K^+\pi^-$ INDISTINGUISHABLE

$\pi^+\pi^-$

NO MASS $\rightarrow$ 3 GENERATIONS INDISTINGUISHABLE $\rightarrow$ DON'T EXIST

NO 3 GENERATIONS $\longrightarrow$ NO $\cancel{CP}$

NO $\cancel{CP}$ $\longrightarrow$ NO MATTER $\neq$ ANTIMATTER

MATTER $\equiv$ MATTER $\longrightarrow$ ONLY γ's IN THE UNIVERSE