ELECTRO-WEAK UNIFICATION

AN UNDERLYING PHILOSOPHY IN PHYSICS IS TO TRY TO UNDERSTAND REALITY IN TERMS OF AS FEW UNDERLYING PRINCIPLES AS POSSIBLE.

A RELATED IDEA IS TO FIND FUNDAMENTAL RELATIONSHIPS BETWEEN APPARENTLY DISPARATE PHENOMENA

CLASSIC EXAMPLE IS MAXWELL'S REALIZATION THAT ELECTRICITY AND MAGNETISM WERE ASPECTS OF THE SAME PHENOMENA

→ LIGHT IS ELECTROMAGNTIC RADIATION

IMPLICATIONS OF THIS DOMINATED PHYSICS FOR 100 YEARS → AND COMPLETELY CHANGED CIVILIZATION

IN THE 1950s MANY THOUGHT THAT QED WAS AN ANOMALY → A QUANTUM FIELD THEORY BASED ON A MASSLESS GAUGE BOSON

IT WAS DOUBTED WHETHER THE WEAK INTERATION & THE STRONG INTERACTION COULD BE DESCRIBED BY QFT.

IN THE 1960s GLASHOW, SALAM (WHO TAUGHT ME QUANTUM MECHANICS) AND WEINBERG PRODUCED A UNIFIED QUANTUM FIELD THEORY OF EM AND WEAK

PREDICTED W, Z^0, Higgs WITH WELL DEFINED COUPLINGS → THEN QCD – A QFT OF THE STRONG INTERACTION

THE WEAK INTERACTION IS PARTLY A VECTOR INTERACTION

→ IT BETTER BE IF WE WANT TO UNIFY IT WITH QED WHICH IS VECTOR

QED MASSLESS SPIN-1 BOSON γ

WEAK TWO SPIN-1 MASSIVE BOSONS $W^{\pm}$ Z^0

SHORT RANGE OF WEAK FORCE

WE WILL COME TO GAUGE SYMMETRY, FOR NOW JUST SAY GAUGE SYMMETRY NEEDS MASSLESS BOSON

TO HAVE GAUGE SYMMETRY WITH MASSIVE BOSONS NEEDS A NEW PRINCIPLE → SPOTANEOUS SYMMETRY BREAKING

Higgs

PROPERTIES OF THE W-BOSON

FOR THE MOMENT, LET'S NOT BOTHER ABOUT WHY THE W IS MASSIVE (~ 80 GeV). LET'S LOOK AT SOME OF IT'S PROPERTIES

WAVE FUNCTION OF W → POLARIZATION 4-VECTOR × PLANE WAVE

$$W_\mu = \varepsilon_\lambda^\mu e^{-ip\cdot x} = \varepsilon_\lambda^\mu e^{i(\vec{p}\cdot\vec{x} - Et)}$$

MASSLESS γ ⟶ 2 POLARIZATION STATES

MASSIVE SPIN-1 ⟶ 3 POLARIZATION STATES

W IN Z-DIRECTION

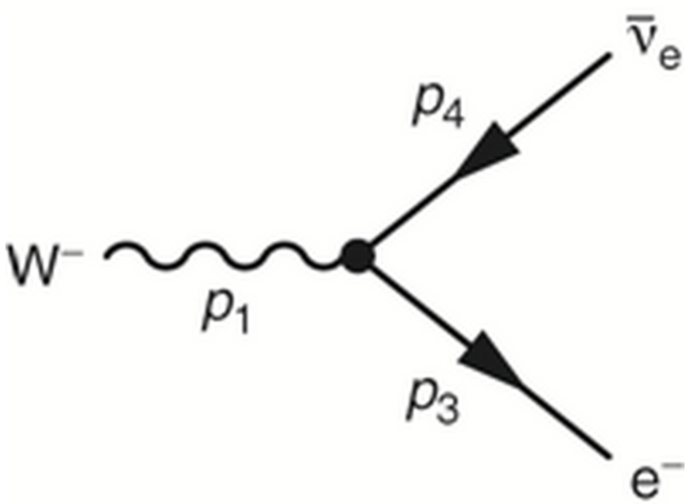
LONGITUDINAL $S_z = 0$

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0) \;, \quad \varepsilon_L = \frac{1}{m_W}(P_z,0,0,E) \;, \quad \varepsilon_+^\mu = \frac{-1}{\sqrt{2}}(0,1,i,0)$$

TRANSVERSE

$S_z = \pm 1$

W-BOSON DECAY



$W^- \to e^- \bar{\nu}_e$

$e^- \to \bar{u}(p_3)$ PARTICLE

$\bar{\nu}_e \to v(p_4)$ ANTIPARTICLE

$W^- \to \varepsilon_\mu^\lambda(p_1)$ POLARIZATION STATE

VERTEX FACTOR $\to$ V-A $\to -i\frac{g_W}{\sqrt{2}}\gamma^\mu(1-\gamma^5)$

FROM FEYNMAN RULES

$$\mathcal{M}_{fi} = \frac{g_W}{\sqrt{2}}\varepsilon_\mu^\lambda(p_1)\,\bar{u}(p_3)\,\underline{\gamma^\mu}\,\frac{1}{2}(1-\gamma^5)\,v(p_4)$$

WE ARE ASSUMING THE NORMAL COUPLING AND LORENTZ STRUCTURE OF THE WEAK INTERACTION

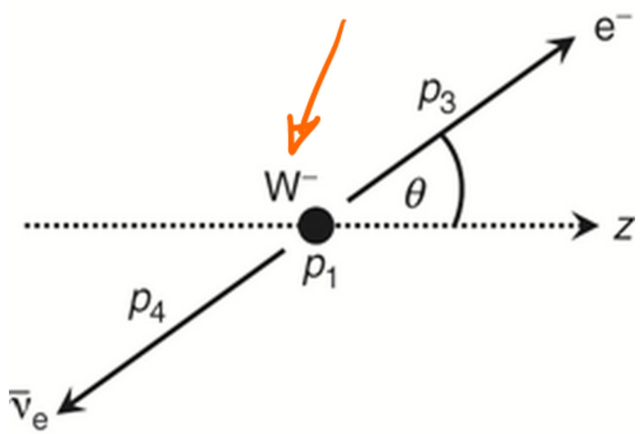
$$\mathcal{M}_{fi} = \frac{g_W}{\sqrt{2}} \varepsilon_\mu^\lambda(p_1)\, \bar{u}(p_3)\, \gamma^\mu \frac{1}{2}(1-\gamma^5)\, v(p_4)$$

$\mathcal{M}_{fi}$ = (W - POLARIZATION) × (LEPTON CURRENT)

$$\mathcal{M}_{fi} = \frac{g_W}{\sqrt{2}} \varepsilon_\mu^\lambda(p_1) \cdot j^\mu$$

$$j^\mu = \bar{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)\, v(p_4)$$

REST FRAME



$m_W >> m_e$

$p_1 = (m_W, 0, 0, 0)$

$p_3 = (E, E\sin\theta, 0, E\cos\theta)$

$p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$

$$E = \frac{m_W}{2}$$

$$j^\mu = \bar{u}(p_3)\gamma^\mu \frac{1}{2}(1-\gamma^5)\, v(p_4)$$

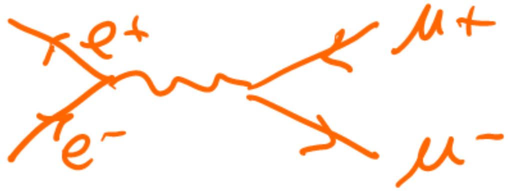
ULTRA-RELATIVISTIC LIMIT, HELICITY STATES $\longrightarrow$ CHIRAL STATES

$\hookrightarrow$ ONLY LEFT HANDED HELICITY PARTICLE STATES

ONLY RIGHT HANDED HELICITY ANTIPARTICLE

$$j^\mu = \bar{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4)$$

THIS LEPTONIC CURRENT SAME AS $\mu^+\mu^-$ CURRENT

IN 

$$j^{\mu, RL} = \bar{u}_\downarrow(p_3)\gamma^\nu v_\uparrow(p_4) \qquad (6.17)$$

$$= 2E\,(0, -\cos\theta, -i, \sin\theta)$$

$\uparrow$ W/2

$$j^\mu = m_W\,(0, -\cos\theta, -i, \sin\theta)$$

$$\mathcal{M}_{fi} = \frac{g_W}{\sqrt{2}} \varepsilon_\mu^\lambda (p_1) j^\mu$$ ← HAVE THIS

3 POLARIZATION STATES ARE:

$$\varepsilon_-^\mu = \frac{1}{\sqrt{2}}(0,1,-i,0) \qquad \varepsilon_L^\mu = (0,0,0,1) \qquad \varepsilon_+^\mu = \frac{-1}{\sqrt{2}}(0,1,-i,0)$$

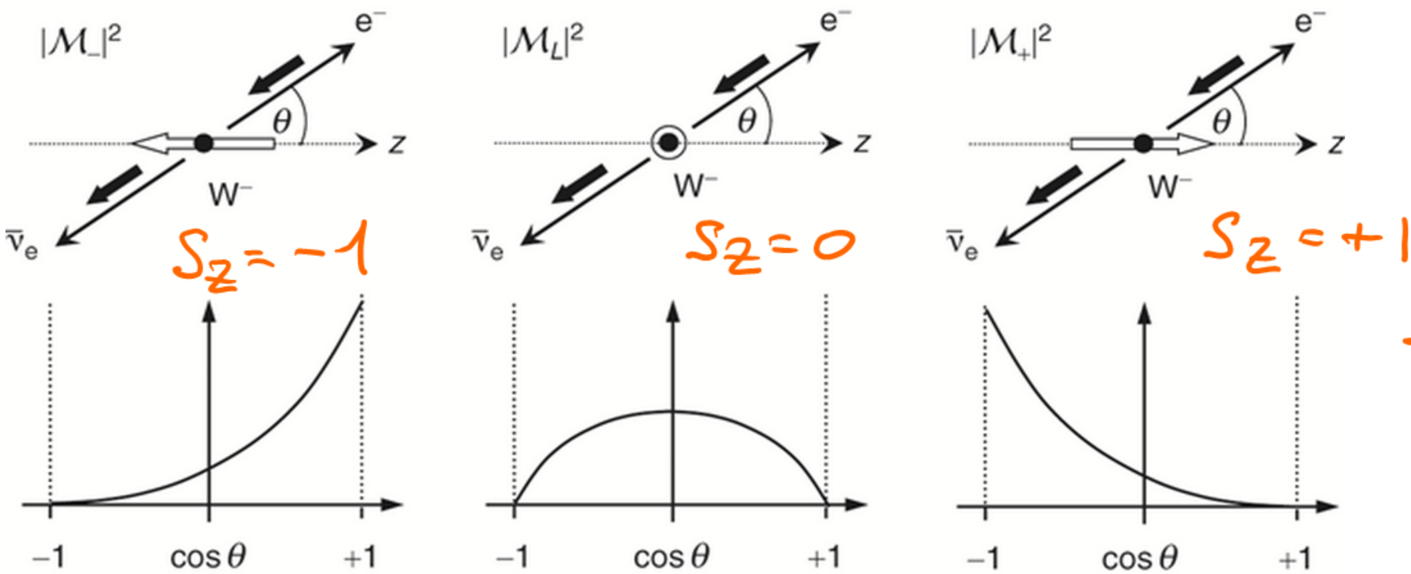
HAVE 3 POSSIBLE POLARIZATIONS → 3 MATRIX ELEMENTS

$$\mathcal{M}_- = \frac{g_W m_W}{2}(0,1,-i,0)\cdot(0,-\cos\theta,-i,\sin\theta) = \frac{1}{2} g_W m_W (1+\cos\theta)$$

(POLARIZATION) • (CURRENT)

$$\mathcal{M}_L = \frac{g_W m_W}{\sqrt{2}}(0,0,0,1)\cdot(0,-\cos\theta,-i,\sin\theta) = -\frac{1}{\sqrt{2}} g_W m_W \sin\theta$$

$$\mathcal{M}_+ = -\frac{g_W m_W}{2}(0,1,i,0)\cdot(0,-\cos\theta,-i,\sin\theta) = \frac{1}{2} g_W m_W (1-\cos\theta)$$



3 POSSIBLE (MATRIX ELEMENT)^2

$$|M_-|^2 = g_w^2 m_w^2 \frac{1}{4}(1+\cos\theta)^2$$

$$|\mathcal{M}_L|^2 = g_w^2 m_w^2 \frac{1}{2}\sin^2\theta$$

$$|\mathcal{M}_+|^2 = g_w^2 m_w^2 \frac{1}{4}(1-\cos\theta)^2$$

ANGULAR DISTRIBUTIONS SAME AS IN $e^+e^- \to \mu^+\mu^-$

AVERAGE OVER THESE 3 MATRIX ELEMENTS

TO GET SPIN AVERAGED

$$\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{1}{3}\left(|\mathcal{M}_-|^2 + |\mathcal{M}_L|^2 + |\mathcal{M}_+|^2 \right)$$

$$= \frac{1}{3} g_W^2 m_W^2 \left[\frac{1}{4}(1+\cos\theta)^2 + \frac{1}{2}\sin^2\theta + \frac{1}{4}(1-\cos\theta)^2 \right]$$

$$= \frac{1}{3} g_W^2 m_W^2$$

IF DECAYING W IS UNPOLARIZED, THEN THE DECAY PRODUCTS ARE ISOTROPICALLY DISTRIBUTED

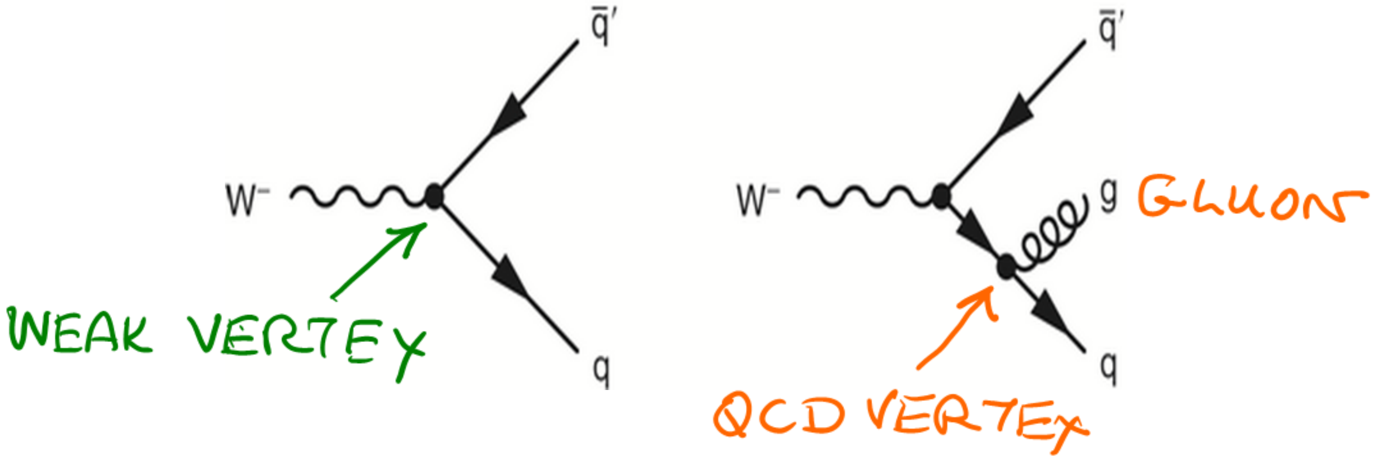
GET DECAY RATE USING USUAL EXPRESSION

$$\Gamma = \frac{p^*}{32\pi^2 m_W^2} \int \langle |\mathcal{M}_{fi}|^2 \rangle \, d\Omega = \frac{p^*}{8\pi m_W^2} \langle |\mathcal{M}_{fi}|^2 \rangle$$

CMS $p^* = \frac{m_W}{2}$

$m_W \gg m_e, m_\nu$

$$\Gamma(W^- \to e^- \bar{\nu}_e) = \frac{g^2 m_W}{48\pi}$$



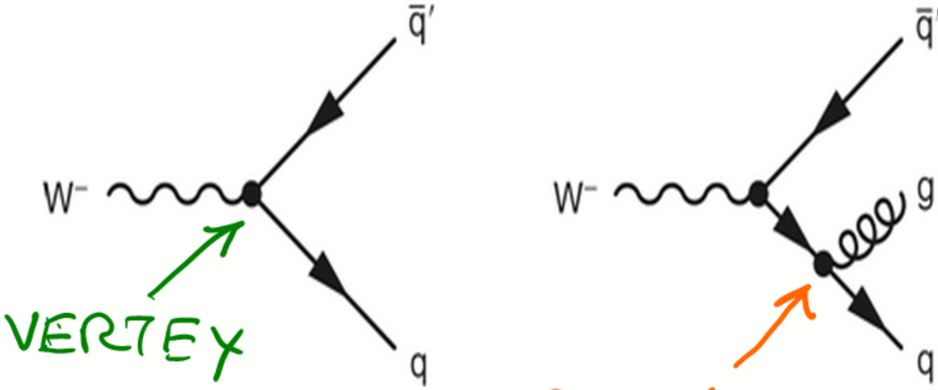
$W^- \to e^- \bar{\nu}_e$ IS JUST ONE DECAY CHANNEL

NEED TO ACCOUNT FOR OTHER LEPTONS AND QUARKS LIGHTER THAN W.

FROM LEPTON UNIVERSALITY

$$\Gamma(W^- \to e^- \bar{\nu}_e) = \Gamma(W^- \to \mu^- \bar{\nu}_\mu) = \Gamma(W^- \to \tau^- \bar{\nu}_\tau)$$

QUARKS ARE A BIT MORE COMPLICATED



$$\Gamma(W^- \to d\bar{u}) = 3\,|V_{ud}|^2\,\Gamma_{e\nu} \quad , \quad \Gamma(W^- \to d\bar{c}) = 3\,|V_{cd}|^2\,\Gamma_{e\nu}$$

3 POSSIBLE QUARK COLORS

SAME g_W

$$\Gamma(W^- \to s\bar{u}) = 3\,|V_{us}|^2\,\Gamma_{e\nu} \quad , \quad \Gamma(W^- \to s\bar{c}) = 3\,|V_{cs}|^2\,\Gamma_{e\nu}$$

$$\Gamma(W^- \to b\bar{u}) = 3\,|V_{ub}|^2\,\Gamma_{e\nu} \quad , \quad \Gamma(W^- \to b\bar{c}) = 3\,|V_{cb}|^2\,\Gamma_{e\nu}$$

ADD ALL PARTIAL WIDTHS $\rightarrow$ TOTAL

UNITARITY OF CKM

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \quad ; \quad |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$

$$\Gamma(W^- \to q\bar{q}') = 6\,\Gamma(W^- \to e^-\bar{\nu}_e)$$

GLUON RADIATION FROM FINAL STATE QUARK

ENHANCEMENT $\quad K_{QCD} = \left[1 + \frac{\alpha_s(m_W)}{\pi}\right] \approx 1.038$

TOTAL $\quad \Gamma_W = (\underbrace{3}_{\text{LEPTONS}} + \underbrace{6K_{QCD}}_{\text{QUARKS}})\,\Gamma(W^- \to e^-\nu_e)$

$$\Gamma_W = 9.2 \times \frac{g_W^2 m_W}{48\pi} = 2.1\,\text{GeV}$$

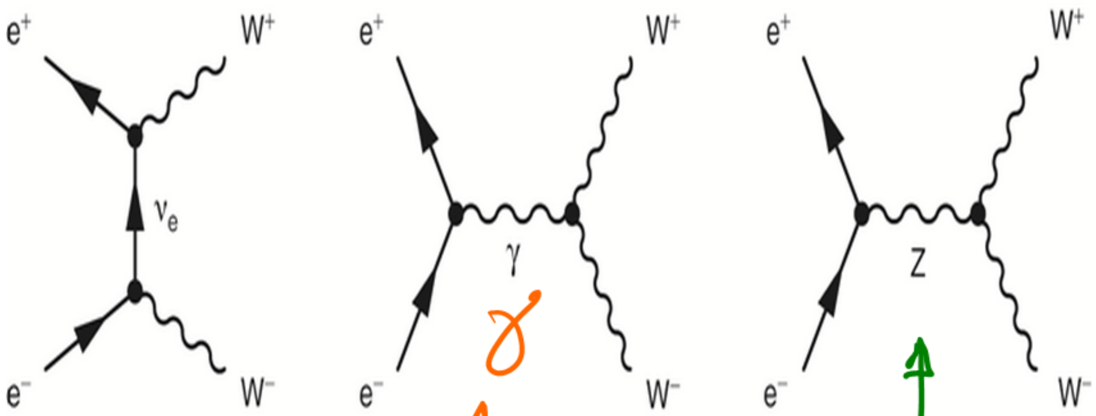
$$BR(W \to q q') = \frac{6K_{QCD}}{3 + 6K_{QCD}} = 67.5\%$$

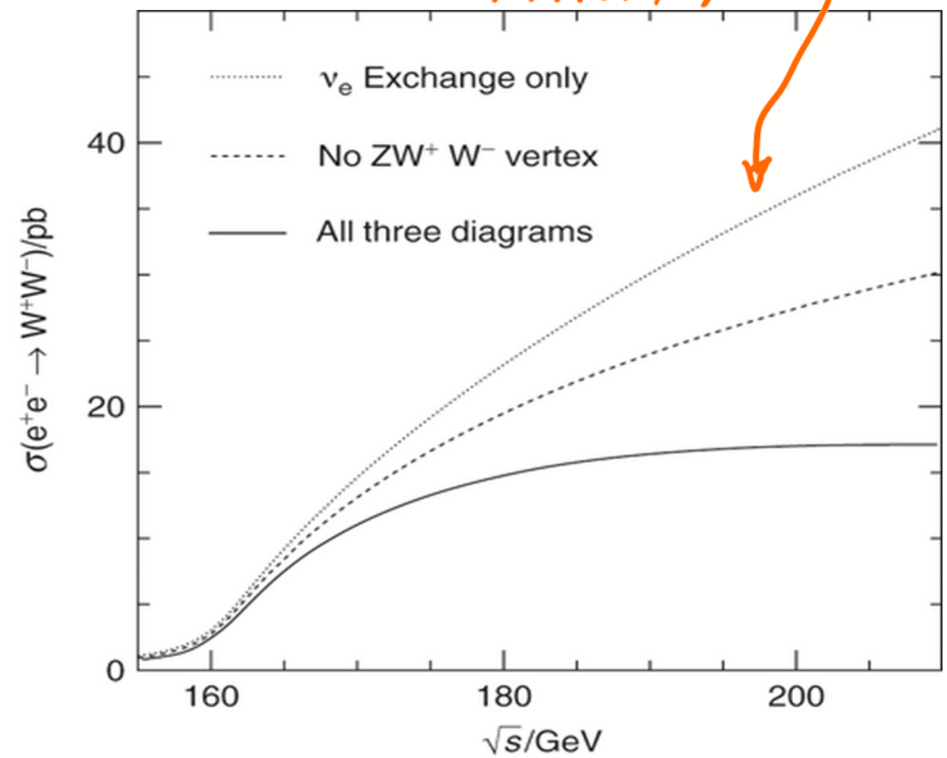
EXPT 2.085 ± 0.042 GeV

$\tau_W \sim O(10^{-25}\,s)$

W PAIR PRODUCTION

VIOLATES UNITARITY





γ

W CHARGED

PRESERVES UNITARITY

↓

INTERFERENCE

CANCELATION ONLY WORKS SINCE THE COUPLINGS OF Z, W, γ ARE RELATED TO EACH OTHER → i.e. NOT ARBITRARY

LOCAL GAUGE PRINCIPLE

MIGHT BE FAMILIAR FROM CLASSICAL E & M

$\bar{E}$ AND $\bar{B}$ CAN BE OBTAINED FROM THE POTENTIALS $\phi, \bar{A}$

$\bar{E}, \bar{B}$ DO NOT CHANGE UNDER TRANSFORMATION

$$\phi \to \phi' = \phi - \frac{\partial \chi}{\partial t}, \quad \bar{A} \to \bar{A} + \bar{\nabla} \chi$$

4-VECTOR $\quad A_\mu \to A'_\mu = A_\mu - \partial_\mu \chi$

IN QM $\quad$ PROB $= \psi(x)\psi^*(x)$

SO PHYSICS MUST BE INVARIANT UNDER

$$\psi(x) \to \psi'(x) = \mathcal{U}\psi(x) = e^{iq\chi(x)}\psi(x)$$

FUCTION OF SPACE-TIME COORDS

FREE DIRAC $\quad i\gamma^\mu \partial_\mu \psi = m\psi$

APPLYING LOCAL GAUGE TRANSFORMATION

$$i\gamma^\mu \partial_\mu \left(e^{iq\chi(x)}\psi\right) = m e^{iq\chi(x)}\psi$$

$$e^{iq\chi} i\gamma^\mu \left[\partial_\mu \psi + iq(\partial_\mu \chi)\psi\right] = e^{iq\chi} m\psi$$

$$i\gamma^\mu \left(\partial_\mu + iq\,\partial_\mu \chi\right)\psi = m\psi$$

DIFFERENT FROM FREE DIRAC

FREE PARTICLE DIRAC DOES NOT EXHIBIT U(1) GAUGE SYMMETRY $\rightarrow$ $e^{iq\chi}$ GENERATES U(1)

IN FACT NO FREE FIELD THEORY CAN MANIFEST GAUGE INVARIANCE $\rightarrow$ COMES FROM PRESENCE OF INTERACTIONS.

FREE DIRAC $\quad i\gamma^\mu \partial_\mu \psi = m\psi$

HYPOTHESIS $\quad i\gamma^\mu(\partial_\mu + iqA_\mu)\psi = m\psi$

$$A_\mu \rightarrow A_\mu' = A_\mu - \partial_\mu \chi$$

THIS CANCELS $-q\gamma^\mu(\partial_\mu \chi)\psi$ ABOVE

WRITE $\quad \partial_\mu + iqA_\mu = D_\mu, \quad i\gamma^\mu D_\mu \psi = m\psi$

NEW DIRAC

IN REQUIRING LOCAL GAUGE INVARIANCE, WE ARE REQUIRED TO ADD INTERACTION TERM

$$q\gamma^\mu A_\mu \psi$$

WE ALREADY SAW THAT THIS CORRESPONDS TO CHARGED PARTICLE INTERACTING WITH A MAGNETIC FIELD

OTHER LOCAL GAUGE SYMMETRIES

QED CORRESPONDS TO THE LOCAL GAUGE SYMMETRY

$$\psi \to \psi' = \psi e^{iq\chi(x)}$$

THE QUANTUM FIELD THEORY OF THE STRONG INTERACTION IS QUANTUM CHROMO DYNAMICS INVARIANCE UNDER SU(3) PHASE TRANSFORMATION

$$\psi(x) \to \psi'(x) = \exp\left[i q_s \vec{\alpha}(x) \cdot \vec{T} \right] \psi(x)$$

$\vec{T} = [T^a]$ 8 GENERATORS OF SU(3) GROUP

$\alpha^a(x) \to$ 8 FUNCTIONS OF SPACE-TIME COORD

$T^a \rightarrow$ GENERATORS OF $SU(3) \rightarrow 3\times 3$ MATRICES

$4 \rightarrow 3$ NEW DEGREES OF FREEDOM

$\hookrightarrow$ COLORS OF QUARKS

IN SIMPLE TERMS SU(3) GAUGE INVARIANCE CORRESPONDS TO THE FACT THAT WE CAN PERMUTE THE THREE COLORS ARBITRARILY AND THERE IS NO PHYSICAL EFFECT

- MORE FORMALLY

QED $\rightarrow$ INVARIANT UNDER 1-D ROTATION $e^{iq\chi(x)} \rightarrow U(1)$

QCD $\rightarrow$ INVARIANT UNDER 3-D ROTATIONS IN COLOR SPACE

$$e^{iq\vec{\chi}(x)\cdot\vec{T}} \rightarrow SU(3)$$

FOR 3-D GAUGE INVARIANCE, DIRAC →

$$i\gamma^\mu\left[\partial_\mu + i g_s (\partial_\mu \bar{\alpha})\cdot \bar{T}\right]\psi = m\psi$$

THIS IS COMPLETELY ANALAGOUS TO iqA_μ IN QCD

IT IS AN INTERACTION TERM

INTRODUCE 8 NEW FIELDS $G_\mu^a(x)$ 1 8

SIMPLY → 3×3 MATRICES → 8 PARAMETERS DUE TO BEING UNITARY

$T_a = \frac{1}{2}\lambda^a$ → 3×3 MATRICES PAGE 225

DIRAC $\quad i\gamma^\mu\left[\partial_\mu + i g_s G_\mu^a T^a\right]\psi - m\psi = 0$

INVARIANT UNDER SU(3) TRANSFORMATIONS

PROVIDED $\quad G_\mu^k \to G_\mu^{k'} = G_\mu^k - \partial_\mu \alpha_k - g_s f_{ijk}\alpha_i G_\mu^j$

LIKE $\quad A_\mu{}' = A_\mu - \partial_\mu \chi$ IN QED

COMPARE FIELD TRANSFORMATIONS IN QED ↔ QCD

QED $\quad A_\mu \to A'_\mu = A_\mu - \partial_\mu \chi$

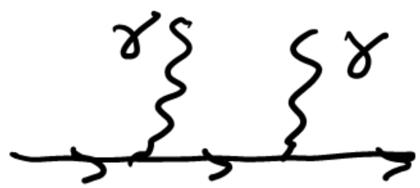
QCD $\quad G^k_\mu \to G'^k_\mu = G^k_\mu - \partial_\mu \alpha_k - g_s f_{ijk} \alpha_i G^j_\mu$

GENERATORS OF SU(3) → MATRICES

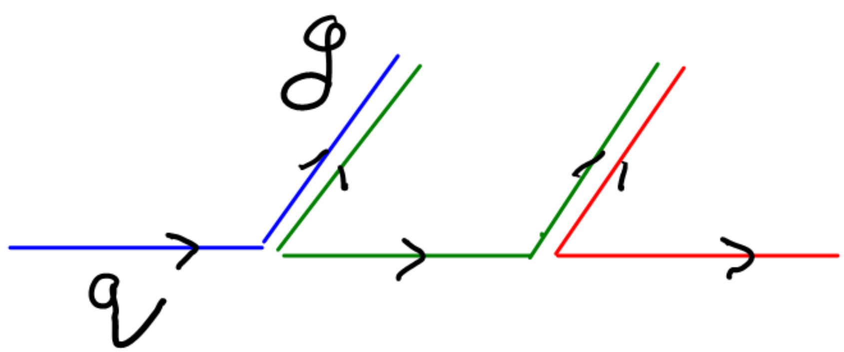
→ DO NOT COMMUTE

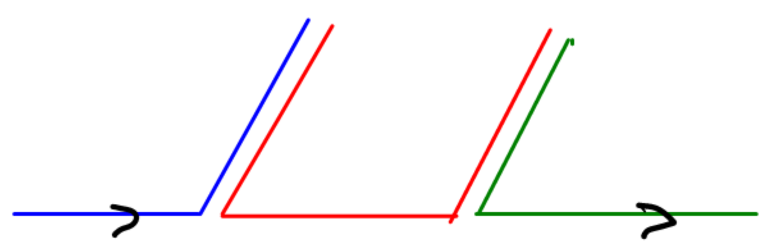
GENERATOR OF U(1) → 1-D ROTATIONS

→ COMMUTE



CAN RADIATE PHOTONS IN ANY ORDER γ → NOT CHARGED





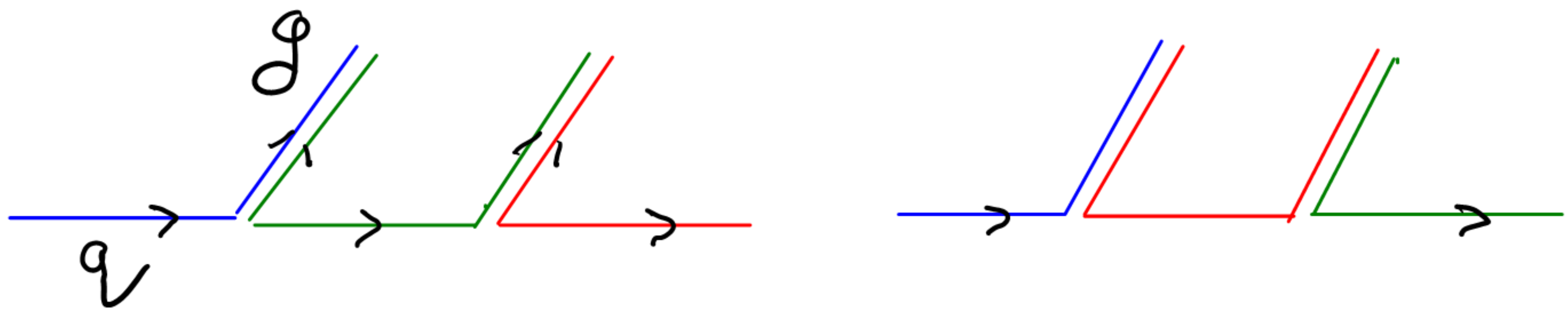
ORDER OF GLUON EMISSION MATTERS

QCD $\quad G_\mu^k \to G_\mu'^k = G_\mu^k - \partial_\mu \alpha_k - g_s f_{ijk} \alpha_i G_\mu^j$

$f_{ijk} \to$ STRUCTURE CONSTANTS OF $SU(3)$

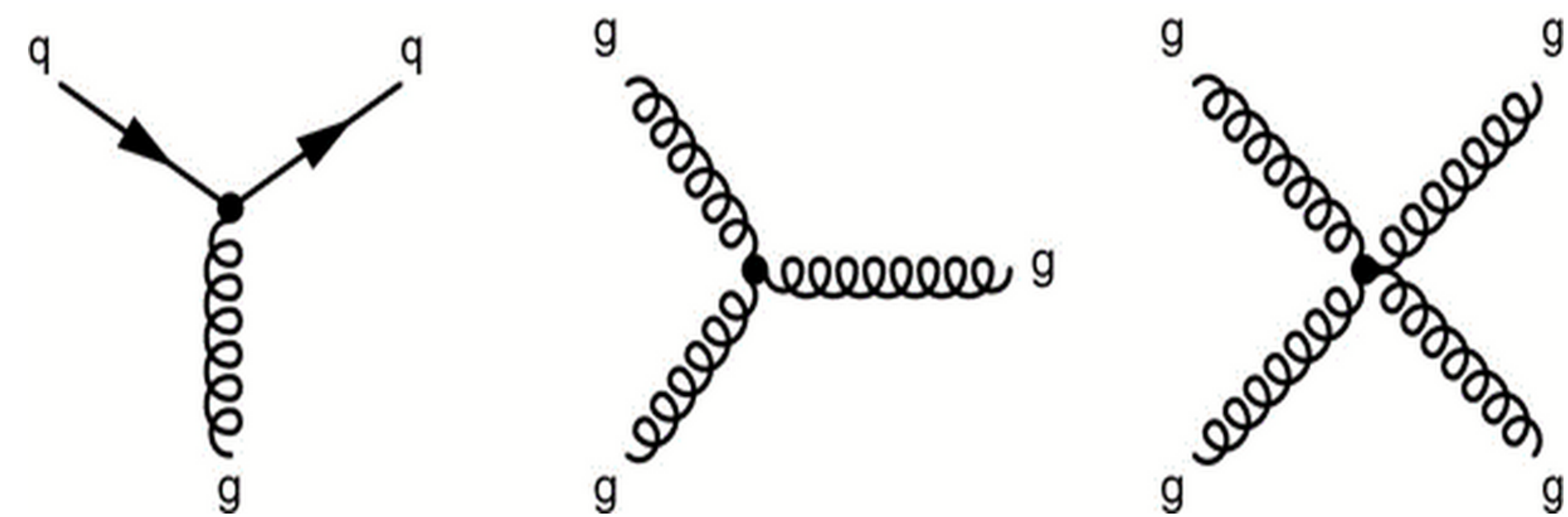
COMMUTATORS $[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k$

$\neq 0$ NON-ABLIAN GROUP



THE ORDER OF EMISSION MATTERS BECAUSE THE GLUONS CARRY COLOR CHARE, UNLIKE UNCHARGED γ

GLUONS DON'T NEED QUARKS — THEY CAN INTERACT AMONG THEMSELVES



$$i\gamma^\mu\left[\partial_\mu + i g_s G^a_\mu T^a\right]\psi - m\psi = 0$$

MEANS $q\,q\,g$ VERTEX IS

$$g_s T^a \gamma^\mu G^a_\mu \psi = g_s \tfrac{1}{2}\lambda^a \gamma^\mu G^a_\mu \psi$$

THE WEAK INTERACTION GAUGE GROUP

FOR QED THE GAUGE GROUP COMES FROM ARBITRARILY CHOOSING THE PHASE OF THE WAVE FUNCTION AT EVERY SPACE-TIME POINT.

FOR QCD IT ARISES FROM BEING ABLE TO ARBITRARILY CHOOSE DEFINTION OF 3 COLORS AT EVERY POINT IN SPACE-TIME

THE WEAK INTERACTION? IF WE "SWITCH OFF" ELECTROMAGNETISM, WE CAN ARBITRARILY CHOOSE WHAT WE CALL AN ELECTRON, AND WHAT WE CALL AN ELECTRON NEUTRINO.

THIS IS INVARIANCE UNDER A 2-DIMENSIONAL PHASE TRANSFORMATION.

SU(2) – INVARIANCE UNDER 2-DIMENSIONAL PHASE TRANSFORMATION

$$\phi(x) \to \phi'(x) = \exp\left[i g_W \vec{\alpha}(x) \cdot \vec{T}\right] \phi(x)$$

$\vec{T} \to$ 3 GENERATORS OF SU(2) GROUP $\to$ PAULI SPIN MATRICES

$$\vec{T} = \frac{1}{2} \vec{\sigma}$$

$\vec{\alpha}(x)$ ARE 3 FUNCTIONS WHICH SPECIFY THE LOCAL PHASE AT EACH POINT IN SPACE-TIME

AS WE DID WITH QED AND QCD WE CAN SATISFY LOCAL PHASE (GAUGE) INVARIANCE BY INTRODUCING INTERACTING FIELDS

↳ IN THIS CASE $\to$ 3 $\to$ W_μ^k , $k = 1, 2, 3$

GAUGE BOSONS $W^{(1)}$ $W^{(2)}$ $W^{(3)}$

GENERATORS OF SU(2) ARE 2x2 MATRICES

→ WAVE FUNCTIONS HAVE 2 COMPONENTS

WE HAVE ALREADY DISCUSSED THAT THE ACTION OF THE W IS

$$\begin{pmatrix} \nu_e \\ \\ e \end{pmatrix} \quad W$$

SO, WE CHOOSE THE 2-COMPONENT WAVE FUNCTION

$$\phi(x) = \begin{pmatrix} \nu_e(x) \\ \\ e^-(x) \end{pmatrix}$$

→ WEAK ISOSPIN DOUBLET

ν_e, AND e ARE "SAME PARTICLE" → 2 DIFFERENT "SPIN" ORIENTATIONS IN ABSTRACT WEAK ISOSPIN SPACE.

$$\phi(x) = \begin{pmatrix} \nu_e(x) \\ e^-(x) \end{pmatrix}$$

IN WEAK ISOSPIN SPACE ν_e, e^- HAVE $I_W = \frac{1}{2}$

TOTAL ISOSPIN

THIRD COMPONENT (Z COMPONENT)

$$I_W^{(3)}(\nu_e) = +\frac{1}{2} \quad , \quad I_W^{(3)}(e^-) = -\frac{1}{2}$$

WEAK CHARGED CURRENT ONLY COUPLES TO LEFT HANDED CHIRAL PARTICLE STATES, AND RIGHT HANDED CHIRAL ANTI PARTICLE STATES

RH PARTICLE, LH ANTIPARTICLE $I_W = 0$ SINGLETS

UNAFFECTED BY SU(2) LOCAL GAUGE TRANSFORMATIONS

$$SU(2) \longrightarrow SU(2)_{LEFT}$$

LEFT HANDED CHIRAL DOUBLETS COUPLING TO THE GAUGE BOSONS OF $SU(2)_L$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \quad \begin{matrix} I_W^{(3)} = +\frac{1}{2} \\ I_W^{(3)} = -\frac{1}{2} \end{matrix}$$

RIGHT HANDED CHIRAL PARTICLE STATES

$$e^-_R \quad \mu^-_R \quad \tau^-_R \qquad u_R \quad c_R \quad t_R \quad d_R \quad s_R \quad b_R$$

THESE RH STATES DO NOT COUPLE TO GAUGE BOSONS OF $SU(2)_L \rightarrow W$

AGAIN BY REQUIRING LOCAL GAUGE INVARIANCE WE INTRODUCE A NEW INTERACTION TERM

$$i g_W T_k \gamma^\mu W_\mu^k = i g_W \frac{1}{2} \sigma_k \gamma^\mu W_\mu^k \varphi_L$$

$$i g_W T_k \gamma^\mu W_\mu^k = i g_W \frac{1}{2} \sigma_k \gamma^\mu W_\mu^k \varphi_L$$

$\varphi_L \rightarrow$ WEAK ISOSPIN DOUBLET OF CHIRAL LEFT HANDED PARTICLES

INTERACTION $\rightarrow$ 3 WEAK CURRENTS $\rightarrow$ 3 W^k

FOR THE ELECTRON DOUBLET

$$\varphi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

THE 3 WEAK CURRENTS, CORRESPONDING TO $3 \times \sigma_i$

$$j_1^\mu = \frac{g_W}{2} \bar{\varphi}_L \gamma^\mu \sigma_1 \varphi_L , \quad j_2^\mu = \frac{g_W}{2} \bar{\varphi}_L \gamma^\mu \sigma_2 \varphi_L , \quad j_3^\mu = \frac{g_W}{2} \bar{\varphi}_L \gamma^\mu \sigma_3 \varphi_L$$

$$\bar{\varphi}_L = (\bar{\nu}_L , e_L)$$

THE WEAK CHARGED CURRENTS ARE RELATED TO THE WEAK ISOSPIN RAISING AND LOWERING OPERATORS

THESE STEP BETWEEN STATES IN A WEAK ISOSPIN DOUBLET

↳ THIS IS EXACTLY WHAT WE WANT THE W TO DO

$$\begin{pmatrix} \nu \\ e \end{pmatrix}$$

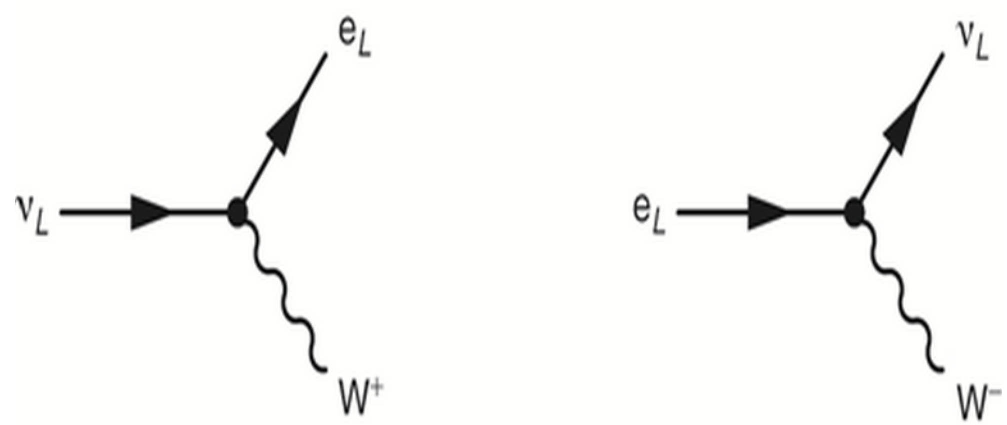
$W^{\pm} \sim \sigma_{\pm}$

$\sigma_{\pm} = \frac{1}{2}(\sigma_1 \pm i \sigma_2)$

THE 4-VECTOR CURRENTS CORRESPONDING TO $W^{\pm}$ EXCHANGE, ARE:

$$j_{\pm}^{\mu} = \frac{1}{\sqrt{2}}\left(j_1^{\mu} + j_2^{\mu}\right) = \frac{g_W}{\sqrt{2}} \bar{\varphi}_L \gamma^{\mu} \frac{1}{2}(\sigma_1 \pm i \sigma_2) \varphi_L$$

$$= \frac{g_W}{\sqrt{2}} \bar{\varphi}_L \gamma^{\mu} \sigma_{\pm} \varphi_L .$$



THE PHYSICAL W-BOSONS ARE THE LINEAR COMBINATIONS

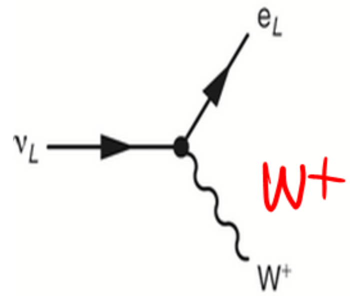
$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}\left(W^{(1)}_{\mu} \mp W^{(2)}_{\mu}\right)$$

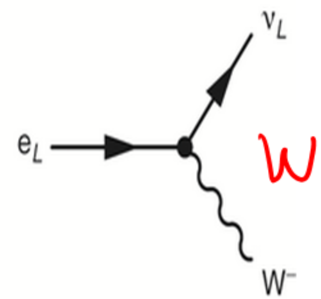
$$\vec{J}^{\mu} \cdot \vec{W}_{\mu} = J_1^{\mu} W^{(1)}_{\mu} + J_2^{\mu} W^{(2)}_{\mu} + J_3^{\mu} W^{(3)}_{\mu} = \underbrace{J_+^{\mu} W^+_{\mu} + J_-^{\mu} W^-_{\mu}}_{\text{CHARGED CURRENTS}} + \underbrace{J_3^{\mu} W^3_{\mu}}_{\text{NEUTRAL CURRENT}}$$

W^+ W^-

CHARGED CURRENTS

NEUTRAL CURRENT

J_+^μ  W^+

 W^- J_-^μ

J_+^μ CORRESPONDS TO W^+ EXCHANGE $\rightarrow$

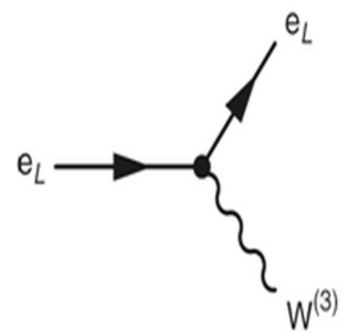
$$J_+^\mu = \frac{g_W}{\sqrt{2}} \bar{\varphi}_L \gamma^\mu \sigma_+ \varphi_L = \frac{g_W}{\sqrt{2}} (\bar{\nu}_L, \bar{e}_L) \gamma^\mu \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

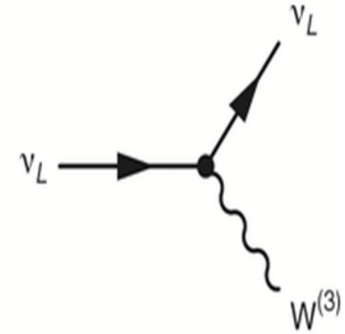
$$= \frac{g_W}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L = \frac{g_W}{\sqrt{2}} \bar{\nu} \gamma^\mu \frac{1}{2}(1-\gamma^5) e$$

$$J_-^\mu = \frac{g_W}{\sqrt{2}} \bar{\varphi}_L \gamma^\mu \sigma_- \varphi_L = \frac{g_W}{\sqrt{2}} (\bar{\nu}_L, \bar{e}_L) \gamma^\mu \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= \frac{g_W}{\sqrt{2}} \bar{e}_L \gamma^\mu \nu_L = \frac{g_W}{\sqrt{2}} \bar{e} \gamma^\mu \frac{1}{2}(1-\gamma^5) \nu$$

$SU(2)_L$ SYMMETRY $\rightarrow$ WEAK CHARGED CURRENTS





$$\vec{J}^{\mu} \cdot \vec{W}_{\mu} = J_{+}^{\mu} W_{\mu}^{+} + J_{-}^{\mu} W_{\mu}^{-} + J_{3}^{\mu} W_{\mu}^{3}$$

$SU(2)_L \rightarrow$ WEAK NEUTRAL CURRENT

$$J_3^{\mu} = g_w \bar{\varphi}_L \gamma^{\mu} \frac{1}{2} \sigma_3 \varphi_L$$

$$J_3^{\mu} = g_w \frac{1}{2} (\bar{\nu}_L , \bar{e}_L) \gamma^{\mu} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= g_w \frac{1}{2} \bar{\nu}_L \gamma^{\mu} \nu_L - g_w \frac{1}{2} \bar{e}_L \gamma^{\mu} e_L$$

$$J_3^{\mu} = I_W^{(3)} g_w \bar{f} \gamma^{\mu} \frac{1}{2} (1 - \gamma^5) f$$

$$J_3^\mu = I_W^{(3)} g_W \bar{f} \gamma^\mu \frac{1}{2}(1-\gamma^5) f$$

$I_W^{(3)} \rightarrow$ 3RD COMPONENT OF WEAK ISOSPIN

$\rightarrow$ COMPLETELY ANALOGOUS TO S_z, Z COMPONENT OF SPIN.

RIGHT HANDED PARTICLES AND LEFT HANDED ANTIPARTICLES ARE IN ISOSPIN SINGLETS

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad e_R \leftarrow I_W = 0$$

$\downarrow$ DOES NOT COUPLE TO $W^{(3)}$