ELECTROWEAK UNIFICATION AND THE Z^0

NEUTRAL CURRENT FROM $SU(2)_L$

$$J_3^\mu = I_W^{(3)} g_W \bar{f} \gamma^\mu \frac{1}{2}(1-\gamma^5) f$$

THIS DECOUPLES FROM RIGHT HANDED PARTICLES

BUT OBSERVED Z^0 DOES COUPLE TO RH PARTICLES

HAVE TWO NEUTRAL FIELDS A_μ (γ) Z_μ (Z)

MIX $W_\mu^{(3)} \leftrightarrow B_\mu \longrightarrow Z_\mu \quad A_\mu$

REPLACE ELECTROMAGNETISM $U(1)_{QED}$ BY NEW SYMMETRY $U(1)_Y$ ← HYPERCHARGE

$$\psi(x) \to \psi'(x) = U(x)\psi(x) = \exp\left[i g' \frac{Y}{2} \zeta(x) \right] \psi(x)$$

$$\psi(x) \to \psi'(x) = U(x)\psi(x) = \exp\left[ig'\frac{Y}{2}\zeta(x)\right]\psi(x)$$

AS USUAL WHEN IMPOSE LOCAL GAUGE SYMMETRY

→ GET AN INTERACTION TERM

$$g'\frac{Y}{2}\gamma^{\mu}B_{\mu}\psi$$

NEW FIELD CORRESPONDING TO NEW LOCAL GAUGE SYMMETRY

↳ $U(1)$

COMPARE THIS TO QED $U(1)$

$$Qe\gamma^{\mu}A_{\mu}\psi$$

MAKE γ AND Z LINEAR COMBINATIONS B_{μ} $W_{\mu}^{(3)}$

$$\begin{cases} A_{\mu} = +B_{\mu}\cos\theta_W + W_{\mu}^{(3)}\sin\theta_W \\ Z_{\mu} = -B_{\mu}\sin\theta_W + W_{\mu}^{(3)}\cos\theta_W \end{cases}$$

↑ WEAK MIXING ANGLE

MIXING NATURAL IN HIGGS MECHANISM

γ MASSLESS

Z MASSIVE

MIXING GIVES THE PHYSICAL CURRENTS

$$j^\mu_{em} = j^\mu_Y \cos\theta_W + j^\mu_3 \sin\theta_W$$

$$j^\mu_Z = -j^\mu_Y \sin\theta_W + j^\mu_3 \cos\theta_W$$

WHAT DO WE ACTUALLY MEAN BY UNIFICATION?

↳ COUPLINGS OF WEAK AND EM RELATED

CONSIDER e ν_e

$W^{(3)}$ WEAK NEUTRAL CURRENT IS

LH → $$j^\mu_3 = \frac{1}{2} g_W \bar{\nu}_L \gamma^\mu \nu_L - \frac{1}{2} g_W \bar{e}_L \gamma^\mu e_L$$

LH+RH → $$j^\mu_Y = \frac{1}{2} g' Y_{e_L} \bar{e}_L \gamma^\mu e_L + \frac{1}{2} g' Y_{e_R} \bar{e}_R \gamma^\mu e_R + \frac{1}{2} g' Y_{\nu_L} \bar{\nu}_L \gamma^\mu \nu_L + \frac{1}{2} g' Y_{\nu_R} \bar{\nu}_R \gamma^\mu \nu_R$$

$$g' \frac{Y}{2} \gamma^\mu B_\mu \psi$$

$$j^\mu_{em} = Q_e e\, \bar{e}_L \gamma^\mu e_L + Q_e e\, \bar{e}_R \gamma^\mu e_R$$

+ NO ν PART

$$j^\mu_{em} = j^\mu_Y \cos\theta_W + j^\mu_3 \sin\theta_W$$

$$j^\mu_{em} = \tfrac{1}{2} g' Y_{e_L} \bar{e}_L \gamma^\mu e_L \cos\theta_W + \tfrac{1}{2} g' Y_{e_R} \bar{e}_R \gamma^\mu e_R \cos\theta_W$$

$$+ \tfrac{1}{2} g' Y_{\nu_L} \bar{\nu}_L \gamma^\mu \nu_L \cos\theta_W + \tfrac{1}{2} g' Y_{\nu_R} \bar{\nu}_R \gamma^\mu \nu_R \cos\theta_W$$

$$+ \tfrac{1}{2} g_W \bar{\nu}_L \gamma^\mu \nu_L \sin\theta_W - \tfrac{1}{2} g_W \bar{e}_L \gamma^\mu e_L \cos\theta_W$$

COMPARE COEFFICIENTS IN TWO j^μ_{em} EXPRESSION

$$\bar{e}_L \gamma_\mu e_L \rightarrow Q_e e = \tfrac{1}{2} g' Y_{e_L} \cos\theta_W - \tfrac{1}{2} g_W \sin\theta_W$$

$$\bar{\nu}_L \gamma^\mu \nu_L \rightarrow 0 = \tfrac{1}{2} g' Y_{\nu_L} \cos\theta_W + \tfrac{1}{2} g_W \sin\theta_W$$

$$\bar{e}_R \gamma^\mu e_R \rightarrow Q_e e = \tfrac{1}{2} g' Y_{e_R} \cos\theta_W$$

$$\bar{\nu}_R \gamma^\mu \nu_R \rightarrow 0 = \tfrac{1}{2} g' Y_{\nu_R} \cos\theta_W$$

RELATES QED COUPLINGS TO $U_Y(1)$ COUPLINGS

IN STANDARD MODEL, GAUGE SYMMETRY

$$U_Y(1),\quad SU(2)_L \longrightarrow U_Y(1)\times SU(2)_L$$

FOR SIMULTANEOUS INVARIANCE

$$\begin{pmatrix} \nu_e \\ e^-_L \end{pmatrix} \leftarrow \text{SAME HYPERCHARGE } Y$$

IF NOT SAME Y

$$U_Y\begin{pmatrix} \nu_e \\ e^-_L \end{pmatrix} \rightarrow \begin{pmatrix} \nu_e\, e^{i\alpha(Y_{\nu_e})} \\ e^-_L\, e^{i\beta(Y_{e_L})} \end{pmatrix}$$

THIS WOULD OBVIOUSLY NOT BE INVARIANT UNDER $SU(2)$, THE WAY WAS

THE SIGNIFICANCE OF HYPERCHARGE IS THAT FERMIONS HAVE BOTH ELECTRIC CHARGE AND WEAK CHARGE → REFLECTED IN WEAK ISOSPIN DOUBLETS

$$Y = \alpha Q + \beta I_W^{(3)}$$

eg $\begin{pmatrix} \nu_e \\ e^-_L \end{pmatrix} \rightarrow$

	ELECTRIC CHARGE Q	TOTAL WEAK ISOSPIN $\vec{I}$	$I_W^{(3)}$
ν_e	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-_L	-1	$+\frac{1}{2}$	$-\frac{1}{2}$

COULD SAY

$$\begin{pmatrix} \nu_e \\ e_L^- \end{pmatrix} \rightarrow \begin{pmatrix} +\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix} \xrightarrow{\text{CONVENTION}} \begin{pmatrix} +1 \\ -1 \end{pmatrix}$$

$$Y = 2\left(Q - I_W^{(3)}\right)$$

HAD

$$Q_e e = \tfrac{1}{2} g' Y_{e_L} \cos\theta_W - \tfrac{1}{2} g_W \sin\theta_W$$

$$0 = \tfrac{1}{2} g' Y_{\nu_L} \cos\theta_W + \tfrac{1}{2} g_W \sin\theta_W$$

PUT $Y_{e_L} = Y_{\nu_L} = -1$

$$-e = g'/2\,(-1)\cos\theta_W - \tfrac{1}{2} g_W \sin\theta_W$$

$$0 = g'/2\,(-1)\cos\theta_W + \tfrac{1}{2} g_W \sin\theta$$

SUBTRACT

$$e = g_W \sin\theta_W$$

RELATES WEAK
EM COUPLING

ADD $Q_e \cdot e = \frac{1}{2} g' (Y_{e_L} + Y_{\nu_L}) \cos\theta_W$

$-1 \qquad -1 \qquad -1$

$e = g' \cos\theta_W$

$Y = 2(Q - I_W^{(3)})$ ← $= 0$ FOR RIGHT HANDED

$Y_{e_R} = -2$, $Y_{\nu_R} = 0$ ← DOESN'T COUPLE

IN TERMS OF COUPLING CONSTANTS

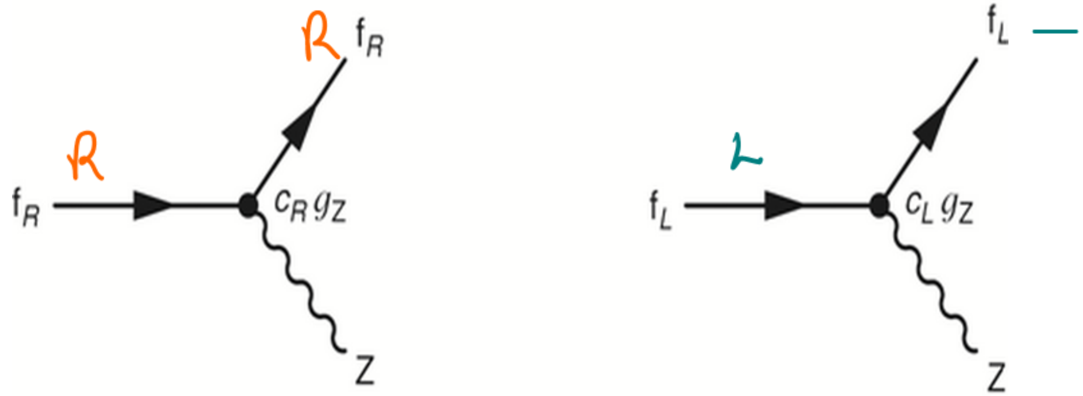
$$\frac{e^2}{g_W^2} = \frac{\alpha}{\alpha_W} = \sin^2\theta_W \quad (= 0.23146 \pm 0.00012)$$

FOR THIS TO MEAN ANYTHING $\sin^2\theta_W$ MUST BE UNIVERSAL → IN ALL INTERACTIONS

$\nu N \to \nu N$, $\nu N \to \mu N$

$$e^+e^- \to \begin{matrix} Z \\ \gamma \end{matrix} \to \begin{matrix} f \\ \bar{f} \end{matrix}$$

Z BOSON COUPLING



HAD $\quad J_Z^\mu = -J_Y^\mu \sin\theta_W + J_3^\mu \cos\theta_W$

$$J_Z^\mu = -\tfrac{1}{2} g' \sin\theta_W \left[Y_{f_L} \bar{u}_L \gamma^\mu u_L + Y_{f_R} \bar{u}_R \gamma^\mu u_R \right] + I_W^{(3)} g_W \cos\theta_W \left[\bar{u}_L \gamma^\mu u_L \right]$$

$$Y = 2(Q - I_W^3), \quad RH \; I_W^{(3)} = 0$$

$$J_Z^\mu = -g' \sin\theta_W \left[(Q_f - I_W^{(3)}) \bar{u}_L \gamma^\mu u_L + Q_f \bar{u}_R \gamma^\mu u_R \right] + I_W^{(3)} g_W \cos\theta_W \left[\bar{u}_L \gamma^\mu u_L \right]$$

$$J_Z^\mu = \left[-g'\left(Q_f - I_W^{(3)}\right)\sin\theta_W + I_W^{(3)} g_W \cos\theta_W\right] \bar{u}_L \gamma^\mu u_L$$

$$- \left[g' \sin\theta_W Q_f\right] \bar{u}_R \gamma^\mu u_R$$

$$g' = g_W \tan\theta_W$$

$$J_Z^\mu = g_W\left[-\left(Q_f - I_W^{(3)}\right)\frac{\sin^2\theta_W}{\cos\theta_W} + I_W^{(3)}\cos\theta_W\right] \bar{u}_L \gamma^\mu u_L$$

$$- g_W \left[\frac{\sin^2\theta_W}{\cos\theta_W} Q_f\right] \bar{u}_R \gamma^\mu u_R$$

DEFINE $g_Z = \dfrac{g_W}{\cos\theta_W} = \dfrac{e}{\sin\theta_W \cos\theta_W}$

$$J_Z^\mu = g_Z\left(I_W^{(3)} - Q_f \sin^2\theta_W\right)\bar{u}_L \gamma^\mu u_L - g_Z\left(Q_f \sin^2\theta_W\right)\bar{u}_R \gamma^\mu u_R$$

$$J_Z^\mu = g_Z\left(c_L \bar{u}_L \gamma^\mu u_L + c_R \bar{u}_R \gamma^\mu u_R\right)$$

$c_L = I_W^3 - Q_f \sin^2\theta_W$ $\qquad$ $c_R = -Q_f \sin^2\theta_W$

$$c_L = I^3_w - Q_f \sin^2\theta_w \qquad c_R = -Q_f \sin^2\theta_w$$

THE $W^\pm$ ONLY COUPLE LEFT-HANDED

THE Z HAS BOTH RIGHT HANDED AND LEFT HANDED COUPLINGS

↳ THIS REFLECTS THAT IT IS A MIXTURE OF W → LEFT, AND U(1) → LEFT + RIGHT.

RECALL $\bar{u}_L \gamma^\mu u_L = \bar{u} \gamma^\mu \frac{1}{2}(1-\gamma^5) u$

$$\bar{u}_R \gamma^\mu u_R = \bar{u} \gamma^\mu \frac{1}{2}(1+\gamma^5) u$$

$$J_Z^\mu = g_Z \bar{u} \gamma^\mu \left[c_L \frac{1}{2}(1-\gamma^5) + c_R \frac{1}{2}(1+\gamma^5) \right] u$$

$$= g_Z \bar{u} \gamma^\mu \frac{1}{2} \left[(c_L + c_R) - (c_L - c_R) \gamma^5 \right] u$$

Table 15.1 The charge, $I_W^{(3)}$ and weak hypercharge assignments of the fundamental fermions and their couplings to the Z assuming $\sin^2\theta_W = 0.23146$.

fermion	Q_f	$I_W^{(3)}$	Y_L	Y_R	c_L	c_R	c_V	c_A
ν_e, ν_μ, ν_τ	0	$+\frac{1}{2}$	-1	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-\frac{1}{2}$	-1	-2	-0.27	$+0.23$	-0.04	$-\frac{1}{2}$
u, c, t	$+\frac{2}{3}$	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{4}{3}$	$+0.35$	-0.15	$+0.19$	$+\frac{1}{2}$
d, s, b	$-\frac{1}{3}$	$-\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{2}{3}$	-0.42	$+0.08$	-0.35	$-\frac{1}{2}$

$$J_Z^\mu = g_Z \bar{u} \gamma^\mu \frac{1}{2}\left[(c_L + c_R) - (c_L - c_R)\gamma^5\right] u$$

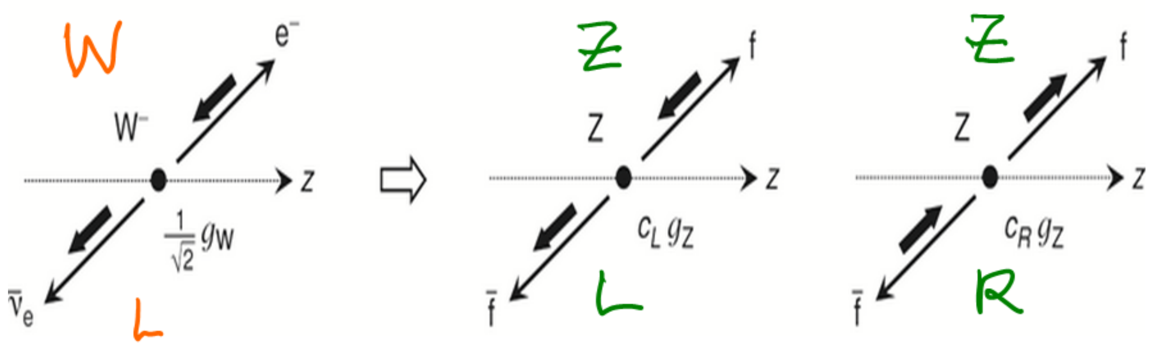
$$= \frac{1}{2} g_Z \bar{u} \left(c_V \gamma^\mu - c_A \gamma^\mu \gamma^5 \right) u$$ ← PARITY VIOLATION

$$c_V = (c_L + c_R) = I_W^{(3)} - 2Q\sin^2\theta_W$$

$$c_A = (c_L - c_R) = I_W^{(3)}$$

VERTEX $\quad -i\frac{1}{2} g_Z \gamma^\mu \left[c_V - c_A\right]$

DECAYS OF THE Z



$m_f \ll m_Z$

← ONLY THESE CONTRIBUTE

EVEN THOUGH Z COUPLES TO LEFT AND RIGHT, ITS V-A NATURE STILL EXCLUDES SOME CHIRAL STATES

$$\bar u_R \gamma^\mu (c_V - c_A \gamma^5) v_R = u^\dagger \tfrac{1}{2}(1+\gamma^5)\gamma^0\gamma^\mu (c_V - c_A\gamma^5) \times \tfrac{1}{2}(1-\gamma^5)$$

$$= \tfrac{1}{4} u^\dagger \gamma^0 (1-\gamma^5)(1+\gamma^5)\gamma^\mu (c_V - c_A\gamma^5) v$$

$$= \tfrac{1}{4} \bar u \gamma^\mu \underbrace{P_L P_R}_{0} (c_V - c_A\gamma^5) v = 0$$

FOR W DECAY, WE GOT

$$\langle |\mathcal{M}_{fi}|^2 \rangle = \frac{1}{3}\left(|\mathcal{M}_-|^2 + |M_L|^2 + |\mathcal{M}_+|^2 \right)$$

NOT $\sin\theta_W$!

$$= \frac{1}{3} g_W^2 m_W^2 \left[\frac{1}{4}(1+\cos\theta)^2 + \frac{1}{2}\sin^2\theta + \frac{1}{4}(1-\cos\theta)^2 \right]$$

$$= \frac{1}{3} g_W^2 m_W^2$$

CAN JUST USE THIS FOR $Z \to LH_{PARTICLE} + RH_{ANTIPARTICLE}$

$\frac{1}{2} g_W^2 \to g_Z^2 c_L^2 \to \langle |\mathcal{M}_L|^2 \rangle = \frac{2}{3} c_L^2 g_Z^2 m_Z^2$

$Z \to RH_{PART} + LH_{ANTI}$ $\quad c_L \to c_R$

$$\langle |\mathcal{M}|^2 \rangle = \langle |\mathcal{M}_L|^2 + |\mathcal{M}_R|^2 \rangle = \frac{2}{3}\left(c_L^2 + c_R^2\right) g_Z^2 m_Z^2$$

$$\langle |\mathcal{M}|^2 \rangle = \langle |\mathcal{M}_L|^2 + |\mathcal{M}_R|^2 \rangle = \frac{2}{3}(C_L^2 + C_R^2)\, g_Z^2\, m_Z^2$$

$C_V = C_L + C_R$, $C_A = C_L - C_R$ $\longrightarrow$ $C_V^2 + C_A^2 = 2(C_L^2 + C_R^2)$

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{3}(C_V^2 + C_A^2)\, g_Z^2\, m_Z^2$$

USING USUAL FORMULA

$$\Gamma(Z \to f\bar{f}) = \frac{g_Z^2 m_Z}{48\pi}(C_V^2 + C_A^2)$$

Z WIDTH AND BRANCHING RATIOS

$m_Z = 91.2\ \text{GeV}$

$$g_Z^2 = \frac{g_W^2}{\cos\theta_W} = \frac{8 m_W^2}{\sqrt{2}\cdot\cos^2\theta_W}\cdot G_F \approx 0.55$$

FOR A PARTICULAR FERMION FLAVOR → C_V, C_A FOR THAT FLAVOR

eg. $Z \to \nu_e \bar{\nu}_e$, $C_V = C_A = \frac{1}{2}$

$$\Gamma(Z\to\nu_e\bar{\nu}_e) = \frac{g_Z^2 m_Z}{48\pi}\left(\frac{1}{4}+\frac{1}{4}\right) = 167\ \text{MeV}$$

$$\Gamma_{TOT} = \sum_f \Gamma(Z\to f\bar{f})$$ ← EXCEPT TOP

NOT TOP?

$$\Gamma_{TOT} = 3\Gamma(Z\to\nu_e\bar{\nu}_e) + 3\Gamma(Z\to e^+e^-) + 3\times 2\Gamma(Z\to u\bar{u}) + 3\times 3\Gamma(Z\to d\bar{d})$$

$$\Gamma_Z \approx 2.5\ \text{GeV}$$

BRANCHING RATIOS

$$Br(Z \to \nu_e \bar{\nu}_e) = Br(Z \to \nu_\mu \bar{\nu}_\mu) = Br(Z \to \nu_\tau \bar{\nu}_\tau) \approx 6.9\%$$

$$Br(Z \to e^+ e^-) = Br(Z \to \mu^+ \mu^-) = Br(Z \to \tau^+ \tau^-) \approx 3.5\%$$

$$Br(Z \to u\bar{u}) = Br(Z \to c\bar{c}) \approx 12\%$$

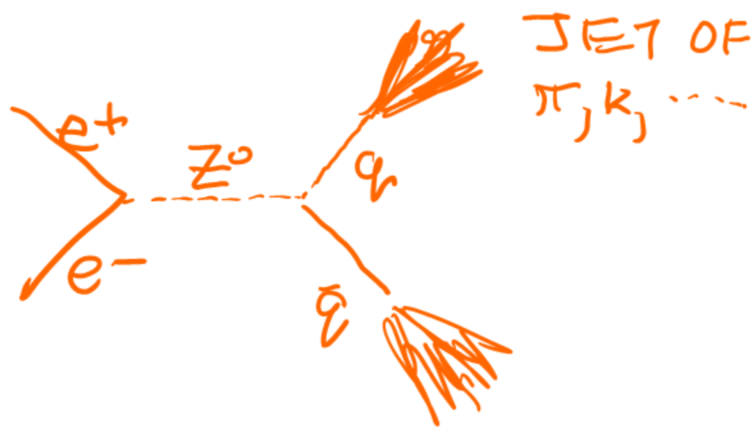
$$Br(Z \to d\bar{d}) = Br(Z \to s\bar{s}) = Br(b\bar{b}) \approx 15\%$$

$$Br(Z \to \nu\bar{\nu}) \simeq 21\%$$

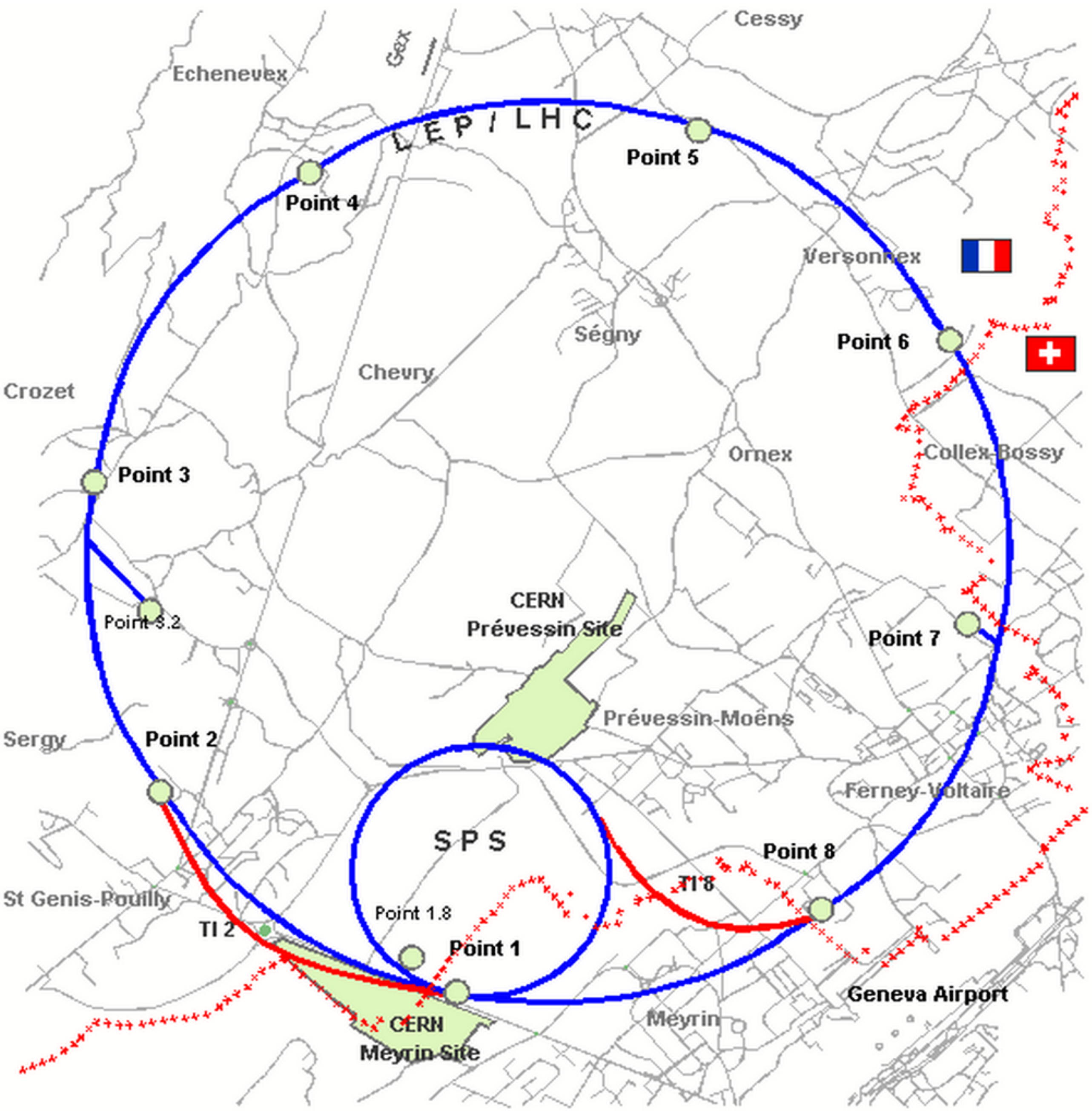
$$Br(Z \to \ell\bar{\ell}) = 10\%$$

$$Br(Z \to \text{HADRONS}) \approx 69\%$$

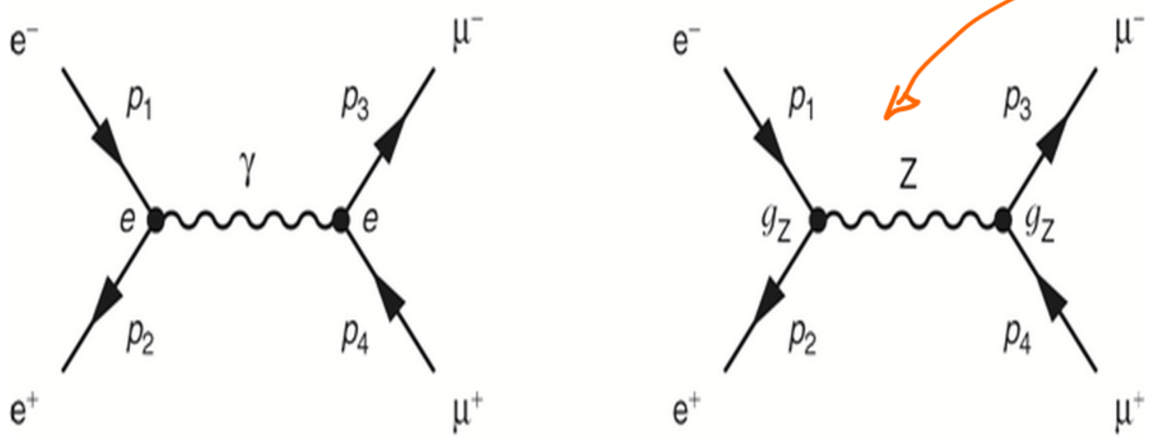
→ JETS



LARGE ELECTRON POSITRON COLLIDER



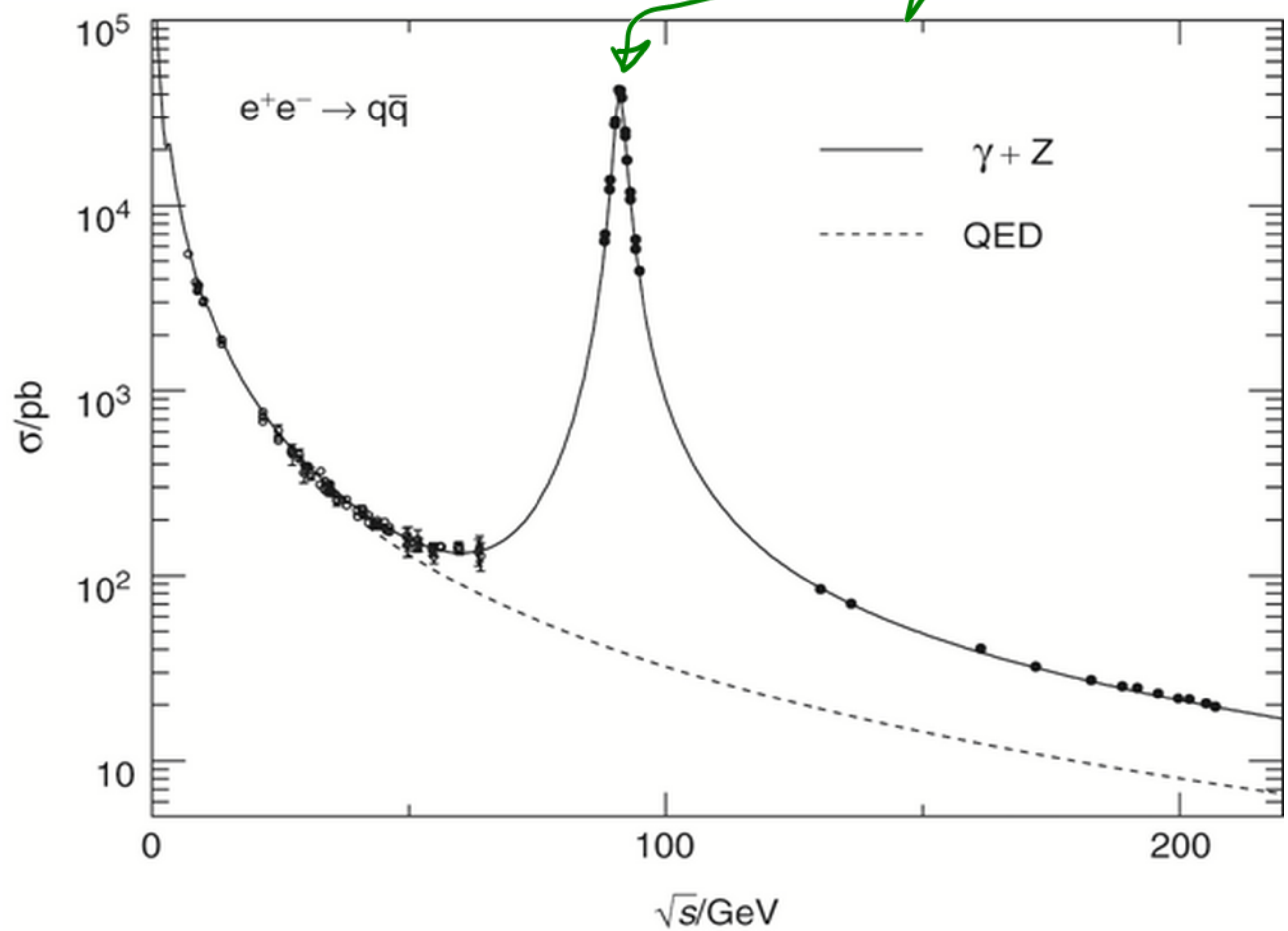
Map of CERN sites and LHC access points

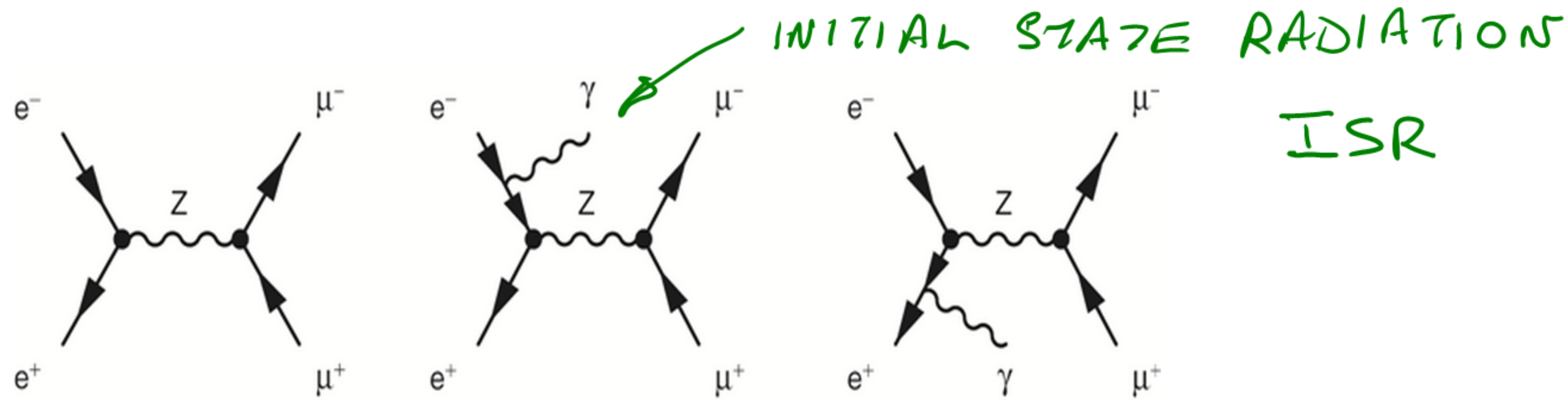


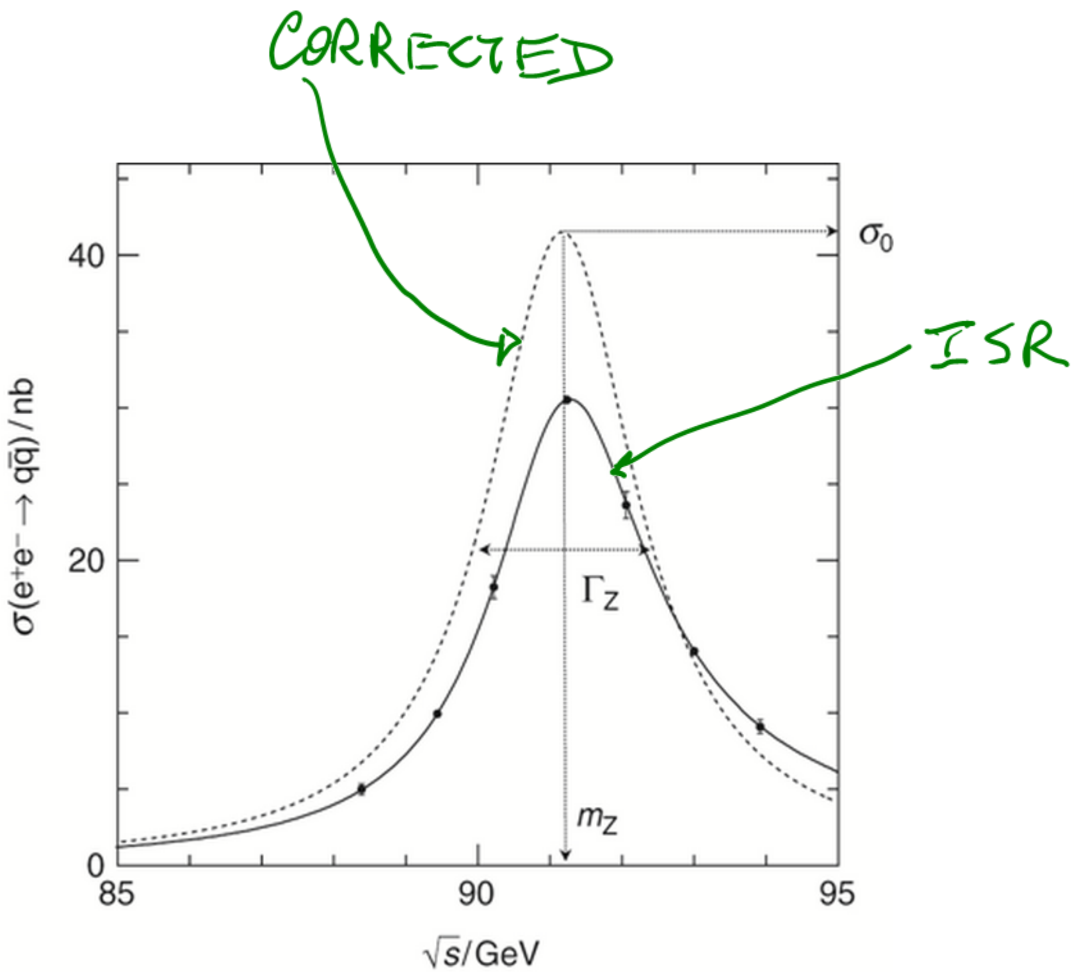
PROPAGATOR

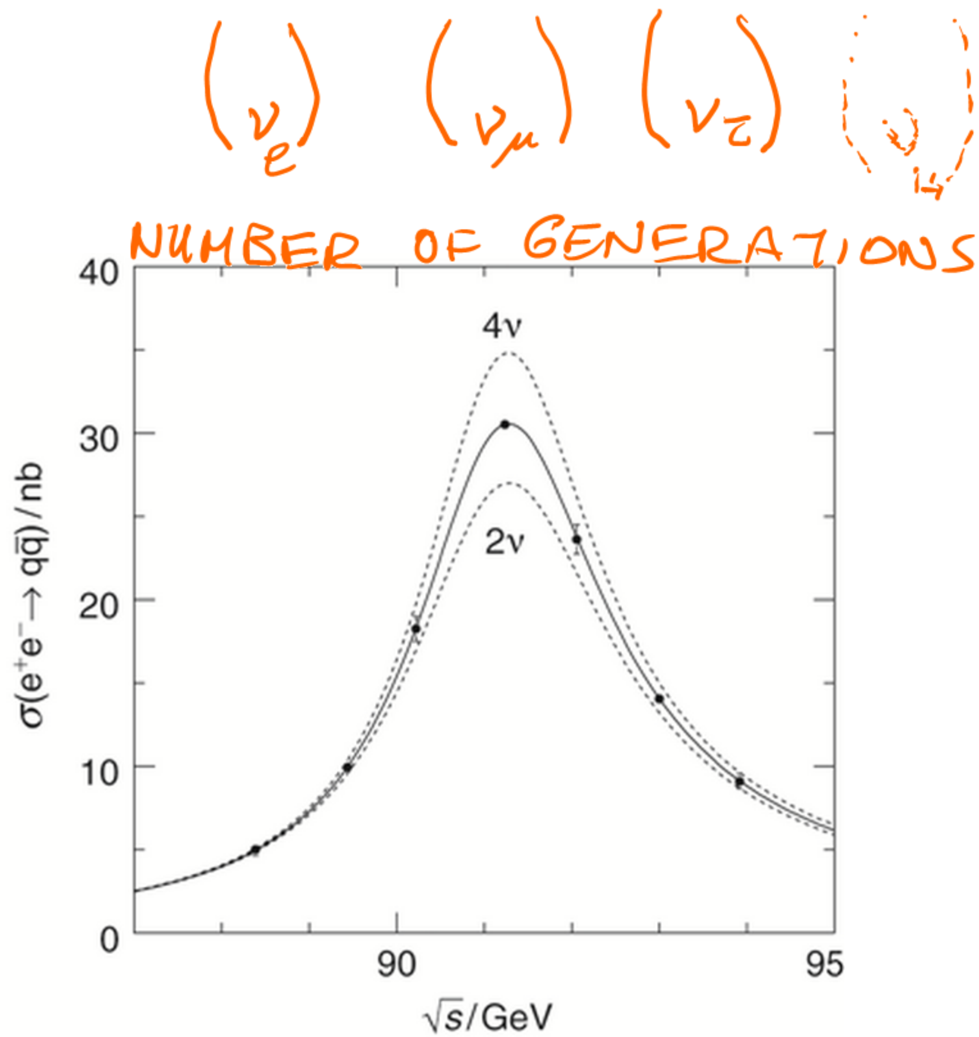
$$\frac{1}{q^2 - m_Z^2} \rightarrow \frac{1}{q^2 - m_Z^2 + i m_Z \Gamma_Z}$$

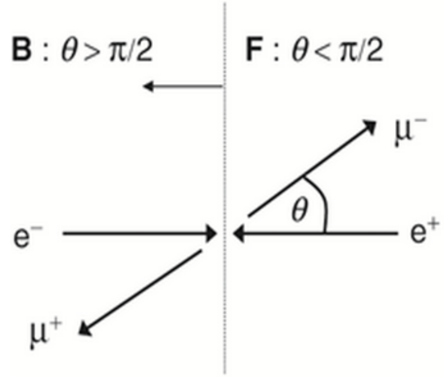
$q^2 = m_Z^2$

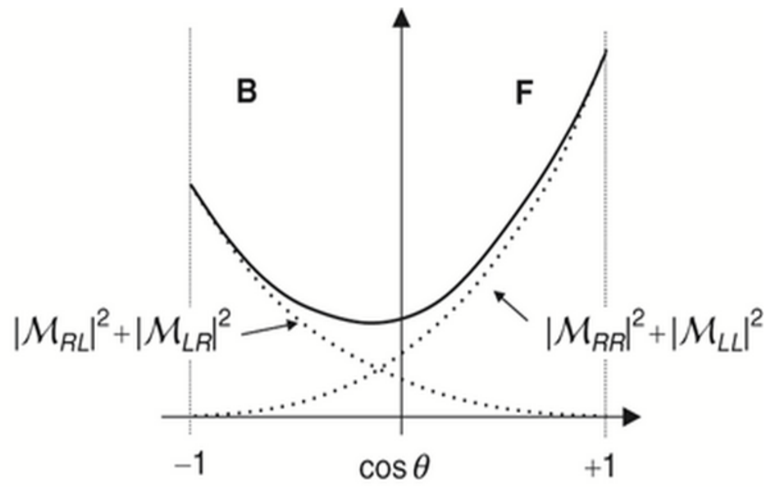



INITIAL STATE RADIATION
ISR
e⁻
e⁺
Z
μ⁻
μ⁺
γ


CORRECTED
ISR
σ_0
Γ_Z
m_Z
$\sigma(e^+e^- \to q\bar{q})$ / nb
$\sqrt{s}$/GeV
0
20
40
85
90
95

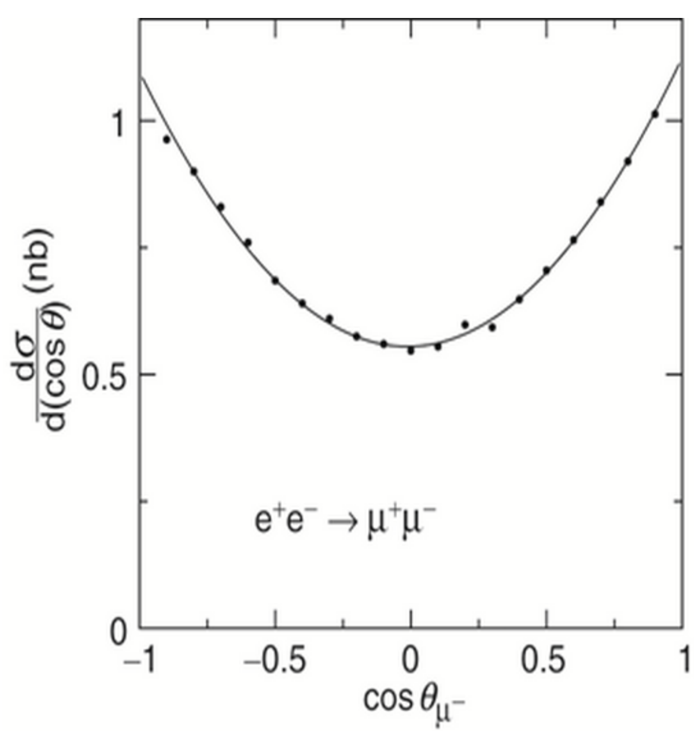

ν_e
ν_μ
ν_τ
NUMBER OF GENERATIONS
4ν
2ν
$\sigma(e^+e^- \to q\bar{q})$ / nb
$\sqrt{s}$/GeV
0
10
20
30
40
90
95

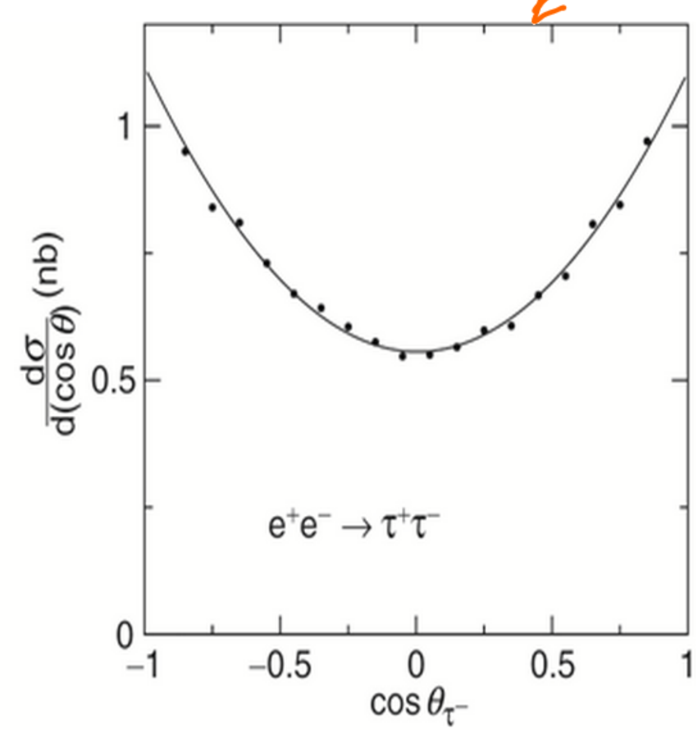


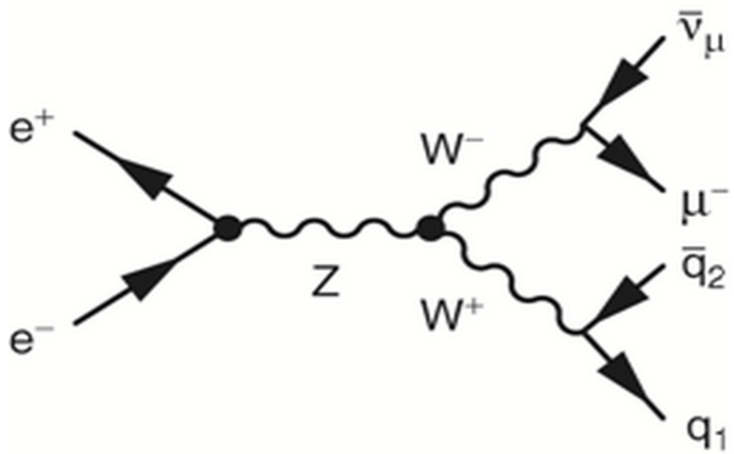


$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{C_V^{\ell}}{C_A^{\ell}} = 1 - 4\sin^2\theta_W$$

MEASURE







TRIPLE GAUGE
BOSON COUPLING