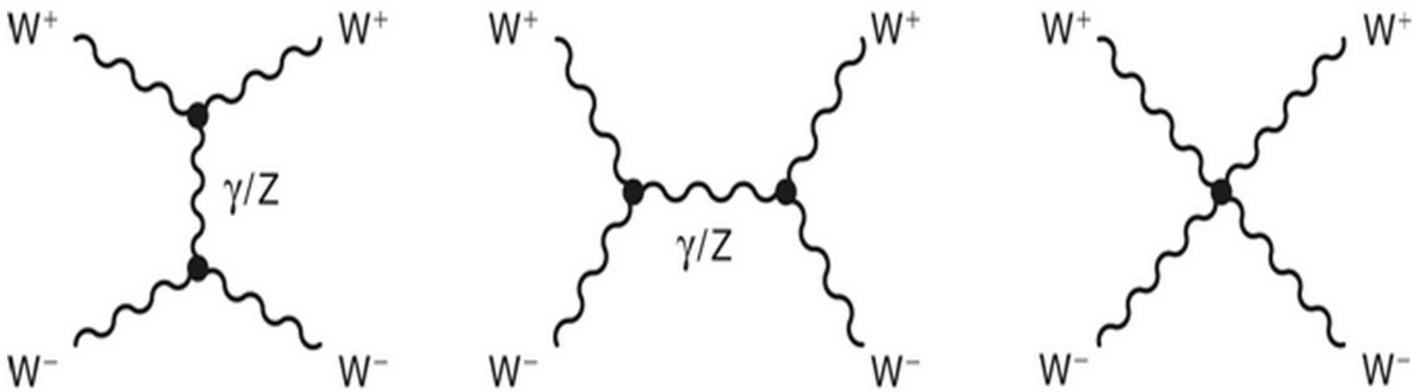
HIGGS BOSON

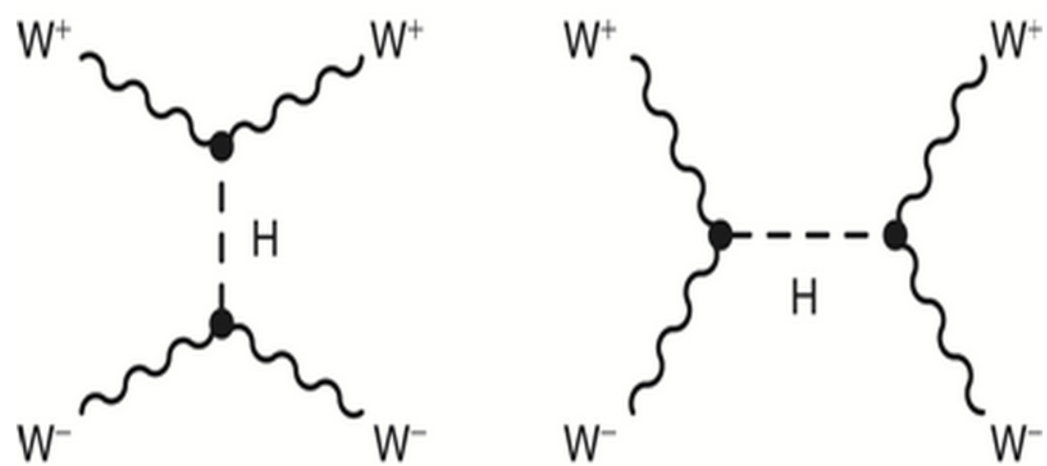


UNITARITY VIOLATION IN $e^+e^- \to W^+W^-$ SOLVED BY INTRODUCTION OF Z BOSON

$W^+W^- \to W^+W^-$ VIOLATES UNITARITY AT 1TeV ~ LHC

COMES FROM $W_L^+W_L^- \to W_L^+W_L^-$ ← MASSIVE PARTICLES

CANCELLED BY SCALAR



HIGGS MECHANISM → CENTRAL TO STANDARD MODEL

WE'VE SEEN THAT GAUGE SYMMETRY IS CENTRAL TO UNDERSTANDING WHERE INTERACTIONS COME FROM

GERHARD 't HOOFT SHOWED (WHILE A GRADUATE STUDENT) THAT ONLY LOCALLY GAUGE INVARIANT THEORIES ARE RENORMALIZABLE (FREE FROM UNCONTROLLABLY INFINITIES)

MASSIVE BOSONS $W^{\pm}$ Z BREAK LOCAL GAUGE INVARIANCE

↳ HIGGS MECHANISM SOLVES THIS

A LOCALLY GAUGE INVARIANT THEORY, WITH MASSIVE GAUGE BOSONS

LAGRANGIANS AND QUANTUM FIELD THEORY

HIGGS MECHANISM DEVELOPED IN TERMS OF THE LAGRANGIAN OF THE STANDARD MODEL.

WE HAVE TO INTRODUCE SOME OF THE IDEAS OF QFT.

I AM ESSENTIALLY ONLY GOING TO QUOTE SOME RESULTS WITHOUT DERIVATION. SOME OF THESE RESULTS ARE EXPANDED IN CHAP 17 OF THE TEXT BOOK

IF YOU ARE REALLY KEEN I RECOMMEND "QUANTUM FIELD THEORY OF THE GIFTED AMATEUR" — BY LANCASTER & BLUNDELL

CLASSICAL FIELDS

THE LAGRANGIAN APPROACH WAS DEVELOPED BEFORE QUANTUM MECHANICS

A CLASSICAL EXAMPLE OF AN EQUATION OF MOTION

$$F = m\ddot{x}$$

CAN FORMALLY DERIVE THIS FROM LAGRANGIAN

$$L = T - V$$

POTENTIAL ENERGY

KINETIC ENERGY

$$L \equiv L(q_i, \dot{q}_i)$$

$q_i \rightarrow$ GENERALIZED COORDINATES

LEAST ACTION

$L \rightarrow$ EQUATIONS OF MOTION VIA $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$

EULER-LAGRANGE

SIMPLE EXAMPLE

ONE DIMENSIONAL MOTION OF A PARTICLE

$$L = T - V = \frac{1}{2} m \dot{x}^2 - V(x)$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \qquad \frac{\partial L}{\partial x} = -\frac{\partial V}{\partial x}$$

EULER-LAGRANGE

$$m\ddot{x} = -\frac{\partial V(x)}{\partial x}$$

FORCE IS GRADIENT OF POTENTIAL

$$m\ddot{x} = F$$

NEWTON

DISCRETE SYSTEM → CONTINUOUS SYSTEM

$$L\left(q_i, \frac{dq_i}{dt}\right) \rightarrow \mathcal{L}(\phi_i, \partial_\mu \phi_i)$$

LAGRANGIAN DESITY

FIELD $\phi_i(t, x, y, z)$

FIELDS ARE CONTINUOUS FUNCTIONS OF

$$L = \int \mathcal{L}\, d3\bar{x}$$

AGAIN, LEAST ACTION GIVES EULER-LAGRANGE

$$\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\right) - \frac{\partial \mathcal{L}}{\partial \phi_i} = 0$$

ALL TEXTBOOKS USE VIBRATIONS OF A 1-DIMENSIONAL STRING AS AN EXAMPLE → READ IT.

RELATIVISTIC FIELDS

SINGLE PARTICLE WAVE FUNCTIONS $\rightarrow$ MULTIPARTICLE EXCITATIONS OF A CONTINUOUS QUANTUM FIELD

SCALAR FIELD

SPIN-0 PARTICLES ARE DESCRIBED BY EXCITATIONS OF A SCALAR FIELD $\phi(x) \longrightarrow$ KLEIN GORDON

SCALAR LAGRANGIAN

$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu \phi)(\partial_\mu \phi) - \frac{1}{2} m^2 \phi^2$$

$$= \frac{1}{2}\Big((\partial_0\phi)(\partial_0\phi) - (\partial_1\phi)(\partial_1\phi) - (\partial_2\phi)(\partial_2\phi) - (\partial_3\phi)(\partial_3\phi) - \frac{1}{2} m^2 \phi^2$$

IN EULER-LAGRANGE

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi, \quad \frac{\partial \mathcal{L}}{\partial(\partial_0 \phi)} = \partial_0 \phi = \partial^0 \phi, \quad \frac{\partial \mathcal{L}}{\partial(\partial_k \phi)} = -\partial_k \phi = \partial^k \phi$$

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

KLEIN-GORDON FOR SCALAR FIELD

RELATIVISTIC SPIN-$\frac{1}{2}$ FIELDS

DIRAC LAGRANGIAN IS:

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\psi - m\bar{\psi}\psi$$

PARTIAL DERIVATIVES W.R.T. $\bar{\psi}$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi}_i)} = 0 \quad ; \quad \frac{\partial \mathcal{L}}{\partial \bar{\psi}_i} = i\gamma^\mu\partial_\mu\psi - m\psi$$

EULER-LAGRANGE $\rightarrow$

$$-\frac{\partial \mathcal{L}}{\partial \bar{\psi}_i} = 0$$

$$i\gamma^\mu(\partial_\mu\psi) - m\psi = 0 \qquad \text{DIRAC}$$

RELATIVISTIC SPIN-1 FIELDS

MAXWELL'S EQUATIONS $\rightarrow$ $A^\mu = (\phi, \vec{A})$

$$\partial_\mu F^{\mu\nu} = j^\nu$$

FIELD STRENGTH TENSOR

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$j = (\rho, \vec{J})$

FOR $j^\mu = 0$

NO SOURCES

$$\mathcal{L}_{EM} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$\mathcal{L}_{EM} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

PUTTING IN FORM OF $F^{\mu\nu}$

$$\mathcal{L}_{EM} = \frac{1}{2}\left(\vec{E}^2 - \vec{B}^2\right) \rightarrow H_{EM} = \frac{1}{2}\left(\vec{E}^2 + \vec{B}^2\right)$$

IF γ HAD MASS

$$\mathcal{L}_{PROCA} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_\gamma^2 A^\mu A_\mu$$

$\hookrightarrow$ LAGRANGIAN FOR MASSIVE SPIN-1 PARTICLE

NÖTHER'S THEOREM

SYMMETRY OF LAGRANGIAN $\longrightarrow$ CONSERVED QUANTITY

EXAMPLE

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

UNCHANGED BY GLOBAL $U(1)$ $\leftarrow$ DOES NOT VARY IN SPACE TIME

$$\psi \rightarrow \psi' = e^{i\theta}\psi$$

CORRESPONDS TO CONSERVED CURRENT

$$j^\mu = \bar{\psi}\gamma^\mu\psi \longrightarrow \partial_\mu j^\mu = 0$$

CONSERVATION OF ELECTRIC CHARGE.

BACK TO LOCAL GAUGE INVARIANCE

HAVE LOOKED AT INVARIANCE OF DIRAC EQUATION UNDER A LOCAL PHASE TRANSITION

$$\psi \to \psi'(x) = e^{iq\chi(x)}\psi(x)$$

$$\to \partial_\mu(\chi(x)) \neq 0$$

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

$$\mathcal{L} \to \mathcal{L}' = ie^{-iq\chi}\bar{\psi}\gamma^\mu\left[e^{iq\chi}\partial_\mu\psi + iq(\partial_\mu\chi)e^{iq\chi}\psi\right] - me^{-iq\chi}\bar{\psi}e^{iq\chi}\psi$$

$$= \mathcal{L} - q\bar{\psi}\gamma^\mu(\partial_\mu\chi)\psi$$

$\to \mathcal{L}$ NOT INVARIANT.

$$\mathcal{L} - q\bar{\Psi}\gamma^\mu(\partial_\mu \chi)\psi$$

CAN RESTORE INVARIANCE BY DEFINING COVARIANT DERIVATIVE

$$\partial_\mu \to D_\mu = \partial_\mu + iqA_\mu$$

$\hookrightarrow A_\mu \to A_\mu' = A_\mu - \partial_\mu \chi(x)$

$\hookrightarrow$ HAVE TO INTRODUCE INTERACTION WITH FIELD A_μ

GAUGE INVARIANT LAGRANGIAN FOR SPIN-$\frac{1}{2}$ FERMION

$$\mathcal{L} = \underbrace{\bar{\Psi}(i\gamma^\mu\partial_\mu - m)\psi}_{\text{PROPAGATING ELECTRON}} - \underbrace{q\bar{\Psi}\gamma^\mu A_\mu \psi}_{\text{INTERACTION WITH EM FIELD}}$$

$$\mathcal{L} = \underbrace{\bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi}_{\text{PROPAGATING ELECTRON}} - \underbrace{q\,\bar{\Psi}\gamma^\mu A_\mu \Psi}_{\text{INTERACTION WITH EM FIELD}}$$

WHAT ABOUT THE PHOTON, IT IS ALSO CONTRIBUTING KINETIC ENERGY

THE PHOTON IS JUST THE EM FIELD $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$$\mathcal{L}_{QCD} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m_e)\Psi + e\bar{\Psi}\gamma^\mu \Psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

EULAR ~ LEGRANGE EQUATIONS GIVE EQUATION OF MOTIONS

$$\partial_\mu F^{\mu\nu} = j^\nu \leftarrow \text{MAXWELL'S EQUATION}$$

ALL OF ELECTROMAGNETISM IS IN U(1) SYMMETRY.

WEAK INTERACTION

REQUIRE INVARIANCE UNDER LOCAL SU(2)

$\partial_\mu \longrightarrow D_\mu$ JUST "LIKE" QED

$$\partial_\mu \to D_\mu = \partial_\mu + i g_W \bar{T} \cdot \tilde{W}_\mu \quad (2)$$

$\bar{T} \longrightarrow \frac{1}{2}\bar{\sigma} \to$ GENERATORS OF SU(2)

$\tilde{W}_\mu \to$ 3 NEW GAUGE FIELDS,

BUT NOT PHYSICAL $W^\pm$, Z

$\longrightarrow$ YET!

PARTICLE MASS PROBLEM

LOCAL GAUGE INVARIANCE BROKEN BY TERMS IN THE LAGRANGIAN CORRESPONDING TO PARTICLE MASSES

eg $\rightarrow$ MASSIVE γ

$$\mathcal{L}_{QED} \rightarrow \bar{\psi}(i\gamma^\mu \partial_\mu - m_e)\psi + e\bar{\psi}\gamma^\mu A_\mu \psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

UNDER LOCAL $U(1)$ $A_\mu \rightarrow A_\mu' = A_\mu - \partial_\mu \chi$

$$\frac{1}{2}m_\gamma^2 A_\mu A^\mu \rightarrow \frac{1}{2}m_\gamma^2 (A_\mu - \partial_\mu \chi)(A^\mu - \partial^\mu \chi) \neq \frac{1}{2}m_\gamma^2 A_\mu A^\mu$$

MASS TERM NOT GAUGE INVARIANT

CAN ONLY HAVE GAUGE INVARIANCE FOR MASSLESS GAUGE BOSONS

QED ✓

WEAK ✗

QCD ✓

PROBLEM OF PARTICLE MASS NOT JUST GAUGE BOSONS

ELECTRON SPINOR $\psi_e \rightarrow$ CALL IT "e"

$$-m_e \bar{e} e = -m_e e \left[\frac{1}{2}(1-\gamma^5) + \frac{1}{2}(1+\gamma^5) \right] e$$

$$= -m_e e \left[\frac{1}{2}(1-\gamma^5) e_L + \frac{1}{2}(1-\gamma^5) e_R \right]$$

$$= -m_e \left(\bar{e}_R e_L + \bar{e}_L e_R \right)$$

UNDER $SU(2)_L$ $\quad e_L \longrightarrow$ ISOSPIN DOUBLET

$e_R \longrightarrow$ ISOSPIN SINGLET

MASS TERM $-m_e \bar{e} e$ BREAKS GAUGE INVARIANCE

INTERACTING SCALAR FIELDS

GAUGE INVARIANCE CAN BE MAINTAIND BY INVOKING A BROKEN SYMMETRY

LOOK AT A SIMPLE 1-DIMENSIONAL MODEL

IN QED $\quad i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi$ ELECTRON KINETIC ENERGY

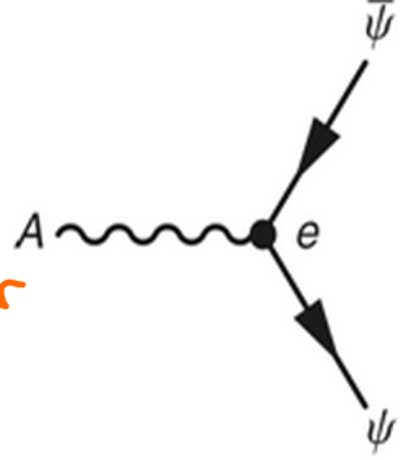
$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ PHOTON KINETIC ENERGY

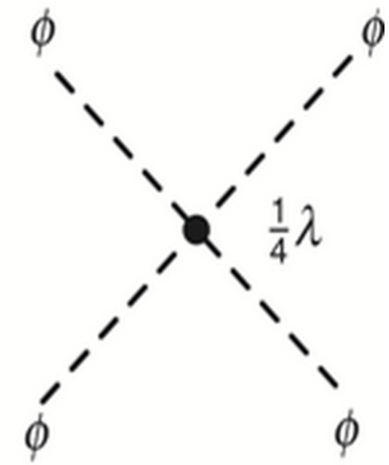
VERTEX FACTOR $\quad e\bar{\psi}\gamma^{\mu}\psi A_{\mu}$ POTENTIAL

STRENGTH OF COUPLING BETWEEN FIELD IS GENERALLY GIVEN BY

$$\bar{\psi}\psi A$$

QED INTERACTION





INTERACTING SCALAR FIELDS

SCALAR FIELD ϕ WITH POTENTIAL GIVEN BY

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)$$

$$= \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

KINETIC ENERGY OF FIELD

MASS OF PARTICLE

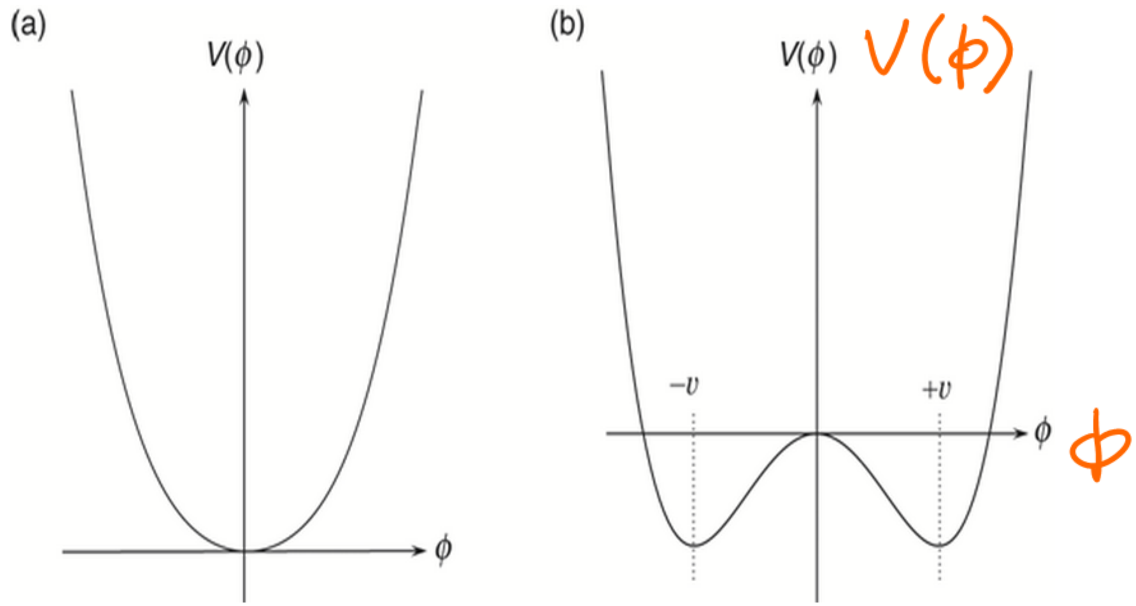
SELF INTERACTION OF FIELD

μ^2 +ve

MINIMUM AT

$\phi = 0$

$\lambda > 0$



μ^2 −ve

$\lambda > 0$

$$\phi = \pm\sqrt{\frac{-\mu^2}{\lambda}}$$

VACUUM → LOWEST ENERGY OF ϕ → MINIMUM OF $\phi(x)$

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$$

$$\frac{dV}{d\phi} = \mu^2\phi + \lambda\phi^3 = 0$$

$\phi = 0, \quad \mu^2 + \lambda\phi^2 = 0$

$$\phi^2 = \frac{-\mu^2}{\lambda}$$

$$\frac{d^2V}{d\phi^2} = \mu^2 + 3\lambda\phi^2 = \text{+VE}$$

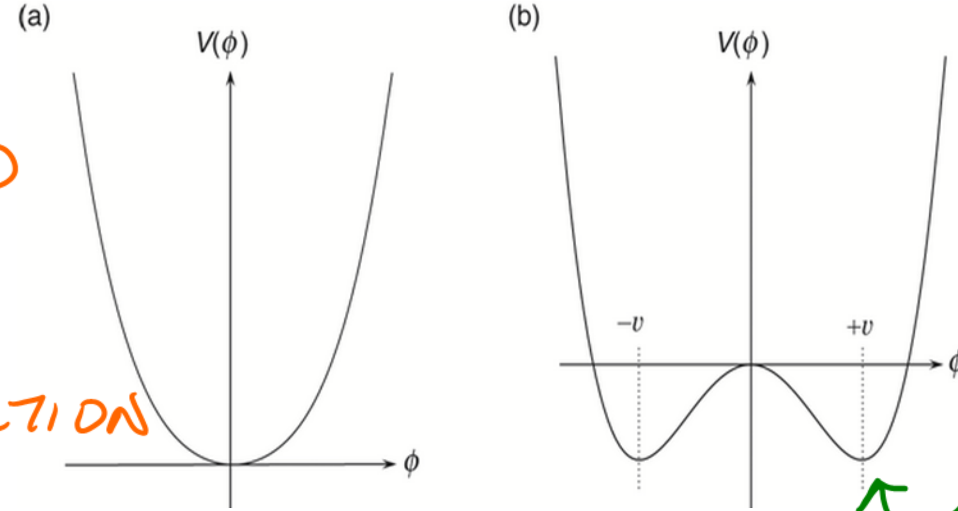
MINIMUM

$\lambda \rightarrow$ +VE

VACUUM

FIELD $\phi = 0$

$\mathcal{L} \to$ 4 POINT INTERACTION



$\mu^2 < 0$

$\phi = \pm \sqrt{\dfrac{-\mu^2}{\lambda}}$

← VACUUM

FOR $\mu^2 < 0$ $\quad -\frac{1}{2}\mu^2\phi^2$ IS NO LONGER MASS

POTENTIAL MINIMUM $\phi = \pm\sqrt{\dfrac{-\mu^2}{\lambda}}$

LOWEST ENERGY $\phi \neq 0$ NON ZERO VACUUM EXPECTATION VALUE

2 DEGENERATE POSSIBLE VACUUA

SYSTEM ENDS IN ONE OR OTHER

$\left.\begin{array}{l}\text{VACUUM DOES NOT HAVE}\\ \text{SYMMETRY OF } \mathcal{L}\end{array}\right\}$ SPONTANEOUS SYMMETRY BREAKING.

PARTICLES ARE EXCITATIONS OF QUANTUM FIELD

AFTER SPONTANEOUS SYMMETRY BREAKING THE VACUUM STATE IS $v = \pm\sqrt{\frac{-\mu^2}{\lambda}}$

PARTICLES IN THIS "THEORY" CAN BE "FOUND" BY CONSIDERING EXCITATIONS AROUND THE VACUUM

I.E. PERTURBATIONS AROUND $\phi(x) = v + \eta(x)$

$$\partial_\mu \phi(x) = \partial_\mu \eta(x)$$

CONSTANT.

THEN

$$\mathcal{L}(x) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda\phi^4$$

BECOMES

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2}\mu^2(v+\eta)^2 - \frac{1}{4}\lambda(v+\eta)^4$$

$$\mu^2 = -\lambda v^2$$

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4 + \frac{1}{4}\lambda v^4$$

COMPARE THIS TO:

$$\mathcal{L}_s = \frac{1}{2}(\partial_\mu \phi)(\partial_\mu \phi) - \frac{1}{2} m^2 \phi^2$$

K.G.

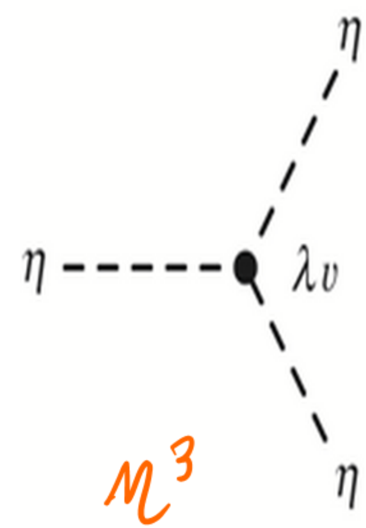
MASS

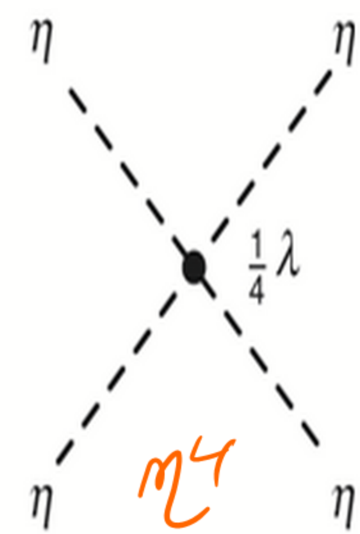
$$m_\eta = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

MASSIVE SCALAR FIELD

$\frac{\lambda v^4}{4} \rightarrow$ CONSTANT

VANISHES AFTER DIFF IN EULAR LEGRANGE





AFTER SPONTANEOUS SYMMETRY BREAKING

EXPAND ABOUT "BROKEN" VACUUM

$$\mathcal{L}(\eta) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2} m_\eta^2 \eta^2 - V(\eta)$$

cf $\mathcal{L}_S(\phi) = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \frac{1}{2} m^2 \phi^2$

$$V=(\eta) = \lambda v \eta^3 + \frac{1}{4}\lambda \eta^4$$

THIS IS SAME LAGRANGIAN AS:

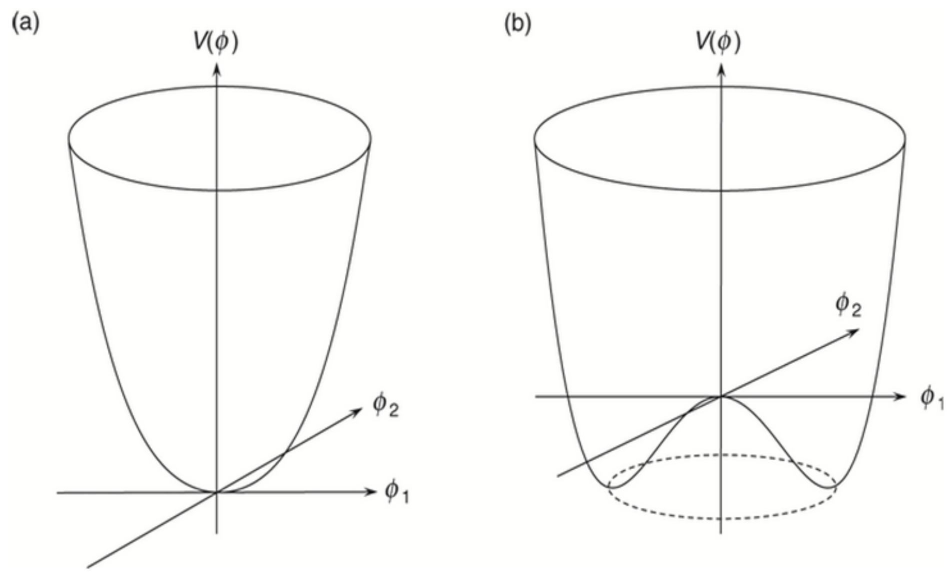
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}\mu^2 \phi^2 - \frac{1}{4}\lambda \phi^4$$

BOTH HAVE SAME PHYSICS CONTENT

SMALL PERTURBATIONS AROUND VACUUM $\Big\}$ $\rightarrow$ PERTURBATION THEORY

SYMMETRY BREAKING IN A COMPLEX SCALAR FIELD

NEXT STEP TOWARDS REAL WORLD



COMPLEX

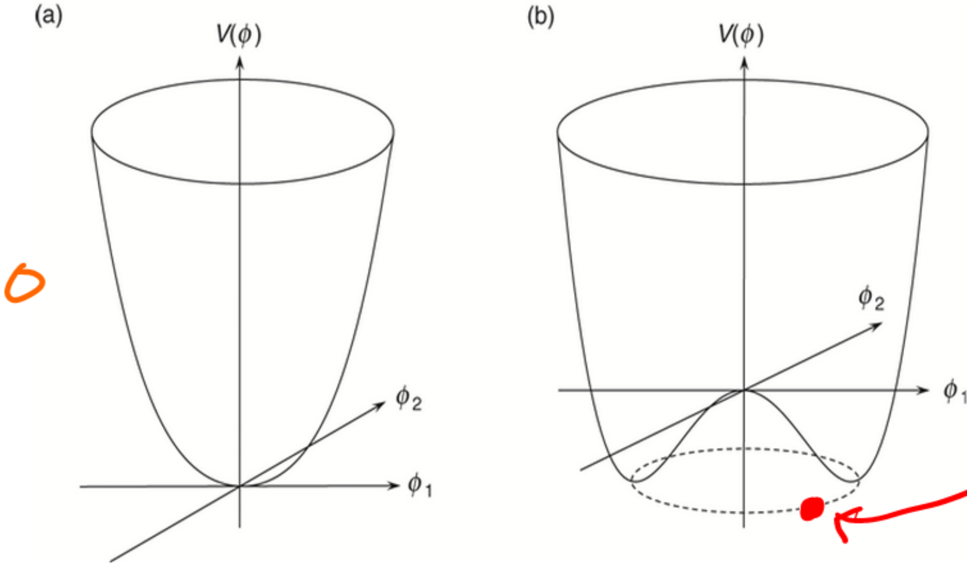
$$\phi = \frac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right)$$

REAL

$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - V(\phi)$$

$$V(\phi) = \mu^2 (\phi^* \phi) + \lambda (\phi^* \phi)^2$$

$\mu^2 > 0$



$\mu^2 < 0$

"BROKEN" VACUUM

IN TERMS OF TWO REAL SCALAR FIELDS

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)(\partial^\mu \phi_1) + \frac{1}{2}(\partial_\mu \phi_2)(\partial^\mu \phi_2) - \frac{1}{2}\mu^2(\phi_1^2 + \phi_2^2) - \frac{1}{4}\lambda(\phi_1^2 + \phi_2^2)$$

FOR A MINIMUM $\lambda > 0$

INVARIANT UNDER $\phi \to \phi' = e^{i\alpha}\phi \to \phi'^* \phi' = \phi^* \phi$

$\to$ GLOBAL $U(1)$

$\mu^2 > 0$

V_{MIN} $\phi_1 = \phi_2 = 0$

$\mu^2 < 0$ $\phi_1^2 + \phi_2^2 = \frac{-\mu^2}{\lambda} = v^2$

$\to$ DASHED CIRCLE

VACUUM BREAKS $U(1)$

CHOOSE $(\phi_1, \phi_2) = (v, 0)$ ANY WHERE ON DASHED CIRCLE

EXPAND ABOUT VACUUM

$$\phi_1(x) = \eta(x) + v \quad , \quad \phi_2(x) = \xi(x)$$

$$\phi = \frac{1}{\sqrt{2}}(\eta + v + i\xi)$$

POTENTIAL

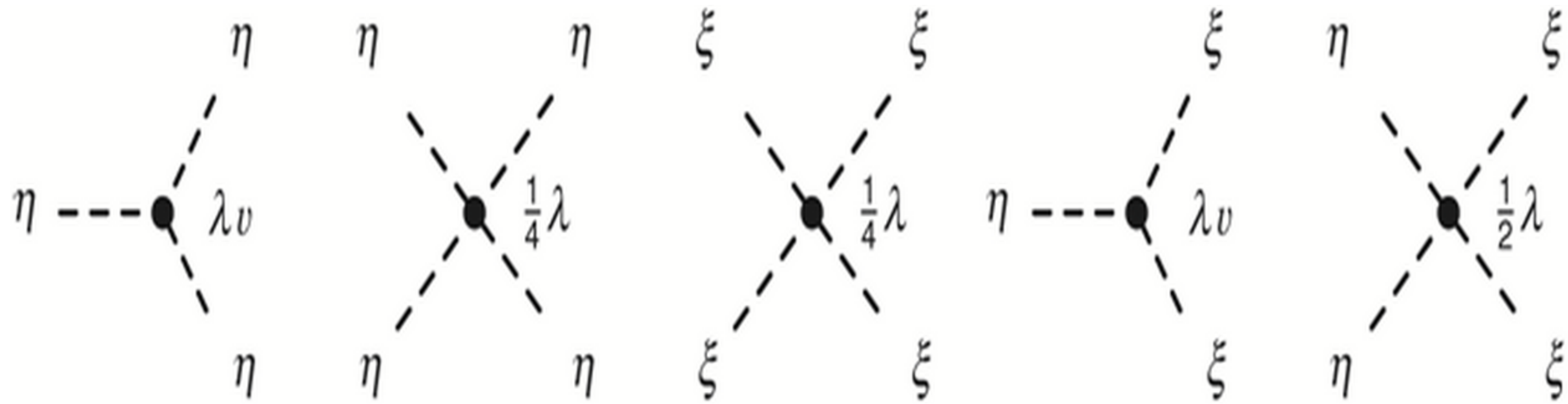
$$V(\eta, \xi) = \mu^2 \phi^2 + \lambda \phi^4, \quad \phi^2 = \phi\phi^* = \frac{1}{2}\left[(v+\eta)^2 + \xi^2\right]$$

AGAIN PUT $\mu^2 = -\lambda v^2$

$$V(\eta, \xi) = -\frac{1}{2}\lambda v^2\left[(v+\eta)^2 + \xi^2\right] + \frac{1}{4}\lambda\left[(v+\eta)^2 + \xi^2\right]$$

$$V(\eta, \xi) = -\frac{1}{4}\lambda v^4 + \underbrace{\lambda v^2 \eta^2}_{\text{MASS}} + \lambda v \eta^3 + \frac{1}{4}\lambda\eta^4 + \lambda v \eta \xi^2 + \frac{1}{2}\lambda \eta^2 \xi^2$$

OTHER TERMS INTERACTIONS $\eta \leftrightarrow \xi$



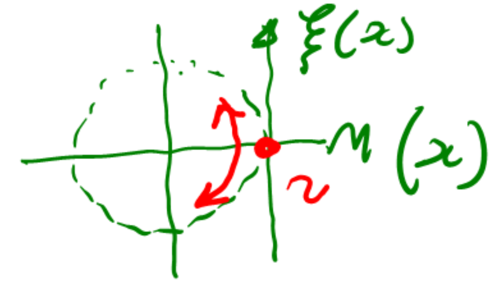
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) + \frac{1}{2} m_\eta^2 \eta^2 + \frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) - V_{INT}(\eta, \xi)$$

$$m_\eta = \sqrt{2\lambda v^2}$$

$$V_{INT} = \lambda v \eta^3 + \frac{1}{4}\lambda \eta^4 + \frac{1}{4}\lambda \xi^4 + \lambda v \eta \xi^2 + \frac{1}{2}\lambda \eta^2 \xi^2$$

$\eta \rightarrow$ MASSIVE SCALAR

$\xi \rightarrow$ MASSLESS SCALAR



GOLDSTONE BOSON

ALWAYS GET IT FOR A BROKEN SYMMETRY

EXTEND TO LOCAL GAUGE INVARIANCE

LAST TWO EXAMPLES SPONTANEOUS SYMMETRY BREAKING IN GLOBAL GAUGE INVARIANCE

STANDARD MODEL IS A LOCAL GAUGE THEORY $SU(2)_L$

LOOK AT LOCAL $U(1)$ FIRST.

FOR COMPLEX SCALAR FIELD HAD

$$\mathcal{L} = (\partial_\mu \phi)^*(\partial^\mu \phi) - \mu^2 (\phi^* \phi) + \lambda (\phi^* \phi)^2$$

GLOBAL $U(1)$, BUT NOT LOCAL $\Rightarrow \partial_\mu \; \partial^\mu$

LOCAL $\rightarrow$ $\phi(x) \rightarrow \phi'(x) = e^{ig\chi(x)} \phi(x)$

AS USUAL $\partial_\mu \rightarrow D_\mu = \partial_\mu + igB_\mu$

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - V(\phi^2)$$

$$\mathcal{L} = (D_\mu \phi^*)(D^\mu \phi) - V(\phi^2)$$

GAUGE INVARIANT LOCALLY IF

$$B_\mu \to B_\mu' = B_\mu - \partial_\mu \chi(x)$$

COMBINED LAGRANGIAN FOR $\phi(x)$, B

$$\mathcal{L} = -\frac{1}{4} \underbrace{F^{\mu\nu} F_{\mu\nu}} + (D_\mu \phi)^* (D^\mu \phi) - \mu^2 \phi^2 - \lambda \phi^4$$

$\longrightarrow F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$

KINETIC TERM, LIKE EM.

B is MASSLESS $\to \frac{1}{2} m_B B_\mu B^\mu$

BREAKS GAUGE INVARIANCE

$$(D_\mu\phi)^*(D^\mu\phi) = (\partial_\mu - igB_\mu)\phi^*(\partial^\mu + igB^\mu)\phi$$

$$= (\partial_\mu\phi)^*(\partial^\mu\phi) - igB_\mu\phi^*(\partial_\mu\phi) + ig(\partial_\mu\phi^*)B^\mu\phi$$

$$+ g^2 B_\mu B^\mu \phi^*\phi$$

"AS USUAL" FOR $\mu^2 < 0$ HAVE DEGERATE VACUUM STATE $\rightarrow$ CHOICE OF VACUUM BREAKS SYMMETRY

CHOOSE $\phi_1 + i\phi_2 = v$, EXPAND AROUND THIS VACUUM

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + ig\,\xi(x))$$

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda^2 v^2 \eta^2}_{\text{MASSIVE } \eta} + \underbrace{\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)}_{\text{MASSLESS } \xi}$$

$$+ \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2 v^2 B_\mu B^\mu}_{\text{MASSIVE GAUGE FIELD}} + V_{INT} + g v B_\mu(\partial^\mu\xi)$$

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda^2 v^2 \eta^2}_{\text{MASSIVE } \eta} + \underbrace{\tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)}_{\text{MASSLESS } \xi}$$

$$+ \underbrace{\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} g^2 v^2 B_\mu B^\mu}_{\text{MASSIVE GAUGE FIELD}} + \underbrace{V_{INT}}_{\text{3, 4 POINT INTERACTION}} + g v B_\mu (\partial^\mu \xi)$$

SYMMETRY BREAKING:

- MASSIVE SCALAR η
- MASSLESS GOLDSTONE BOSON ξ
- MASSLESS GAUGE FIELD B $\rightarrow$ ACQUIRES MASS TERM $\frac{1}{2} g^2 v^2 B_\mu B^\mu$

THE LAGRANGIAN HAS SAME PHYSICS

ie SYMMETRY, BUT NOT MANIFESTLY

PROBLEMS

ORIGINAL $\mathcal{L}$ $\Bigg\}$ 4 DEGREES OF FREEDOM

ϕ_1, ϕ_2

$B \rightarrow$ 2 TRANSVERSE POLARIZATN.

GAUGE BOSON B BECOMES MASSIVE

$\rightarrow$ GAINS LONGITUDINAL POLARIZATION

$g v B_\mu (\partial^\mu \xi)$

SPIN 1 $\dashrightarrow$ SPIN 0

$\downarrow$

PHYSICALLY NO MASSLESS GOLDSTONE BOSON EXISTS

GET RID OF THE GOLDSTONE BOSON

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda^2 v^2 \eta^2}_{\text{MASSIVE } \eta} + \boxed{\underbrace{\tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)}_{\text{MASSLESS } \xi}}$$

$$+ \underbrace{\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \boxed{\tfrac{1}{2} g^2 v^2 B_\mu B^\mu}}_{\text{MASSIVE GAUGE FIELD}} - V_{INT} + \boxed{g v B_\mu (\partial^\mu \xi)}$$

3, 4 POINT INTERACTION

NOTICE THAT

$$\tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + g v B_\mu (\partial^\mu \xi) + \tfrac{1}{2} g^2 v^2 B_\mu B^\mu = \tfrac{1}{2} g^2 v^2 \left| B_\mu + \tfrac{1}{gv}(\partial_\mu \xi) \right|^2$$

LAGRANGIAN IS GAUGE INVARIANT

SO DO A GAUGE TRANSFORMATION

$$B_\mu(x) \longrightarrow B'_\mu(x) = B_\mu(x) + \frac{1}{gv} \partial_\mu \xi(x)$$

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda^2 v^2 \eta^2}_{\text{MASSIVE } \eta} + \underbrace{\tfrac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)}_{\text{MASSLESS } \xi}$$

$$+ \underbrace{\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} g^2 v^2 B_\mu B^\mu}_{\text{MASSIVE GAUGE FIELD}} + V_{INT} + g v B_\mu (\partial^\mu \xi)$$

$\uparrow$ 3, 4 POINT INTERACTION

AFTER THE GAUGE TRANSFORMATION

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2}_{\text{MASSIVE } \eta} - \underbrace{\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} g^2 v^2 B'_\mu B^{\mu'}}_{\text{MASSIVE GAUGE FIELD}} - V_{INT}.$$

THERE IS A "GAUGE" WHERE THE GOLDSTONE BOSON NO LONGER APPEARS IN THE LAGRANGIAN HOW CAN THAT BE?

THE GAUGE WE CHOSE CORRESPONDS TO:

$$\chi(x) = \frac{-\xi(x)}{g v}$$

GAUGE TRANSFORMATION ON ORIGINAL COMPLEX SCALAR ϕ

$$\phi(x) \rightarrow \phi'(x) = \exp\left(-g \cdot \frac{\xi(x)}{g v}\right) \cdot \phi(x) = \exp\left(-\frac{i\,\xi(x)}{v}\right) \cdot \phi(x)$$

SCALAR FIELD EXPANDED ABOUT VACUUM

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + \eta(x) + i\,\xi(x)\right)$$

TO 1^{st} ORDER $\rightarrow$ $\approx \frac{1}{\sqrt{2}}\left[v + \eta(x)\right] e^{i\xi(x)/v}$

$$\phi(x) \rightarrow \phi'(x) = \frac{1}{\sqrt{2}} e^{-i\xi/v}\left[v + \eta(x)\right] e^{i\xi/v} = \frac{1}{\sqrt{2}}\left(v + \eta(x)\right)$$

UNITARY GAUGE $\longrightarrow$ $= \frac{1}{\sqrt{2}}\left(v + h(x)\right)$

"Higgs"

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \lambda v^2 \eta^2}_{\text{MASSIVE } \eta} - \underbrace{\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} g^2 v^2 B_\mu B^\mu}_{\text{MASSIVE GAUGE FIELD}} - V_{INT.}$$

NO CROSS TERMS LIKE $B_\mu(\partial^\mu \xi) \rightarrow$ PHYSICAL FIELDS

"GOLDSTONE BOSON EATEN BY B $\rightarrow$ B BECOMES FAT"

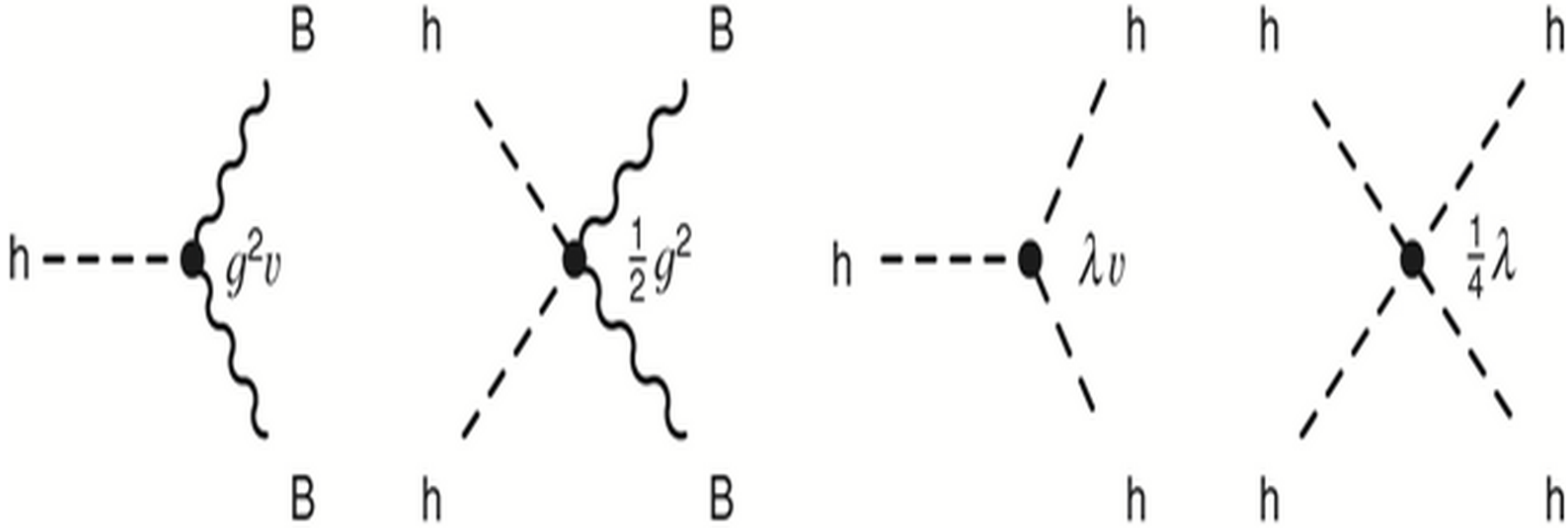
PUT $\mu^2 = -\lambda v^2$, $\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))$

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) - \lambda v^2 h^2}_{\text{MASSIVE HIGGS SCALAR}} - \underbrace{\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} g^2 v^2 B_\mu B^\mu}_{\text{MASSIVE GAUGE BOSON}}$$

$$+ \underbrace{g^2 v B_\mu B^\mu h + \tfrac{1}{2} g^2 B_\mu B^\mu}_{h,B \text{ COUPLING}} - \underbrace{\lambda v h^3 - \tfrac{1}{4}\lambda h^4}_{\text{HIGGS SELF COUPLING}}$$

$$m_B = g v \quad , \quad m_H = \sqrt{2\lambda}\, v$$

VACUUM EXPECTATION VALUE


h
B
B
$g^2 v$
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$\frac{1}{4}\lambda$