

SOLUTIONS PS #1

①

1.1) a), d), f), j), n), o) allowed

b) ν_e is neutral - cannot couple to $\gamma \rightarrow$ EM boson.

c) violates electric charge conservation. Also γ cannot change particle into antiparticle

e) γ^μ cannot change flavor.

g) Z does not change flavor.

h) W can do $e^- \rightarrow \nu_e$, not $e^- \rightarrow \nu_\mu$

i) e does not have color charge \therefore does not couple to gluon.

k) gluon does not carry flavor.

ok $u \rightarrow s$ ~~not~~

not ok $u \rightarrow \bar{u}$ etc.

l) no $\gamma\gamma$ vertex

m) $W^\pm \rightarrow$ must flip in generation $\begin{pmatrix} u \\ d \end{pmatrix}$

p) no vertex coupling 2 γ to a fermion line.

1.3) a) $\mu^- \rightarrow e^+ e^- e^+$ Violates lepton flavor.

what happens is $\mu^- \rightarrow e^- \bar{\nu}_e$
 correct ν oscillations

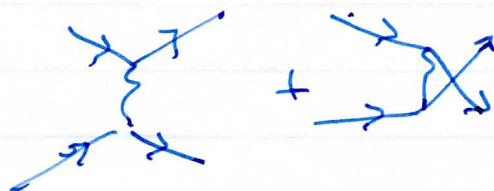
b) $\nu_\tau + p \rightarrow \mu^- n$ — electron charge not conserved.

c) $\nu_\tau + p \rightarrow \tau^+ n$ ν_τ particle \rightarrow τ^+ Anti particle

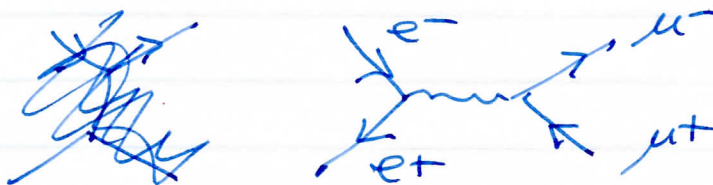
d) $\pi^+(u\bar{d}) + \pi^-(d\bar{u}) \rightarrow n(udd) + \pi^0(u\bar{u})$
 equal # of particle, antiparticle \rightarrow # particles \neq antiparticles (quarks)

~~$e^+ e^- \rightarrow e^+ e^-$~~ 

1.6) $e^- e^- \rightarrow e^- e^-$

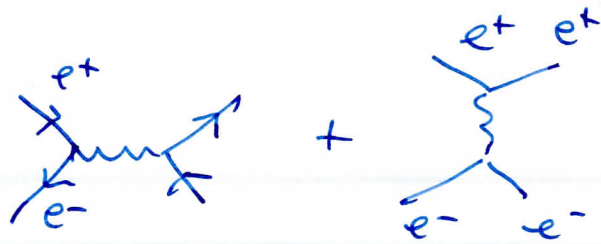


$e^+ e^- \rightarrow \mu^+ \mu^-$

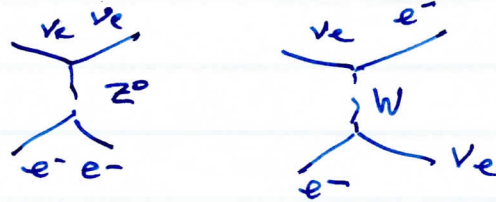


(3)

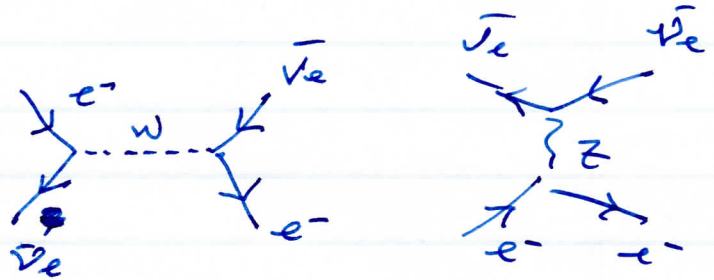
$$e^+e^- \rightarrow e^+e^-$$



$$e^- \nu_e \rightarrow e^- \nu_e$$



$$e^- \bar{\nu}_e \rightarrow e^- \bar{\nu}_e$$



1.10) Fixed target to get CMS 14 TeV

$$\text{LHC} \quad \sqrt{s} = 14 \text{ TeV}$$

$$\text{Fixed} \quad s = p^2 = \sum_i E_i^2 - \sum_i \vec{p}_i^2 = (E + m_p)^2 - \vec{p}^2$$

\uparrow 4 moun \uparrow Beam energy \uparrow Stationary proton \uparrow Beam mom.

$$s = E^2 + 2m_p E + m_p^2 - (E^2 - m_p^2)$$


$$s = 2m_p E$$

\uparrow Beam energy

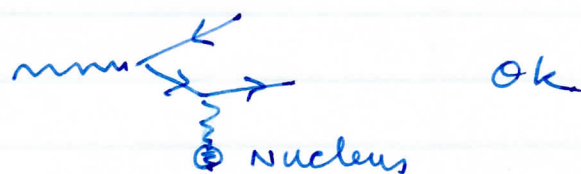
$$E = \frac{(14 \text{ TeV})^2}{2 \times 1 \times 10^{-6}}$$

\uparrow m_p :-

$$\approx 1 \times 10^5 \text{ TeV}$$

2.3) $\gamma \rightarrow e^+ e^-$ ~~is~~  not ok

$$p_\gamma^2 = (p_{e^+} + p_{e^-})^2$$



$$0 = m_e^2 + m_e^2 + \underbrace{2p_{e^+} \cdot p_{e^-}}_{\text{always +ve or 0}}$$

$$0 = 2m_e^2 \quad \text{---} \quad ??$$

2.5) $E' = \gamma(E - \beta p_z), \quad p'_x = p_x, \quad p'_y = p_y$
 $p'_z = \gamma(p_z - \beta E)$

$$(E')^2 - (p')^2 = \gamma^2 (E - \beta p_z)^2 - p_x^2 - p_y^2 - \gamma^2 (p_z - \beta E)^2$$

$$= \gamma^2 (E^2 - 2E\beta p_z + \beta^2 p_z^2) - p_x^2 - p_y^2$$

$$- \gamma^2 p_z^2 + \gamma^2 \beta^2 E^2 + 2\gamma^2 p_z \beta E$$

$$= \gamma^2 E^2 - 2\gamma^2 E\beta p_z + \gamma^2 \beta^2 p_z^2 - p_x^2 - p_y^2$$

$$- \gamma^2 p_z^2 - \gamma^2 \beta^2 E^2 + 2\gamma^2 p_z \beta E$$

$$= \gamma^2 E^2 \underbrace{(1 - \beta^2)}_{=1} - \gamma^2 p_z^2 (1 - \beta^2) - p_x^2 - p_y^2$$

$$= \gamma^2 E^2 - \gamma^2 p_z^2 - p_x^2 - p_y^2$$

$$= E^2 - p^2$$

$$(E')^2 = (p')^2$$

$$= \gamma(E - \beta p_z)^2 - p_x^2 - p_y^2 - \gamma^2(p_z - \beta E)^2$$

$$= \gamma(E^2 - 2\beta p_z E + \beta^2 p_z^2) - p_x^2 - p_y^2 - \gamma^2 p_z^2 + \gamma^2 p_z^2 \beta^2 E^2$$

$$= \gamma E^2 - 2\gamma\beta p_z E + \gamma\beta^2 p_z^2 - \gamma^2 p_z^2 + \gamma^2 p_z^2 \beta^2 E^2 - p_x^2 - p_y^2$$

$$= \gamma^2 E^2 (1 - \beta^2)$$

$$\gamma^2(E - \beta p_z)^2 - p_x^2 - p_y^2 - \gamma^2(p_z - \beta E)^2$$

$$= \gamma^2(E^2 - 2E\beta p_z + \beta^2 p_z^2) - p_x^2 - p_y^2 - \gamma^2(p_z^2 - 2p_z \beta E + \beta^2 E^2)$$

$$= \cancel{\gamma^2 E^2} - \cancel{2\gamma^2 E\beta p_z} - \cancel{\gamma^2 \beta^2 p_z^2} - p_x^2 - p_y^2 + \cancel{2\gamma^2 p_z \beta E} - \cancel{\gamma^2 \beta^2 E^2}$$

$$= \gamma^2 E^2 - 2\gamma^2 E\beta p_z - p_x^2 - p_y^2 + \gamma^2 p_z^2 \beta^2 E^2 - \gamma^2 \beta^2 E^2$$

$$= \gamma^2 E^2 - \gamma^2 \beta^2 E^2 + \gamma^2 p_z^2 - \gamma^2 p_z^2$$

$$= \gamma^2 E^2 (1 - \beta^2) + \gamma^2 p_z^2 (1 - \beta^2)$$

(5)

6

$$2.15) a) \quad L_x = y p_z - z p_y, \quad L_y = z p_x - x p_z \\ L_z = x p_y - y p_x$$

$$\begin{aligned} [L_x, L_y] &= [y p_z - z p_y, z p_x - x p_z] \\ &= [y p_z, z p_x] + [z p_y, x p_z] \\ &= y [p_z, z p_x] + p_y [z, x p_z] \\ &= y p_x [p_z, z] + p_y x [z, p_z] \\ &= -i \cancel{y} p_x + i p_y x \\ &= i (p_y x - y p_x) \\ &= i (x p_y - y p_x) \\ &= i L_z \end{aligned}$$

(5)

6

$$2.15) a) \quad L_x = y p_z - z p_y, \quad L_y = z p_x - x p_z$$

$$L_z = x p_y - y p_x$$

$$[L_x, L_y] = [y p_z - z p_y, z p_x - x p_z]$$

$$= [y p_z, z p_x] + [z p_y, x p_z]$$

$$= y [p_z, z p_x] + p_y [z, x p_z]$$

$$= y p_x [p_z, z] + p_y x [z, p_z]$$

$$= -i y p_x + i p_y x$$

$$= i (p_y x - y p_x)$$

$$= i (x p_y - y p_x)$$

$$= i L_z$$

$$L_x L_y = L_y L_x + i L_z$$

7

(b)

$$[L^2, L_x] = [L_x L_x, L_x] + [L_y L_y, L_x] + [L_z L_z, L_x]$$

$$= L_y L_y L_x - L_x L_y L_y + L_z L_z L_x - L_x L_z L_z$$

$$\rightarrow = L_y (L_x L_y - i L_z) - (L_y L_x + i L_z) L_y \\ + L_z (L_x L_z + i L_y) - (L_z L_x - i L_y) L_z$$

(c)

$$L_- L_+ = (L_x - i L_y)(L_x + i L_y)$$

$$= L_x^2 + L_y^2 - i [L_y, L_x]$$

$$= L_x^2 + L_y^2 - i [-i L_z]$$

$$L_- L_+ = L_x^2 + L_y^2 + L_z$$

$$\text{then } L^2 = L_x^2 + L_y^2 + L_z^2$$

$$L^2 = L_+ L_- + L_z + L_z^2$$

$$2.16) a) [\hat{S}_x, \hat{S}_y] =$$

$$= \frac{1}{4} \left[\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right]$$

$$= \frac{1}{4} \left[\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \right]$$

$$= \frac{1}{4} \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix}$$

$$= \frac{i}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = i \hat{S}_z$$

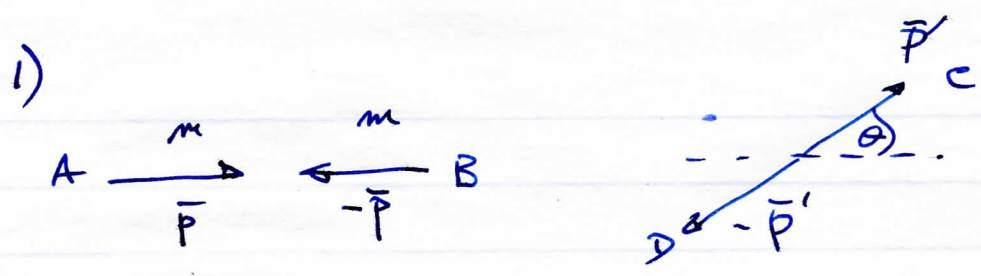
$$b) \quad \sigma_k^2 = I \quad k = \text{any of them}$$

$$\text{So } \hat{S}^2 = \frac{1}{4} (\sigma_x^2 + \sigma_y^2 + \sigma_z^2) = \frac{3}{4} I$$

Commutators same as angular momentum
 \therefore ~~the~~ states in vector space of \hat{S}
 acts on \hat{S} are labeled $|S, m\rangle$

$$\hat{S}^2 |S, m\rangle = S(S+1) |S, m\rangle$$

$$\hat{S}^2 |S, m\rangle = \frac{3}{4} |S, m\rangle$$



$$|\bar{p}| = |\bar{p}'| \rightarrow \text{ELASTIC}$$

$$E = (m^2 + |\bar{p}|^2)^{1/2} = (m^2 + |\bar{p}'|^2)^{1/2}$$

$$s = (p_A + p_B)^2 = (2E, 0)^2 = 4(m^2 + |\bar{p}|^2)$$

$$t = (p_A - p_B)^2 = (0, \bar{p} - \bar{p}')^2 = |\bar{p}|^2 + |\bar{p}'|^2 - 2\bar{p} \cdot \bar{p}'$$

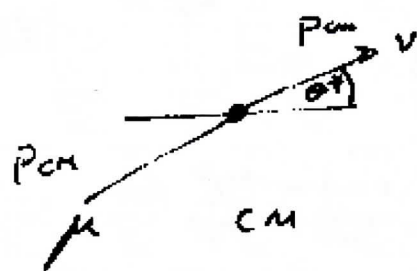
← same.

$$= -2\bar{p}^2 (1 - \cos\theta)$$

$$u = (p_A - p_D)^2 = (0, \bar{p} + \bar{p}')^2 = -2\bar{p}^2 (1 + \cos\theta)$$

Question #2 My Version of #2

Look at components \parallel and \perp to v motion



LORENTZ xform

$$\textcircled{1} \quad \bar{p}_{xLAB}^v = (\bar{p}_{xcm}^v + \beta E_{cm}^v) \gamma^v = (\bar{p}_{cm} \cos \theta^* + \beta E_{cm}^v) \gamma^v$$

$$\text{But } \bar{p}_{xLAB}^v = 0 \quad \therefore \quad \bar{p}_{cm} \cos \theta^* = -\beta E_{cm}^v$$

$$\begin{aligned} \textcircled{2} \quad \bar{p}_{xLAB}^\mu &= (\bar{p}_{xcm}^\mu + \beta E_{cm}^\mu) \gamma^v \\ &= (-\bar{p}_{cm} \cos \theta^* + \beta E_{cm}^\mu) \gamma^v \end{aligned}$$

But from above have:

$$|\bar{p}_{cm}| \cos \theta = -\beta E_{cm}^v$$

$$\begin{aligned} |\bar{p}_{xLAB}^\mu| &= (\beta E_{cm}^v + \beta E_{cm}^\mu) \gamma \\ &= \beta \gamma^v m_{\pi} \quad \textcircled{3} \end{aligned}$$

②

Now we want

$$\tan \theta_\mu = \frac{p_y^\mu}{p_x^\mu} \Big|_{LAB}$$

just calculated that.

get p_y^μ

calculate p_x

$$\begin{aligned} \bar{p}_y^\mu &= \bar{p}_y^\nu = \bar{p}_y^{\nu cm} \\ &= \bar{p}^\nu \sin \theta^* \end{aligned}$$

in CM.
4 vec $\longrightarrow p_\pi = p_\nu + p_\mu$

$$p_\nu = p_\pi - p_\mu$$

$$p_\nu^2 = (p_\pi - p_\mu)^2$$

better write

$$p_\mu^2 = (p_\pi - p_\nu)^2$$

$$m_\mu^2 = \underbrace{p_\pi^2}_{m_\pi^2} + \underbrace{p_\nu^2}_0 - 2 \vec{p}_\pi \cdot \vec{p}_\nu$$

$$= m_\pi^2 - 2 \underbrace{E_\pi E_\nu}_{m_\pi^2} + \vec{p}_\pi \cdot \vec{p}_\nu$$

$\vec{p}_\pi \cdot \vec{p}_\nu = 0$ in CM

$$m_\mu^2 = m_\pi^2 - 2 E_\nu m_\pi$$

$$m_\mu^2 = m_\pi^2 - 2 m_\pi E_\nu$$

But for ν $E_\nu = |\vec{p}_\nu|$

$$m_\mu^2 = m_\pi^2 - 2 m_\pi |\vec{p}_\nu|$$

$$\begin{aligned} 2 m_\pi |\vec{p}_\nu| &= m_\pi^2 - m_\mu^2 \\ &= m_\pi^2 \left(1 - \frac{m_\mu^2}{m_\pi^2} \right) \end{aligned}$$

$$|\vec{p}_\nu| = m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2} \right) \quad (4)$$

so $p_{\mu}^{\mu} \text{ LAB} : p_{\mu}^{\mu} \sin \theta^* = p_{\mu}^{\nu} \sin \theta^*.$

so $\tan \theta_{\mu}^{\text{LAB}} : \frac{p_{\mu}^{\mu}}{p_{\mu}^x} \Big|_{\text{LAB}} = \frac{\sin \theta^* (1 - m_\mu^2/m_\pi^2) m_\pi}{\beta \gamma m_\pi}$

$\sin^2 \theta^* ?$

$$\sin^2 \theta^* = 1 - \cos^2 \theta^*$$

$$= 1 - \frac{\beta^2 E_{\text{cm}}^2}{p_{\text{cm}}^2} \quad E_\nu = p_\nu$$

$$= 1 - \beta^2$$

$$\sin \theta^* = \sqrt{1 - \beta^2} = \frac{1}{\gamma}$$

$$\therefore \tan \theta_{\mu}^{\text{LAB}} = \left(1 - \frac{m_\mu^2}{m_\pi^2} \right) \frac{1}{\beta \gamma^2}$$

$$K^+ \rightarrow \mu^+ \gamma : \tan \theta = \frac{1}{2(0.8)(1.467)^2} \left(1 - \frac{106^2}{494^2} \right) \quad \textcircled{10}$$

$$\approx 0.215$$

$$\theta \sim 12.1^\circ$$

$$\pi^+ \rightarrow \mu^+ \gamma \quad \tan \theta = \frac{1}{2(0.8)(1.667)^2} \left(1 - \frac{106^2}{140^2} \right)$$

$$= 0.096$$

$$\theta \sim 5.5^\circ$$

$$c) \quad \gamma = |\bar{p}| = \gamma m_\pi |\beta| = \gamma m_\pi \frac{\sqrt{\gamma^2 - 1}}{\gamma} = m_\pi \sqrt{\gamma^2 - 1}$$

$$|\bar{p}| \gg m \quad \gamma = \sqrt{\frac{p^2}{m_\pi^2} + 1} = \sqrt{\frac{(209)^2}{0.14^2} + 1} \approx 1429$$

$$\text{Time to travel } 340 \text{ m } @ \beta \sim 1 \quad t = \frac{340}{3 \cdot 10^8} = 1.13 \mu\text{s}$$

$$\text{Decay fraction} = 1 - \frac{N(t=1.13 \mu\text{s})}{N(t=0)}$$

$$= 1 - \exp\left(-\frac{1.13}{37}\right) = 3\%$$

$$\begin{aligned} \tau' &= \tau \gamma = 1429 \times 2.6 \times 10^{-8} \\ \text{LAB} \quad &= 37 \mu\text{s} \end{aligned}$$