3.2, 3.4, 3.8, 3.9, 4.3, 4.4, 4.6, 4.7 (1)
4.12

PROB SET #2 SOLUTIONS

3.2 $a \rightarrow 1+2$

$$p^* = \frac{1}{2m_a}\left(\left[m_a^2-(m_1+m_2)^2\right]\left[m_a^2-(m_1-m_2)^2\right]\right)^{\frac{1}{2}}$$

$$m_a = E_1 + E_2 \qquad \text{(in CMS)}$$

$$m_a - E_2 = E_1$$

$$m_a^2 - 2m_aE_2 + m_2^2 + p^{*2} = m_1^2 + p^{*2}$$

so $m_a^2 + (m_2^2 - m_1^2) = 2m_a \underbrace{E_2}_{\sqrt{m_2^2+p^{*2}}}$

$$m_a^4 + 2m_a^2(m_2^2-m_1^2) - (m_2^2-m_1^2)^2 = 4m_a^2(m_2^2+p^{*2})$$

$$m_a^4 - 2m_a^2\left[(m_1+m_2)^2+(m_1-m_2)^2\right] + (m_1-m_2)^2(m_1+m_2)^2 = 4m_a^2 \cancel{m_2} p^{*2}$$

$$m_a^4 - 2m_a^2\left[(m_1+m_2)^2+(m_1-m_2)^2\right] + (m_1-m_2)^2(m_1+m_2)^2 = 4m_a^2p^{*2}$$

$$\left[m_a^2-(m_2+m_1)^2\right]\left[m_a^2-(m_1-m_2)^2\right] = 4m_a^2p^{*2}$$

$$p^* = \frac{1}{2m_a}\left(\left[m_a^2-(m_1+m_2)^2\right]\left[m_a^2-(m_1-m_2)^2\right]\right)^{\frac{1}{2}}$$

3.4) ? How many Higgs in 5 years.

Total number of events $N = \int \sigma \mathcal{L} \, dt$

Event rate $= \sigma \mathcal{L}$

Rate $= \sigma \mathcal{L} = (250 \times 10^{-39}) \times (2 \times 10^{34}) = 0.005 \, s^{-1}$

for 5 years at 50% running

TOTAL $N = 0.005 \times 0.5 \times 365.25 \times 86400$

$= 394000 \quad e^+e^- \to HZ$

3.8 Show rest frame flux $F = 4 m_b p_a$.

Independent of rest frame, Lorentz invariant flux is:

$$F = 4\left[(p_a \cdot p_b)^2 - m_a^2 m_b^2\right]^{\frac{1}{2}}$$

In this particular case

$$p_a = (E_a, 0, 0, \bar{p}_a)$$

$$p_b = (m_b, 0, 0, 0)$$ ← at rest in this frame.

So $$F = 4\left(E_a^2 m_b^2 - m_a^2 m_b^2\right)^{\frac{1}{2}}$$

$$= 4\left[(p_a^2 + m_b^2) m_b^2 - m_a^2 m_b^2\right]^{\frac{1}{2}}$$

$$= 4 p_a m_b$$ ✓

3.9) initial state momenta in 2-body scatter.

$$p_i^{*2} = \frac{1}{4s}\left[s-(m_1+m_2)^2\right]\left[s-(m_1-m_2)^2\right]$$

In CMS $\quad \sqrt{s} = E_1^* + E_2^*$

$$(\sqrt{s}-E_1^*)^2 = E_2^{*2}$$

$$s - 2\sqrt{s}\,E_1^* + E_1^{*2} = E_2^{*2}$$

$$s - 2\sqrt{s}\,E_1^* + m_1^{*2} + p_i^{*2} = m_i^{*2} + p_i^{*2}$$

$$2\sqrt{s}\,E_1^* = s + (m_1^2 - m_2^2)$$

$$4sE_1^{*2} = s^2 + 2s(m_1^2 - m_2^2) - (m_1^2 - m_2^2)^2$$

$$4s(p_i^{*2} + m_1^{*2}) = s^2 + 2s(m_1^2 - m_2^2) - (m_1^2 - m_2^2)^2$$

$$4sp_i^{*2} = s^2 + 2sm_1^2 - 2sm_2^2 - 4sm_1^2$$

$$= s^2 + 2s(m_1^2 + m_2^2) + (m_1 - m_2)^2(m_1+m_2)^2$$

$$= \underbrace{\left[s-(m_1+m_2)^2\right]\left[s-(m_1-m_2)^2\right]}$$

$$p_i^{*2} = \frac{1}{4s}$$

4.3

~~3-9~~) Verify that $E^2 = p^2 + m^2$ is recovered if any Dirac Spinor $\rightarrow$ Dirac eqn. $(\gamma^\mu p_\mu - m)u = 0$

$\gamma^\mu p_\mu - m$ in matrix rep is:

$$= E\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} - p_x\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} - p_y\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$$- p_z\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} - m\mathbb{I}$$

$$= \begin{pmatrix} E-m & 0 & -p_z & -p_x + ip_y \\ 0 & E-m & -p_x - ip_y & p_z \\ p_z & p_x - ip_y & -(E+m) & 0 \\ p_x + ip_y & -p_z & 0 & -(E+m) \end{pmatrix}$$

$$\downarrow$$

$(\gamma^\mu p_\mu - m)u = 0$

$$\rightarrow \begin{pmatrix} E-m & 0 & -p_z & -p_x + ip_y \\ 0 & E-m & -p_x - ip_y & p_z \\ p_z & p_x - ip_y & -(E+m) & 0 \\ p_x + ip_y & -p_z & 0 & -(E+m) \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix} = 0$$

$$\begin{pmatrix} (E-m)+0-\dfrac{p_z^2}{E+m}+\dfrac{-p_z^2-p_y^2}{E+m} \\ 0+0-\dfrac{p_z}{p_x+ip_y}+\dfrac{p_z}{p_x+ip_y} \\ p_z+0-p_z+0 \\ (p_x+ip_y)+0+0+0 \end{pmatrix} = 0$$

$$\frac{1}{E+m}\begin{pmatrix} E^2-m^2-p_x^2-p_y^2-p_z^2 \\ 0 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\therefore \quad E^2 = p^2 + m^2$$

4.4) Show 4-vector current $j^\mu = (\rho, \bar{j})$ is $j^\mu = 2p^\mu$
also $\rho\ \bar{j}$ consistent with velocity $\beta = p/E$

For arbitrary spinors $\psi\ \phi$

$\mu = 0$ $$\bar{\psi}\gamma^0\phi = \psi_1^*, \psi_2^*, -\psi_3^*, -\psi_4^* \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&-1&0 \\ 0&0&0&-1 \end{pmatrix}$$

$$= \psi_1^*\phi_1 + \psi_2^*\phi_2 + \psi_3^*\phi_3 + \psi_4^*\phi_4$$

$\mu = 1, 2, 3$ $$\bar{\psi}\gamma^1\phi = \psi_1^*\phi_4 + \psi_2^*\phi_3 + \psi_3^*\phi_2 + \psi_4^*\phi_1$$

$$\bar{\psi}\gamma^2\phi = -i(\psi_1^*\phi_4 - \psi_2^*\phi_3 + \psi_3^*\phi_2 - \psi_4^*\phi_1)$$
$$\bar{\psi}\gamma^3\phi = (\psi_1^*\phi_3 - \psi_2^*\phi_4 + \psi_3^*\phi_1 - \psi_4^*\phi_2)$$

eg $$\bar{u}_1\gamma_0 u_1 = (E+m)\left(1 + 0 + \frac{p_z^2}{(E+m)^2} + \frac{(p_x+ip_y)(p_x-ip_y)}{(E+m)^2}\right)$$

$$u_1 = \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+M} \end{pmatrix}\sqrt{E+m}$$

$$= E+m\left(1 + \frac{p_z^2}{(E+m)^2} + \frac{(p_x^2+p_y^2)}{(E+m)^2}\right)$$

$$= (E+m)\left(1 + \frac{p^2}{(E+m)^2}\right)$$

$$= E+m\left(\frac{(E+m)^2+p^2}{(E+m)^2}\right) = \frac{2E^2+2Em}{E+m} = 2E$$

For other 3-components

$\bar{u}_1 \gamma^0 u_1 = 2E, \quad \bar{u}_1 \gamma^1 u_1 = 2p_x \qquad \gamma^2 = 2p_y \quad \gamma^3 = 2p_z$

so $\bar{u}_1 \gamma^0 u_1 = (2E, 2p_x, 2p_y, 2p_z) \Rightarrow 2p^\mu$

for u_2, v_1, and v_2

$\bar{u}_1 \gamma^\mu u_1 = \bar{u}_2 \gamma^\mu u_2 = \bar{v}_1 \gamma^\mu v_1 \cdot \bar{v}_2 \gamma^\mu v_2 = 2p^\mu$

all cross terms like $\bar{u}_1 \gamma^\mu u_2 = 0$

For a particle with $\psi = u(p) e^{i\bar{p}\cdot\bar{x}}$

have $\bar{\psi} = \psi^\dagger \gamma^0 = u(p)^\dagger \gamma^0 e^{-ipx}$

$= \bar{u} e^{-ipx}$

so $j^\mu = \bar{\psi} \gamma^\mu \psi = \bar{u} \gamma^\mu u$

for antiparticle get $j^\mu = \bar{v} \gamma^\mu v$

$u(p)$ can always be expressed as a linear combination of basis spinors $u_1(p), u_2(p)$

$u = \alpha_1 u_1 + \alpha_2 u_2 \qquad |\alpha_1|^2 + |\alpha_2|^2 = 1$

so $\bar{u}\gamma^\mu u = |\alpha_1|^2 \bar{u}_1\gamma^\mu u_1 + |\alpha_2|^2 \bar{u}_2\gamma^\mu u_2 = 2p_\mu.$

For any free particle spinor

$$j^\mu = \bar{u}\gamma^\mu u = 2p^\mu$$

$$j^\mu = (\rho, \bar{j}) \Rightarrow \begin{array}{l} \rho = 2E \\ \bar{j} = 2\bar{p} \end{array}$$

this shows that general spinor $u(p)$ is normalized to $2E$ particles per unit volume

From $E = \gamma m$, $\bar{p} = \gamma m \bar{v}$, the particle flux is $\bar{j} = 2E\bar{v}$, particle flux is

$$\bar{j} = \rho\bar{v}$$

4.6) Show that $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$

for $\mu = \nu = 0$

$$\gamma^0\gamma^0 + \gamma^0\gamma^0 = 2I = 2g^{00} \quad ✓$$

for $\mu = \nu = k$ $\quad \gamma^k\gamma^k + \gamma^k\gamma^k = -2I = 2g^{kk} \quad k = 1, 2, 3$

for $\mu \neq \nu$ anticommutator $\gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu$

$$\Rightarrow \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 0 = 2g^{\mu\nu} \text{ for } \mu \neq \nu.$$

So in each of these cases

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$$

4.7) Prove the components of ψ satisfy K-G

$$(\partial^\mu\partial_\mu + m^2)\psi = 0$$

~~[illegible]~~ Act on with $\gamma^\nu\partial_\nu \times -i$

$$\gamma^\nu\gamma^\mu\partial_\nu\partial_\mu\psi + mi\gamma^\nu\partial_\nu\psi = 0$$

Since ψ satisfies Dirac $\quad i\gamma^\nu\partial_\nu\psi = m\psi$

so $\quad (\gamma^\nu\gamma^\mu\partial_\nu\partial_\mu + m^2)\psi = 0$

order of differentiation doesn't matter

$$\gamma^\nu \gamma^\mu \partial_\nu \partial_\mu = \frac{1}{2}\left(\gamma^\nu \gamma^\mu + \gamma^\mu \gamma^\nu\right) \partial_\nu \partial_\mu$$

$$= g^{\mu\nu} \partial_\nu \partial_\mu$$

So $$\left(g^{\mu\nu} \partial_\nu \partial_\mu + m^2\right)\psi = 0$$

$\hookrightarrow$ $$\left(\partial^\mu \partial_\mu + m^2\right)\psi = 0$$

$\hookrightarrow$ this is K-G ✓

4.12) Show $\hat{h} = \frac{\hat{\Sigma}\cdot\hat{p}}{2p} = \frac{1}{2p}\begin{pmatrix} \bar{\sigma}\cdot\bar{p} & 0 \\ 0 & \bar{\sigma}\cdot\bar{p} \end{pmatrix}$

commutes with $\hat{H}_D = \bar{\alpha}\cdot\bar{p} + \beta m$.

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \qquad \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$$

identity matrix $\rightarrow$ $[\hat{h}, m\beta] = 0$

only need to consider $[\hat{h}, \bar{\alpha}\cdot\bar{p}]$

$$[h, H_D] = \frac{1}{2p}\left[\begin{pmatrix} \bar{\sigma}\cdot\bar{p} & 0 \\ 0 & \bar{\sigma}\cdot\bar{p} \end{pmatrix}\begin{pmatrix} 0 & \bar{\sigma}\cdot\bar{p} \\ \bar{\sigma}\cdot\bar{p} & 0 \end{pmatrix} - \begin{pmatrix} 0 & \bar{\sigma}\cdot\bar{p} \\ \bar{\sigma}\cdot\bar{p} & 0 \end{pmatrix}\begin{pmatrix} \bar{\sigma}\cdot\bar{p} & 0 \\ 0 & \bar{\sigma}\cdot\bar{p} \end{pmatrix}\right]$$

$$= \frac{1}{2p}\left[\begin{pmatrix} 0 & (\bar{\sigma}\cdot\bar{p})^2 \\ (\bar{\sigma}\cdot\bar{p})^2 & 0 \end{pmatrix} - \begin{pmatrix} 0 & (\bar{\sigma}\cdot\bar{p})^2 \\ (\bar{\sigma}\cdot\bar{p})^2 & \end{pmatrix}\right] = 0$$

1)

BR		Partial Trans Rate s^{-1}
0.635	$\times \frac{1}{1.24\times10^{-8}}$	5.1×10^{7}
0.212		1.7×10^{7}
0.056		4.5×10^{7}
0.017		1.3×10^{7}
0.032		2.6×10^{6}
0.048		3.9×10^{6}

Systematic − Σ mass of final State

$\sim$ phase space

$\therefore$ $K\to\mu\nu$ largest

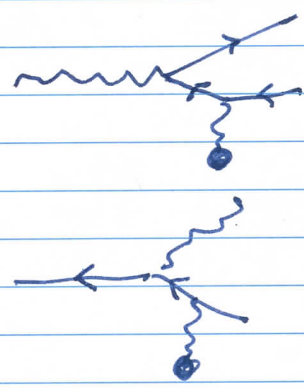
$\to \pi^+\pi^0$ ← π^0 replaces ν

mass ↑ phase space down ↓

$\to 3\pi$ much lower.

$K^+\to\pi^+\mu^+\nu_\mu > 3\pi$ ∵ phase space

2)



$\gamma \to e^+ e^-$

$p_\gamma = p_{e^+} + p_{e^-}$

$p_\gamma^2 = p_{e^+}^2 + p_{e^-}^2 + 2 p_{e^+} p_{e^-}$

$0 = m_e^2 + m_e^2 \quad \uparrow_{+ve}$

$\therefore$ doesn't conserve 4-momentum.

$e^+ \to \gamma e^+$

$p_e^2 = (p_\gamma + p_e)^2$

$m_e^2 = (p_\gamma)^2 + m_e^2 + p_\gamma p_e$

$\cancel{m_e^2} = \cancel{m_e^2} +$ +ve thing.

$\therefore$ doesn't conserve 4-momentum

3) Invariant mass:

$$M^2 = \sum E^2 - \sum \bar{p}^2$$

OR $M^2 = (p_{K_1} + p_{K_2})^2$

$= p_{K_1}^2 + p_{K_2}^2 + 2 p_{K_1} \cdot p_{K_2}$

$= m_1^2 + m_2^2 + 2(E_1\ \bar{p}_1)(E_2\ \bar{p}_2)$

$= m_1^2 + m_2^2 + 2E_1E_2 - 2\bar{p}_1\bar{p}_2$

$= m_1^2 + m_2^2 + 2E_1E_2 - 2|p_1||p_2| \cos\theta$

$m_1^2 + m_2^2 + 2E_1E_2 - 2|\bar{p}_1||\bar{p}_2|\cos\theta$

$E_1^2 = 10^2 + (0.493)^2$

$= 100 + 0.243 = 100.243$

$E_1 = 10.012$ *

$E_2^2 = 5^2 + (0.493)^2 = 25.243$

$E_2 = 5.0242$ *

$W^2 = 2\times(.493)^2 + 2(10.012)(5.0242)$

$- 2\times 10\times 5 \times \cos 6^\circ$

$= 0.486 + 100.60 - 99.452$

$W^2 = 1.634$

$W = \cancel{1.1306}\ 1.278$ GeV

You can't conclude anything from one event – you need a distribution

4) Calc number of scattering centres

$$\frac{10^{-4}\,m^3 \times 60\,kg\,m^{-3}}{1.008} \times 6.022 \times 10^{26}$$

$$= 3.584 \times 10^{24}$$

Rate of interaction $= \sigma \phi N$

xsec   flux   # of scatters

$$= 4.5 \times 10^{-30} \times 10^{7} \times 3.584 \times 10^{24}$$

$$= 161\ s^{-1}$$

π^0 decays by electromagnetic interaction in 10^{-16} → in essence they decay where they are produced.

$$\pi^0 \rightarrow \gamma\gamma$$

$$2 \times 161 = 322\ s^{-1}$$

there is a negligible prob of γ interacting in liquid hydrogen

∴ they all escape