

PHYSICS 489Y - Problem Set #2

Due 18th October

Do problems 3.2, 3.4, 3.8, 3.9 from the textbook. Then do the four simple, practical questions below. Then do problems 4.3, 4.4 4.6, 4.7 and 4.12.

The problems below are to remind us that we have to do experiments for all this to be worthwhile.

1) Assume that positive K meson (K^+) can decay in six different ways (*See table*). We'll learn more about these final state particles. You don't need to know anything about them here; but, look for "systematics" in the decays. Write down any that you notice. Calculate the six different partial transition rates from the following table.

Decay Mode	Branching Fraction	Partial Transition Rate s^{-1}
$K^+ \rightarrow \mu^+ \nu_\mu$	0.635	
$K^+ \rightarrow \pi^+ \pi^0$	0.212	
$K^+ \rightarrow \pi^+ \pi^+ \pi^-$	0.056	
$K^+ \rightarrow \pi^+ \pi^0 \pi^0$	0.017	
$K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$	0.032	
$K^+ \rightarrow \pi^0 e^+ \nu_e$	0.048	

2) Draw the Feynman diagram for pair production,

$$\gamma \rightarrow e^+ e^-$$

and for the process of Bremsstrahlung, which is when an electron radiates a photon, in the field of a heavy nucleus.

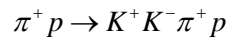
$$e^+ \rightarrow \gamma e^+$$

Why do neither of these processes occur in free space, i.e. why do they only occur in the field of a nucleus? Demonstrate this using 4-vector notation

Hint: This is really simple if you use 4-vector notation. Write down the total 4-momentum squared for the initial state and the final state, remembering that the photon has zero mass. Do this in the rest frame of the photon. Remember that 4-momentum is conserved, as is 3-momentum.

3) In relativistic collisions the kinetic energy of the colliding particles can be transformed into the mass of *new particles*. So neither the number of particles, nor the total mass, is necessarily conserved in a relativistic collision. Very short lived particles can be produced, and we detect their presence by looking for a peak in the invariant mass of the products of the decay.

Say we suspected, or predicted, that there was a particle which decayed to an oppositely charged pair of K mesons. We could search for this particle by firing pions into a liquid hydrogen target.

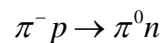


We would then reconstruct the invariant masses of the oppositely charged K mesons, and build up a histogram of the invariant mass distribution over many events. If the invariant mass showed an enhancement at some mass, we could conclude that we had discovered the particle. Consider one event, and assume that the K^+ and the K^- , in the final state, are produced so that they have an angle of 6.3° between them, and that they have momenta $10.0 \text{ GeV}/c$, and $5.0 \text{ GeV}/c$ respectively, in the LAB. What is the *invariant mass* of the $K^+ K^-$ system?

In fact there is a particle known as the ϕ meson, which consists of a s-quark and an anti-s quark bound together by the colour force. The ϕ has a mass of $1020 \text{ MeV}/c^2$. Do you think this one event would persuade you of the existence of the ϕ ? Explain why or why not.

Take the mass of the K meson to be $493 \text{ MeV}/c^2$, independent of its electric charge.

4) A liquid hydrogen target of volume 10^{-4} m^3 and density 60 kg m^{-3} is immersed in a broad, uniform, monoenergetic beam of negative pions. The flux of pions in the beam is 10^7 particles $\text{m}^{-2} \text{ s}^{-1}$. At a sufficiently low enough beam momentum (below around $300 \text{ MeV}/c$) the only reaction we have to consider is



which occurs with a cross section of 45 mb (45 millibarns is equal to $4.5 \times 10^{-30} \text{ m}^2$). The neutral pions decay in the mode $\pi^0 \rightarrow \gamma\gamma$ with a very short lifetime, so that all the decays occur within the target. Neglect any attenuation of the incident beam in the target and calculate the number of γ -rays emitted per second from the target. What elementary force causes the neutral pion to decay?