PHYSICS 489Y - Problem Set #3

Due 1st November

Do problems 5.2, 5.3, 6.2, 6.7, 8.6, 8.7 from the textbook.

Then do the two simple, practical questions below.

1) a) Show that for a spherically symmetrical charge distribution, the form factor is

$$F\left(q^2\right) = 1 - \frac{q^2\left\langle r^2\right\rangle}{6\hbar^2} + O\left(q^4\right).$$

Where $\left\langle r^2\right\rangle$ is the mean of the square of the electric charge distribution.

Outline: First you should write down the general expression for the form factor $F\left(q^2\right)$ in terms of the charge density $\rho\left(\bar{r}\right)$. By assuming spherical symmetry, you will end up with an integral with a term $\sin\left(qr/\hbar\right)$ in it. You can expand this as

$$\frac{\sin\left(qr/\hbar\right)}{qr/\hbar} = \left[1 - \left(\frac{1}{6}\right)\left(\frac{qr}{\hbar}\right)^2 + \ldots\right].$$

b) Show that the form factor corresponding to the charge distribution

$$\rho\left(r\right) = \rho_0 e^{-r/a} / r$$

is

$$F\left(q^2\right) = \frac{1}{\left[1 + \left(q^2 a^2 / \hbar^2\right)\right]}.$$

Note that $\rho\left(r\right)$ is spherically symmetrical, so you can use the integral for $F\left(q^2\right)$ from part **a).**
Remember to normalize the result by dividing through by the integral over the charge density over the volume of the nucleus.

(c) Now let's look at what the form factor does in numerical terms. An electron of momentum 500 MeV/c is scattered through an angle of 11° by an argon nucleus. Assume that there is no recoil and calculate the *momentum transfer* and also the *reduced de Broglie wavelength* of the electron. Further, calculate the *Mott differential cross section*, which corresponds to the case where the argon nucleus could be considered to be point-like. Finally, calculate by how much the *differential cross section* changes if the argon nucleus is considered to be represented by the spatial distribution in part **(a)** of this question. You should take the value of $r = 1.2 \times A^{1/3}\ fm$. *Argon has an Atomic Mass Number of 40, and an Atomic Number of 18.*

2) One can perform deep inelastic scattering experiments using electrons, muons or neutrinos. Since electrons and muons will effectively interact via the electromagnetic interaction, they probe the electric charge distribution within the nucleon. Since neutrinos have no electric charge they will probe the distribution of matter independently of electric charge; in fact they interact via the weak interaction. The reason that neutrinos are interesting is that they can only exist with their spins antiparallel ("left-handed") to their momentum. Antineutrinos have their spins and momentum parallel ("right-handed") Neutrinos and antineutrinos can be used to probe the existence of "right-handed" or "left-handed" objects within the nucleon. Since the weak interaction does not conserve parity (we will discuss what this means) right-handed and left-handed objects have different interactions. *I am assuming here that the neutrinos are completely massless. Since neutrino oscillations demonstrate that they have mass, they must actually have both helicities. But right-handed neutrinos and left-handed anti-neutrinos must be very massive, as we have never observed them.*

i) Why does it not make sense to think of electrons or muons as intrinsically right or left handed?
ii) In deep inelastic scattering, the invariant mass of the final state hadronic system, *W*, is an indication of how inelastic the scattering is, and how "deeply" one is probing the nucleon structure. What are the maximum values of *W* that can be produced with electrons at SLAC and muons at Fermilab. At those q^2 do you think that both of these machines can see scaling behaviour? Why? Make an estimate of the smallest distance scale that each of these machines can probe. *You need to make some assumption about the maximum muon beam energy ... Just assume that the muon energy is 1/3 of the proton energy of the Tevatron at Fermilab. Remember the uncertainty principle.*
iii) The total cross section for deep inelastic scattering of electrons from protons is calculated in the quark-parton model to be just the cross section for scattering off of three quarks with electric charges of either $2/3$ and $-1/3$. One averages over the quark charges, and assumes that the scattering is independent. When the first deep inelastic experiments were performed the fact that quarks carried the *colour* charge was unknown. Show that the quark-parton result is unchanged by the fact that each quark can carry any of three different colour charges, as long as the three colour charges are present in the proton in equal proportions.