TIME REVERSAL

All SYSTEMS HAVE TO BE INVARIANT UNDER

$$CPT$$

CHARGE CONJUGATION (PARTICLES → ANTIPARTICLES)

PARITY

TIME REVERSAL

SINCE WE KNOW NOW THAT WEAK INTERACTIONS ARE NOT INVARIANT UNDER PARITY TRANSFORMS, THAT MUST ALSO VIOLATE SOME COMBINATION OF C, T IN ORDER FOR OVERALL CPT INVARIANCE

$$t \xrightarrow{T} -t$$

$$\vec{r} \longrightarrow \vec{r}$$

$$\vec{p} = m\dot{\vec{r}} \longrightarrow -m\dot{\vec{r}} = -\vec{p}$$

$$\vec{L} = \vec{r} \times \vec{p} \longrightarrow \vec{r} \times (-\vec{p}) = -\vec{L}$$

CPT

→ INTEGER SPIN ARE BOSONS

→ $\frac{1}{2}$-INTEGER SPIN ARE FERMIONS

→ $m_p = m_{\bar{p}}$ P ANY OLD PARTICLE

$\tau_p = \tau_{\bar{p}}$

→ ALL INTERNAL Q. NOS OF P AND $\bar{P}$ ARE OPPOSITE

ALMOST IMPOSSIBLE TO IMAGINE A HAMILTONIAN WHICH IS NOT CPT INVARIANT

Table 3.1. *Tests of the CPT theorem*

Measured quantity	Limit or value
$(M_{K^0} - M_{\bar{K}^0})/(M_{K^0} + M_{\bar{K}^0})$	$< 10^{-19}$
$(M_{e^+} - M_{e^-})/(M_{e^+} + M_{e^-})$	$< 4 \times 10^{-8}$
$(M_\Lambda - M_{\bar{\Lambda}})/(M_{\bar{\Lambda}} + M_\Lambda)$	$(-5 \pm 5) \times 10^{-6}$
$(Q_p - Q_{\bar{p}})/e$	$< 2 \times 10^{-5}$
$\left(\frac{Q_p}{M_p} - \frac{Q_{\bar{p}}}{M_{\bar{p}}}\right) \Big/ \left(\frac{Q_p}{M_p} + \frac{Q_{\bar{p}}}{M_{\bar{p}}}\right)$	$(8 \pm 6) \times 10^{-10}$
$(\mu_{e^+} - \mu_{e^-})/(\mu_{e^+} + \mu_{e^-})$	$-(3 \pm 5) \times 10^{-13}$
$(\tau_{\mu^+} - \tau_{\mu^-})/(\tau_{\mu^+} + \tau_{\mu^-})$	$< 10^{-4}$

Table 3.2. *Effect of T and P operations*

Quantity		Effect of T	Effect of P
position	$\mathbf{r}$	$\mathbf{r}$	$-\mathbf{r}$
momentum	$\mathbf{p}$	$-\mathbf{p}$	$-\mathbf{p}$
spin	$\boldsymbol{\sigma}$, axial vector $(\mathbf{r} \times \mathbf{p})$	$-\boldsymbol{\sigma}$	$\boldsymbol{\sigma}$
electric field	$\mathbf{E}\ (= -\nabla V)$	$\mathbf{E}$	$-\mathbf{E}$
magnetic field	$\mathbf{B}$, axial vector	$-\mathbf{B}$	$\mathbf{B}$
magnetic dipole moment	$\boldsymbol{\sigma} \cdot \mathbf{B}$	$\boldsymbol{\sigma} \cdot \mathbf{B}$	$\boldsymbol{\sigma} \cdot \mathbf{B}$
electric dipole moment	$\boldsymbol{\sigma} \cdot \mathbf{E}$	$-\boldsymbol{\sigma} \cdot \mathbf{E}$	$-\boldsymbol{\sigma} \cdot \mathbf{E}$
longitudinal polarisation	$\boldsymbol{\sigma} \cdot \mathbf{p}$	$\boldsymbol{\sigma} \cdot \mathbf{p}$	$-\boldsymbol{\sigma} \cdot \mathbf{p}$
transverse polarisation	$\boldsymbol{\sigma} \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$	$-\boldsymbol{\sigma} \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$	$\boldsymbol{\sigma} \cdot (\mathbf{p}_1 \times \mathbf{p}_2)$

CLASSICALLY

$$m \frac{d^2 \vec{r}}{dt^2} = \vec{F}$$

IS CLEARLY INVARIANT UNDER $t \to -t$ SINCE IT HAS SECOND ORDER TIME DIFFERENTIAL

ALL CLASSICAL MICROSCOPIC SYSTEMS ARE TIME REVERSAL INVARIANT

MACROSCOPICALLY ENTROPY SEEMS TO DEFINE DIRECTION OF FLOW OF TIME

SCHRÖDINGER TELLS ONE HOW THE TIME EVOLUTION OF A SYSTEM DEPENDS ON THE HAMILTONIAN

$$i\hbar \frac{\partial \psi(\vec{r},t)}{\partial t} = H \psi(\vec{r},t)$$

BECAUSE OF THIS FIRST ORDER TIME DIFFERENTIAL, THIS CANNOT BE INVARIANT UNDER THE TRANSFORM

$$\psi(\vec{r},t) \xrightarrow{T} \psi(\vec{r},-t)$$

IF T IS SYMMETRY OF H

$A = A^{+}$ Herm
$AA^{+} = 1$

$$[H, T] = 0$$

ADD POSTULATE

$$\psi(-t) = T\psi(t)$$

$$i\hbar \frac{\partial [T\psi]}{\partial t} = H T\psi$$ FROM SCHRÖDINGER

$$T\psi(t) \Rightarrow \psi(-t)$$ NOTE $\partial t \rightarrow -\partial t$

$$-i\hbar \frac{\partial \psi(-t)}{\partial t} = H\psi(-t)$$ TRANSFORMED

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi(t)$$ UNTRANSFORMED

$\rightarrow$ <u>NOT</u> SAME SCHRÖDINGER EQUATION

$\therefore$ NOT SAME DYNAMICS

WE CAN MAINTAIN TIME REVERSAL ~~A~~SYMMETRY, BY DEFINING

$$\psi(\vec{r},t) \xrightarrow{T} \psi^*(\vec{r},-t)$$

THEN THE TIME REVERSED SCHRÖDINGER IS

$$i\hbar \frac{\partial[T\psi]}{\partial t} = H\,T\psi$$

$$i\hbar \frac{\partial \psi^*(-t)}{\partial(-t)} = H\psi^*(-t)$$

$$-i\hbar \frac{\partial \psi^*(-t)}{\partial t} = H\psi^*(-t)$$

THEN TAKE COMPLEX CONJUGATE

$$i\hbar \frac{\partial \psi(-t)}{\partial t} = H\psi(-t)$$

THIS IS SAME SCHRÖDINGER EQUATION AS OBEYED BY ψ. $\therefore$ SAME DYNAMICS, AS LONG AS WE USE

$$\psi(t) \rightarrow \psi^*(-t)$$

THIS TRANSFORMATION

$$\psi(t) \xrightarrow{T} \psi^*(-t)$$

IS OK, SINCE PHYSICS DEPENDS ON $\psi\psi^*$

NOTICE THAT WE MAY HAVE EIGENVALUE EQUATIONS FOR PARITY

$$P|\psi_p\rangle = \pi|\psi_p\rangle$$

WHERE $|\psi_p\rangle$ IS AN EIGENFUNCTION OF THE PARITY OPERATOR.

WE CANNOT WRITE DOWN EIGENVALUE EQUATIONS FOR THE TIME REVERSAL OPERATOR, BECAUSE

$$T|\psi\rangle \rightarrow \langle\psi(-t)|$$

EVEN IF TIME REVERSAL IS A SYMMETRY OF THE HAMILTONIAN, IT DOES NOT CORRESPOND TO A SIMPLE CONSERVED QUANTITY

$T \rightarrow$ ANTIUNITARY

ANTILINEAR

TIME REVERSAL IS MOTION REVERSAL

A FREELY PROPAGATING PLANE WAVE STATE

$$\Psi(\vec{r},t) = e^{i(\vec{p}\cdot\vec{r} - Et)/\hbar}$$

$$T\psi(\vec{r},t) = \psi^*(\vec{r},-t)$$

$$= e^{-i(\vec{p}\cdot\vec{r} + Et)/\hbar}$$

$$= e^{i(-\vec{p}\cdot\vec{r} - Et)/\hbar}$$

SO OUR DEFINITION OF THE ACTION OF THE TIME REVERSAL OPERATION, IS TO REVERSE MOMENTUM.

THIS IS EXACTLY THE CLASSICAL IDEA OF "RUNNING THE MOVIE BACKWARDS"

$$T|\vec{p},\vec{J}\rangle \rightarrow |-\vec{p},-\vec{J}\rangle$$

$$\langle -\vec{p},-\vec{J}|$$

TESTS OF T-INVARIANCE

TIME INVARIANCE IMPLIES

$$|M_{i\to f}| = |M_{f\to i}|$$

MATRIX ELEMENT FOR TRANSITION
INITIAL → FINAL

IN STRONG INTERACTIONS, IT HAS BEEN TESTED BY COMPARING

$$p + {}^{27}Al \to \alpha + {}^{24}Mg$$

WITH

$$\alpha + {}^{24}Mg \to p + {}^{27}Al$$

SEEMS GOOD TO ~ 0.3%

NEUTRON ELECTRIC DIPOLE MOMENT LEADS TO MUCH MORE SENSITIVE TEST

AN ELEMENTARY PARTICLE WITH FINITE SPIN SHOULD NOT HAVE AN ELECTRIC DIPOLE MOMENT

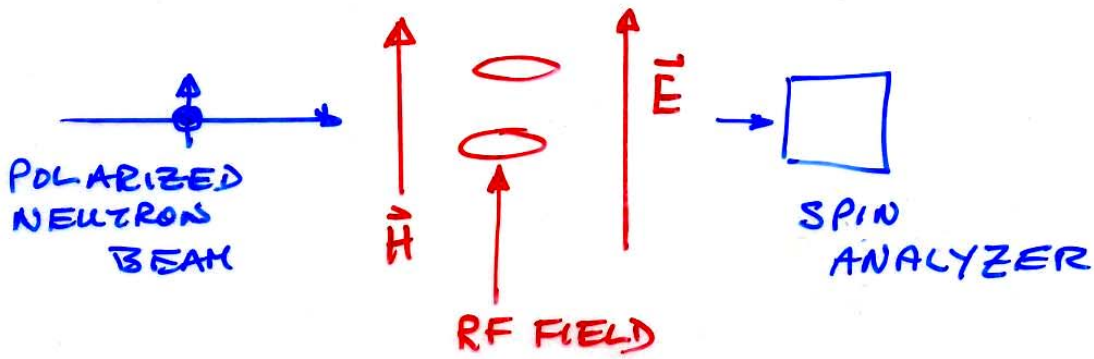
SPIN DEFINES SPATIAL DIRECTION, SO ELECTRIC DIPOLE MOMENT WOULD POINT ALONG SPIN. AN OBSERVABLE QUANTITY WOULD BE

$$\langle \vec{\mu} \cdot \vec{S} \rangle$$

ELECTRIC DIPOLE MOMENT

SPIN

TO MEASURE THIS IS A RATHER COMPLEX EXPERIMENT → SEE PERKINS P. 86



$\vec{H}$ $\vec{E}$ RF WILL DEPOLARIZE BEAM OF NEUTRONS.

WHEN $\vec{E}$ IS FLIPPED IN DIRECTION WILL SEE EFFECT IF DIPOLE MOMENT

UNDER TIME REVERSAL OPERATION

$$\vec{s} \xrightarrow{T} -\vec{s}$$ (ROTATING IN OPPOSITE DIRECTION)

$\vec{\mu}_{eL}$ is CHARGE × DISTANCE OF SEPARATION

$\therefore$ SINCE $\vec{r} \xrightarrow{T} \vec{r}$

$$\vec{\mu} \longrightarrow \vec{\mu}$$

SO $$\vec{\mu}\cdot\vec{s} \xrightarrow{T} -\vec{\mu}\cdot\vec{s}$$

$\vec{\mu}\cdot\vec{s}$ CHANGES SIGN UNDER TIME REVERSAL. IF IT IS TO BE INVARIANT UNDE TIME REVERSAL, ITS EXPECTATION VALUE MUST VANISH

NOTE THAT UNDER PARITY

$$\vec{s} \longrightarrow \vec{s}$$

$$\vec{r} \longrightarrow -\vec{r}$$

SO $$\vec{\mu}\cdot\vec{s} \longrightarrow -\vec{\mu}\cdot\vec{s}$$

HYPOTHESIZE THAT T WAS VIOLATED $\rightarrow$ THEN ONE CAN ESTIMATE HOW LARGE THE NEUTRON ELECTRIC DIPOLE MOMENT COULD BE

$$\mu_{el} \sim \text{CHARGE} \times \text{DISTANCE}$$

$$\sim e \times \text{SIZE OF NEUTRON}$$

$$\sim e \times 10^{-13} \text{ cm}$$

EXPERIMENTALLY, ONE FINDS

$$\mu_{el} \lesssim 10^{-25} \text{ e.cm.}$$

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$$< 4 \times 10^{-27} \text{ e.cm}$$

ABSENCE OF NEUTRON ELECTRIC DIPOLE MOMENT IS A VERY PRECISE TEST OF THE FACT THAT ELECTROMAGNETIC INTERACTIONS ARE INVARIANT UNDER TIME REVERSAL.

CHARGE CONJUGATION SYMMETRY

P + T ARE SYMMETRIES IN SPACE - TIME

C MEANS CHANGING THE CHARGE OF ALL PARTICLES. QUANTUM MECHANICALLY THIS MEANS CHANGING ALL PARTICLES INTO THEIR ANTIPARTICLES

CLASSICALLY

$$Q \xrightarrow{C} -Q$$

$$\vec{E} \longrightarrow -\vec{E}$$

$$\vec{B} \longrightarrow -\vec{B}$$

INTUITIVELY OBVIOUS THAT CLASSICAL ELECTROMAGNETISM IS INVARIANT UNDER SUCH A TRANSFORMATION

QUANTUM MECHANICALLY

$$|\psi(Q,\vec{r},t)\rangle \xrightarrow{C} |\psi(-Q,\vec{r},t)\rangle$$

A PARTICLE CAN BE AN EIGENSTATE OF CHARGE CONJUGATION

eg $\quad C|\pi^0\rangle \longrightarrow |\pi^0\rangle$

HOWEVER, IF THERE ARE OTHER INTERNAL QUANTUM NUMBERS WHICH CHANGE, THEN EVEN AN ELECTRICALLY NEUTRAL PARTICLE MAY NOT BE AN EIGENSTATE OF C

$$C|n\rangle = C|ddu\rangle$$
$$\Rightarrow |\bar{d}\bar{d}\bar{u}\rangle = |\bar{n}\rangle$$

INTERAL QUANTUM NUMBER WHICH CHANGES IS BARYON NUMBER

$$C|K^0\rangle = C|d\bar{s}\rangle \rightarrow |\bar{d}s\rangle$$
$$= \overline{K^0}$$

INTERNAL QUANTUM NUMBER WHICH CHANGES IS STRANGENESS

AS IN PARITY, 2 SUCCESSIVE C OPERATIONS, LEAVE A STATE UNCHANGED $\Rightarrow$ C HAS ± 1 AS EIGENVALS

$$C\,|\vec{E}\rangle \rightarrow |-\vec{E}\rangle$$
$$C\,|\vec{B}\rangle \rightarrow |-\vec{B}\rangle$$

SO $C|\gamma\rangle = -1\,|\gamma\rangle$

THE PHOTON HAS $-$VE "C PARITY"

IF WE ASSUME $[C, H] = 0$, THEN TOTAL C-PARITY IS CONSERVED. FOR EXAMPLE

$$\pi^0 \rightarrow \gamma + \gamma$$
$$\eta_c(\pi^0) = \eta_c(\gamma)\cdot\eta_c(\gamma)$$
$$= (-1)\cdot(-1) = +1$$

HAS OBSERVABLE CONSEQUENCE THAT

$$\pi^0 \not\rightarrow 3\gamma \qquad C = -1$$
$$5\gamma$$
$$\vdots$$

CP SYMMETRY

C IS CONSERVED IN STRONG & ELECTROMAGNETIC INTERACTIONS

C DOES NOT CHANGE EXTERNAL, OR SPACE TIME COORDINATES. SO IT CANNOT CHANGE HELICITY (OR HANDEDNESS) OF ν OR $\bar{\nu}$

C IS NOT A SYMMETRY OF WEAK INTERACTION. WE ALREADY KNOW THAT P IS NOT A SYMMETRY OF THE WEAK INTERACTION

$$|\nu_L\rangle \xrightarrow{C} |\bar{\nu}_L\rangle$$

DOESN'T EXIST!

HOWEVER

$$|\nu_L\rangle \xrightarrow{C} |\bar{\nu}_L\rangle \xrightarrow{P} |\bar{\nu}_R\rangle$$

SO CP DOES END UP IN A PHYSICAL STATE. SO CP CAN BE A SYMMETRY OF WEAK INTERACTION

WE SHOULD WE EXPECT CP SYMMETRY

WE EXPECT ALL THEORIES TO BE

CPT

INVARIANT

WEAK INTERACTION "BREAKS" P. TI ALSO BREAKS C. BUT IT RESPECTS CP.

IF CP WAS NOT A GOOD SYMMETRY

$$[CP, H] \neq 0$$

THIS WOULD IMPLY

TIME REVERSAL INVARIANCE NOT TRUE FOR WEAK INTERACTION