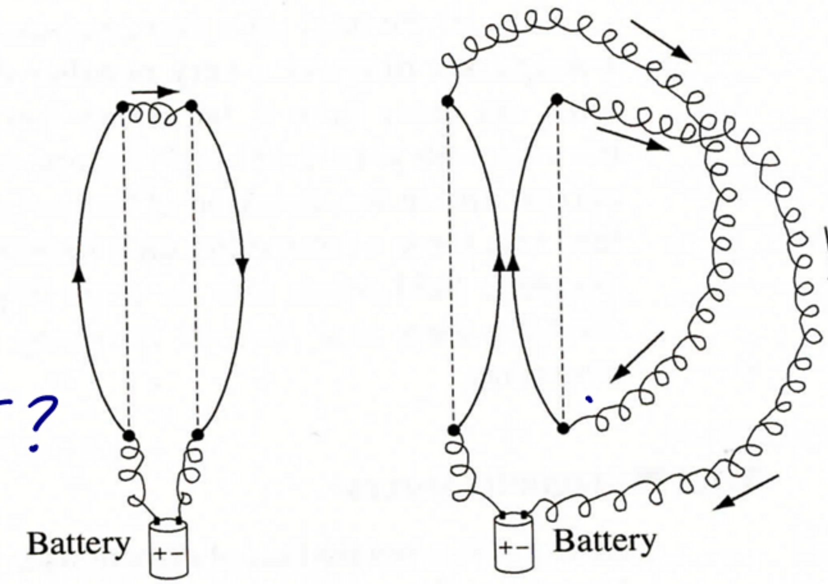
MAGNETOSTATICS

SIMPLE EXPERIMENT

HANGING VERTICAL WIRES

CONNECT TO BATTERY



(a) Currents in opposite directions repel.

(b) Currents in same directions attract.

IS THIS AN ELECTROSTATIC EFFECT?

CHARGES IN WIRES REPEL ??

NO — CURRENTS IN SAME DIRECTION ATTRACT

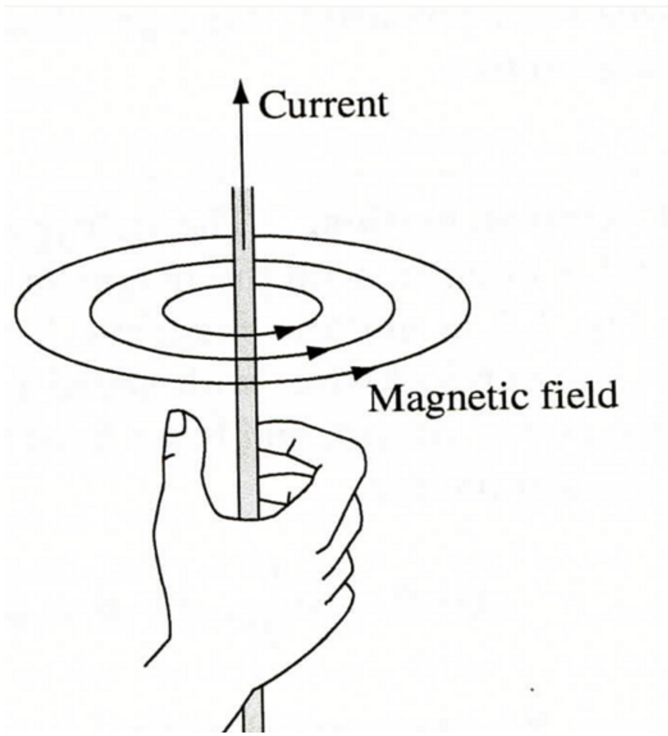
— WIRES ARE NEUTRAL → NO NET CHARGE

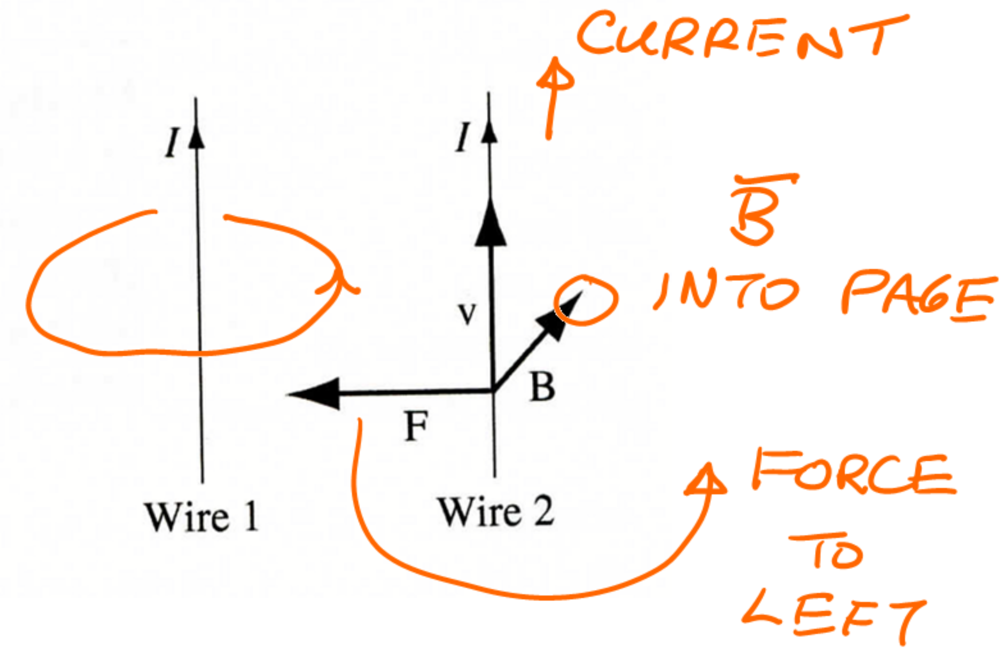
STATIONARY CHARGE → $\bar{E}$ FIELD

MOVING CHARGE → $\bar{E}$ + $\bar{B}$ FIELD

MAGNETIC FIELD

MAGNETIC FIELD





CAN INVESTIGATE
WITH A COMPASS NEEDLE

FIELD CIRCLES AROUND
THE WIRE

CONTRAST TO $\vec{E}$
WHERE $\vec{E}$ & $\vec{F}$
ARE PARALLEL.

MAGNETIC FORCES

MAGNETIC FORCE ON CHARGE Q, VELOCITY $\bar{v}$

$$F = Q(\bar{v} \times \bar{B})$$

LORENTZ FORCE LAW

EXPERIMENTAL FACT
NOT DERIVED FROM THEORY

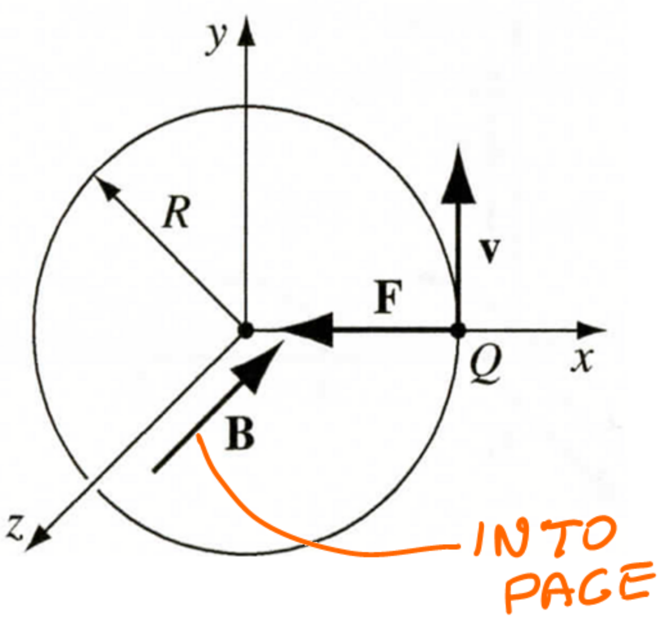
FOR A GENERAL REGION IN WHICH THERE IS AN ELECTRIC FIELD $\bar{E}$ AND A MAGNETIC FIELD $\bar{B}$

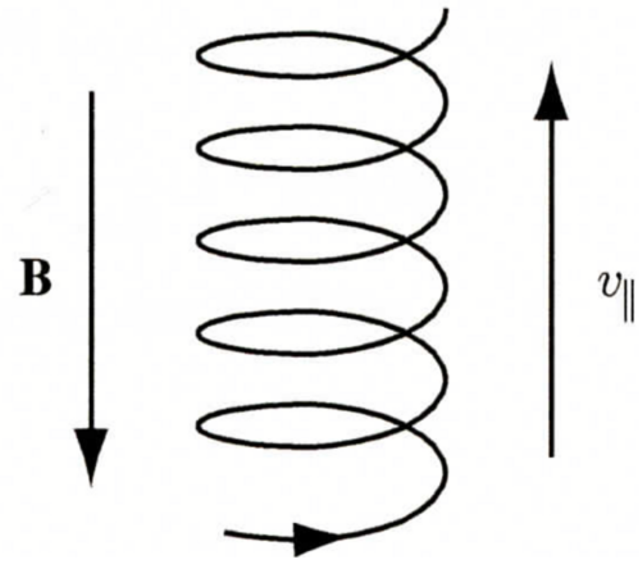
$$\bar{F} = Q[\bar{E} + (\bar{v} \times \bar{B})]$$

$\bar{F}, \bar{E} \rightarrow$ VECTORS
$\bar{B} \rightarrow$ PSEUDO VECTOR

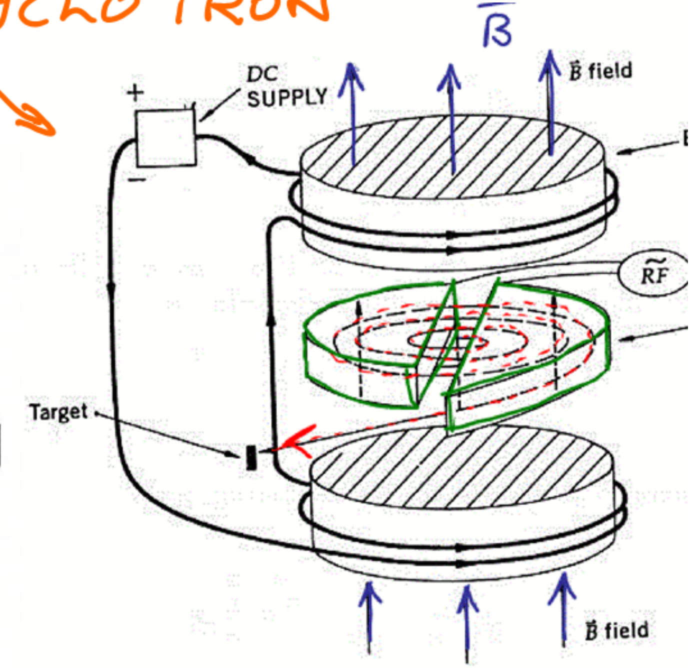
AS I SAID EARLIER THE FORCES ON CHARGES ARE MUCH MORE COMPLEX WHEN THE CHARGES ARE IN MOTION $\rightarrow$ AND WE HAVE NOT EVEN GOT TO ACCELERATION

CYCLOTRON MOTION





CYCLOTRON



CHARGED PARTICLE MOVES IN CIRCULAR PATH

LORENTZ FORCE = CENTRIFUGAL FORCE

$|F| = QvB$ $\qquad\qquad$ mv^2/R

$QvB = mv^2/R$

$QB = mv/R$

$p = QBR$

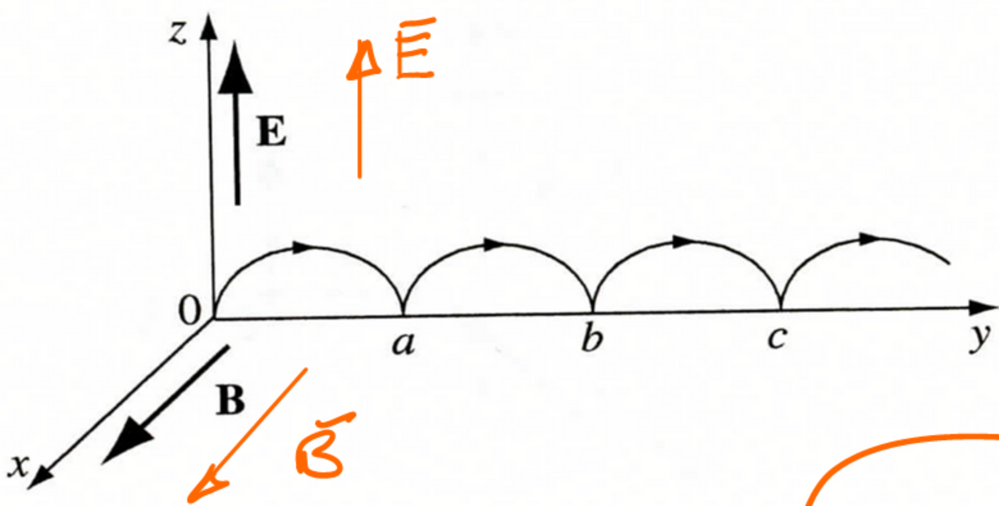
LARGE HADRON COLLIDER

$p = 10^{12}\ eV/c$

$B = 8$ Tesla

$R \sim 3000$ m

COMBINATION OF $\bar{E}$ + $\bar{B}$ CYCLOID MOTION



$\bar{E}$ & $\bar{B}$

RIGHT ANGLES

INITIALLY AT REST → NO $\bar{B}$ FORCE

ACCELATES UNDER $\bar{E}$ → B INCREASE

EVENTUALLY MOVING AGAINST $\bar{E}$

COMES TO REST

NO FORCE IN x → POSITION $(0, y(t), z(t))$

VELOCITY = $\bar{v} = (0, \dot{y}, \dot{z})$ $\quad \dot{y} = \frac{\partial y}{\partial t}$

$$\vec{V} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & \dot{y} & \dot{z} \\ B & 0 & 0 \end{vmatrix} = B\dot{z}\,\hat{y} - B\dot{y}\,\hat{z}$$

APPLYING $F = ma$

$$\vec{F} = Q(\vec{E} + \vec{V} \times \vec{B}) = Q(E\hat{z} + B\dot{z}\,\hat{y} - B\dot{y}\,\hat{z})$$

$$= ma = m(\ddot{y}\,\hat{y} + \ddot{z}\,\hat{z})$$

TREAT y & z COMPONENTS SEPARATELY

$$QB\dot{z} = m\ddot{y} \qquad QE - QB\dot{y} = m\ddot{z}$$

DEFINE $\omega \equiv \dfrac{QB}{m}$ ← CYCLOTRON FREQUENCY

EQUATIONS OF MOTION

$$\ddot{y} = \omega \dot{z} \quad ; \quad \ddot{z} = \omega\left(\frac{E}{B} - \dot{y}\right)$$

JUST QUOTE SOLUTIONS → SEE TEXT.

$$y(t) = C_1 \cos \omega t + C_2 \sin \omega t + \left(\frac{E}{B}\right) t + C_3$$

$$z(t) = C_2 \cos \omega t - C_1 \sin \omega t + C_4$$

INITIAL CONDITIONS

$$\left.\begin{array}{l} \dot{y}(0) = 0 \, , \, \dot{z}(0) = 0 \\ y(0) = z(0) = 0 \end{array}\right\}$$

$$C_4 = \frac{E}{B\omega}$$

$$C_1 = 0$$

$$C_2 = -\frac{E}{B\omega}$$

$$C_3 = 0$$

$$y(t) = \frac{E}{\omega B}(\omega t - \sin\omega t)\,; \quad z(t) = \frac{E}{\omega B}(1 - \cos\omega t)$$

DEFINE $R \equiv E/\omega B$

+ $\sin^2\omega t + \cos^2\omega t = 1$

$$(y - R\omega t)^2 + (z - R)^2 = R^2$$

→ CIRCLE RADIUS R, CENTER $(0, R\omega t, R)$

→ y DIRECTIO

SPEED $u = \omega R = \frac{E}{B}$

PARTICLE MOTION IS PERPENDICULAR TO $\underline{E}$

MAGNETIC FORCES DO NO WORK

THIS FOLLOWS FROM FORM of LORENTZ FORCE

IF Q MOVES $d\bar{l} = \bar{v}\,dt$, WORK DONE IS:

$$dW_{MAG} = \bar{F}_{MAG} \cdot d\bar{l} = Q(\bar{v} \times \bar{B}) \cdot \bar{v}\, dt$$

$\bar{v} \times \bar{B}$ PERPENDICULAR TO $\bar{v}$

$$(\bar{v} \times \bar{B}) \cdot \bar{v} = 0 \rightarrow dW_{MAG} = 0$$

MAGNETIC FORCES CAN CHANGE DIRECTION BUT CANNOT CHANGE THE MAGNITUDE OF THE PARTICLE'S MOMENTUM

CURRENTS:

CURRENT IN A WIRE IS THE CHARGE PER UNIT TIME PASSING A GIVEN POINT

−VE CHARGES MOVING LEFT = +VE MOVING RIGHT

MOST PHENOMENA DEPEND ON CHARGE × VELOCITY

$$q \times \vec{v} = -q \times (-\vec{v})$$

USUALLY −VE ELECTRONS DO THE MOVING

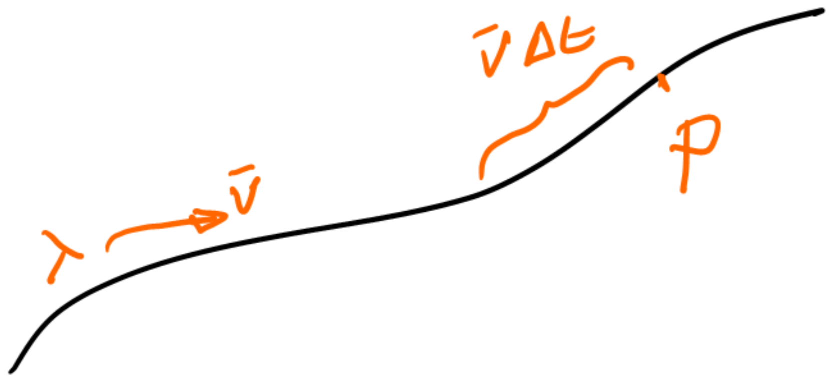
THIS IS OPPOSITE TO CONVENTIONAL CURRENT

CONVENTIONAL CURRENT CHOSEN BY BEN FRANKLIN

UNIT OF CURRENT → COULOMB PER SECOND

→ AMPERE

$$1\,A = 1\,C/s$$



LINE CHARGE λ TRAVELLING DOWN WIRE AT SPEED v

$$I = \lambda v$$

ACTUALLY CURRENT IS A VECTOR $\bar{I} = \lambda \bar{V}$

CURRENT FLOWING IN WIRE $\rightarrow$ CONSTRAINED DIRECTION

$\rightarrow$ VECTOR NATURE SUPERFLUOUS

$\rightarrow$ MATTERS FOR SURFACE / VOLUME

REAL MATTER $\rightarrow$ STATIONARY +VE
MOVING −VE

λ REFERS TO MOVING CHARGES

IF +VE & −VE MOVE $\bar{I} = \lambda_+ \bar{V}_+ + \lambda_- \bar{V}_-$

MAGNETIC FORCE ON SEGMENT of CURRENT CARRYING WIRE

$$\bar{F}_{MAG} = \int (\bar{v} \times \bar{B})\, dq = \int (\bar{v} \times \bar{B})\, \lambda\, d\ell$$

$$= \int (\bar{I} \times \bar{B})\, d\ell$$

$\bar{I}$ AND $d\bar{\ell}$ PARALLEL

$$\bar{F}_{MAG} = \int I\, (d\bar{\ell} \times \bar{B})$$

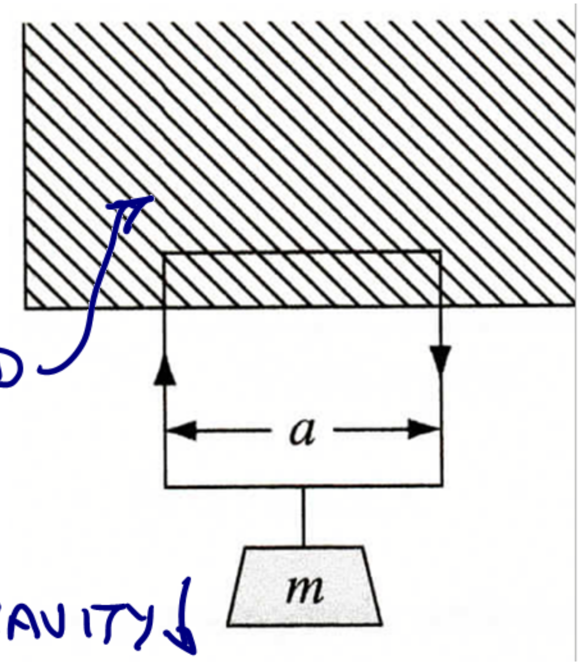
TYPICALLY CURRENT IS CONSTANT IN MAGNITUDE ALONG THE WIRE

$$\bar{F}_{MAG} = I \int d\bar{\ell} \times \bar{B}$$

RECTANGULAR LOOP OF WIRE

SUPPORTING MASS M HANGS

WITH ONE END IN MAGNETIC FIELD



POINTS INTO PAGE

WHAT CURRNT → MAGNETIC FORCE $\uparrow$ = GRAVITY $\downarrow$

$\bar{I} \times \bar{B}$ → CLOCKWISE → MAG FORCE $\uparrow$

$$F_{MAG} = I B a \qquad \left(I \int d\bar{I} \times \bar{B} \right)$$

$$I = \frac{mg}{Ba}$$ ← GRAVITY !

INCREASE CURRET I $\quad F_{MAG}\uparrow > mg\downarrow$

LOOP MOVES UP → WHAT IS DOING WORK?

NOT MAGNETIC FIELD

LOOP RISES → CHARGES NOT MOVING HORIZONTALLY

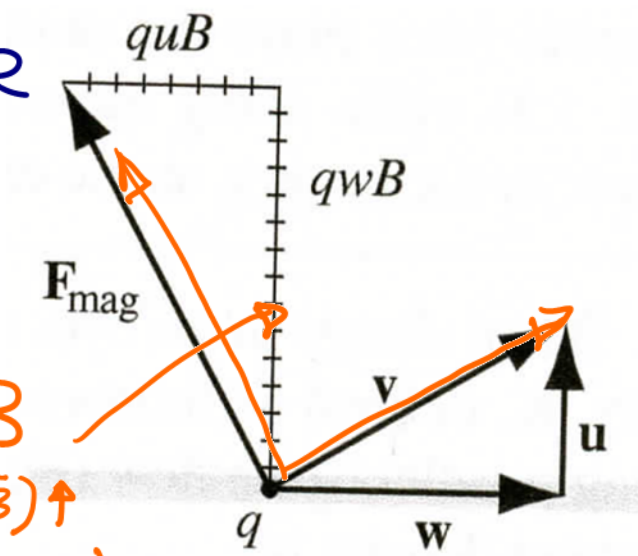
→ VELOCITY ACQUIRES u ↑ COMPONENT

↳ IN ADDITION TO HORIZONTAL ω

$I = \lambda \omega$

MAGNETIC FORCE IS PERPENDICULAR TO VELOCITY OF CHARGES

↳ DOES NO WORK ON q



$\bar{F}_{MAG}$ HAS VERTICAL COMPONENT $q\omega B$

$(\bar{\omega} \times \bar{B}) \uparrow$

NET VERTICAL FORCE ON CHARGES λa

$$F_{VERT} = \lambda a \omega B = I B a$$

$$F_{HORIZ} = \lambda a u B$$ ← OPPOSES FLOW OF CURREN

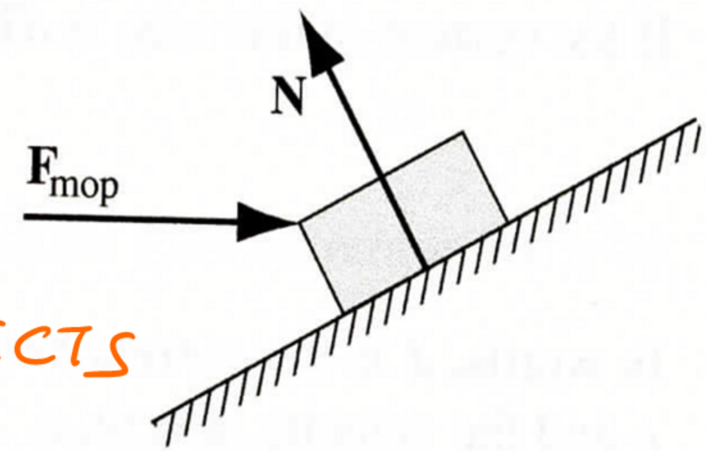
$(\bar{u} \times \bar{B}) \leftarrow$

IN TIME dt CHARGES MOVE HORIZONTALLY $w\,dt$

WORK = FORCE x DISTANCE

$$\underbrace{\int \lambda a u B\, dt \quad \int w\, dt}$$

WORK DONE $W_{BATTERY} = \lambda a B \int u w\, dt = I B a h$



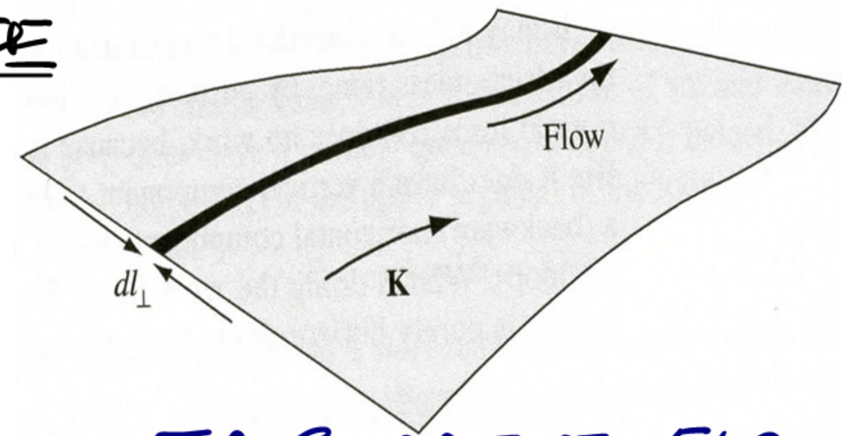
MECHANICAL ANALOGUE

MAGNETIC FORCE REDIRECTS

HORIZONTAL FORCE OF BATTERY

BUT DOES NO WORK

CHARGE FLOW OVER A SURFACE



SURFACE CURRENT DENSITY

RIBBON OF CHARGE

INFINITESIMAL WIDTH $dl_\perp$ PARALLEL TO CURRENT FLOW

CURRENT IN RIBBON dI

$$\vec{K} = \frac{d\vec{I}}{dl_\perp}$$

SURFACE CURRENT DENSITY

K IS CURRENT PER UNIT WIDTH

MOBILE SURFACE CHARGE DENSITY σ

VELOCITY OF CHARGE $\vec{v}$

$$\vec{K} = \sigma \vec{v}$$ ← VARIES OVER SURFACE

$$\vec{F}_{MAG} = \int (\vec{v} \times \vec{B}) \sigma \, da = \int (\vec{K} \times \vec{B}) \, da$$

$\vec{B}$ DISCONTINUOUS AT SURFACE ⟶ USE AVERAGE

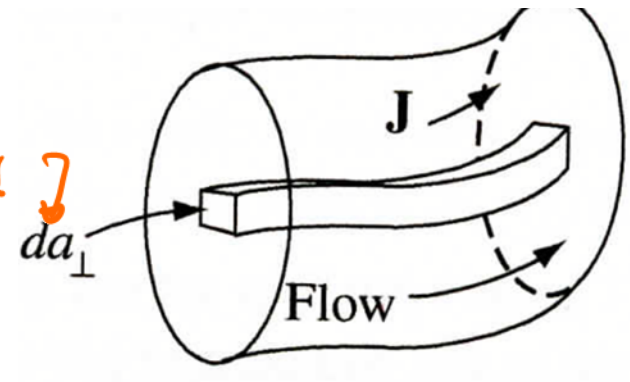
VOLUME CURRENT DENSITY $\bar{J}$

CHARGE IN 3-d REGION

TUBE OF INFITESIMAL X-SECT $da_\perp$

RUNNING PARALLEL TO FLOW

CURRENT IN TUBE IS $d\bar{I}$



VOLUME CURRENT DENSITY $\bar{J} \equiv \dfrac{d\bar{I}}{da_\perp}$

CURRENT PER UNIT AREA

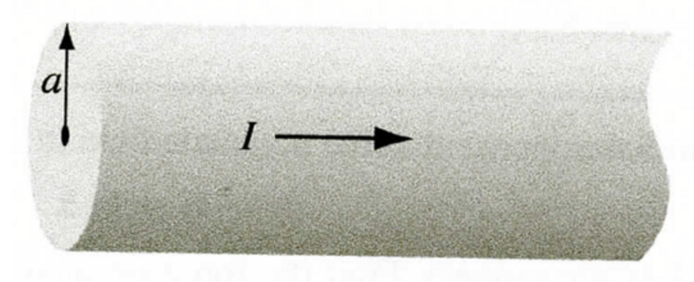
VOLUME CHARGE DENSITY ρ

VELOCITY OF CHARGE $\bar{v}$

$$\left.\begin{matrix}\rho \\ \bar{v}\end{matrix}\right\} \bar{J} = \rho \bar{v}$$

$$\bar{F}_{MAG} = \int (\bar{v} \times \bar{B}) \rho \, d\tau = \int (\bar{J} \times \bar{B}) \, d\tau$$

EXAMPLE (S)

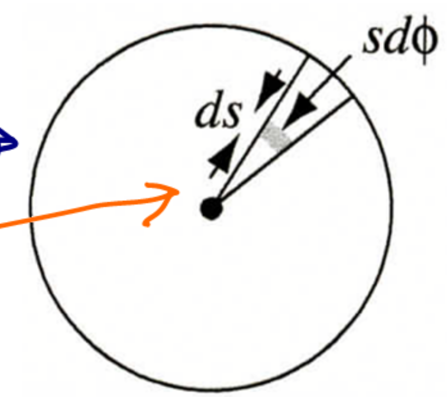


(a) CURRENT UNIFORMLY DISTRIBUTED OVER WIRE OF CIRCULAR X-SECTION RADIUS a, ? VOLUME CURRENT DENSITY

AREA PERPENDICULAR TO FLOW πa^2

$$J = I / \pi a^2$$

(b) CURRENT $\propto$ DISTANCE FROM AXIS →



$$J = ks$$

$$I = \int_S \bar{J} \cdot d\bar{a}_\perp = \int_S \underbrace{(ks)}_{J} \underbrace{(s\,ds\,d\phi)}_{da_\perp}$$

$$= 2\pi k \int_0^a s^2 ds = \frac{2\pi k a^3}{3}$$

GAUSS'S (DIVERGENCE) THEOREM

$$\int_V (\bar{\nabla}\cdot\bar{v})\,d\tau = \oint_S \bar{v}\cdot d\bar{a}$$

$$\oint_S \bar{J}\cdot d\bar{a} = \int_V (\bar{\nabla}\cdot\bar{J})\,d\tau$$

CHARGE CONSERVED → FLOW OUT THRU SURFACE
REDUCES CHARGE INSIDE

$$\int_V (\bar{\nabla}\cdot\bar{J})\,d\tau = -\frac{d}{dt}\int_V \rho\,d\tau = -\int_V \frac{d\rho}{dt}\cdot d\tau$$

TRUE FOR ANY VOLUME

$$\bar{\nabla}\cdot\bar{J} = -\frac{\partial\rho}{\partial t}$$

CONTINUITY EQUATION

LOCAL CHARGE CONSERVATION