# BIOT – SAVARD

## STEADY CURRENTS

STATIONARY CHARGES → $\bar{E}$ CONSTANT, ELECTROSTATICS

STEADY CURRENTS → $\bar{B}$ CONSTANT, MAGNETOSTATICS

$$\frac{\partial \rho}{\partial t} = 0 \quad ; \quad \frac{\partial \bar{J}}{\partial t} = \bar{0}$$

OBVIOUSLY THIS IS AN IDEALIZATION

CHARGE IS QUANTIZED IN POINTS

SO, IN REALITY CURRENT ALWAYS

VARIES → WE CALL THIS

"SHOT NOISE" IN ELECTRONICS

IN ELECTROSTATICS → STARTED FROM POINT CHARGES

↓ GENERALIZE

CHARGE DISTRIBUTIONS

MAGNETOSTATICS → START FROM MOVING CHARGES

IN A WIRE $|\bar{I}|$ MUST BE SAME EVERYWHERE

↳ CHARGE DOES NOT PILE UP SOMEWHERE

$$\bar{\nabla} \cdot \bar{J} = -\frac{\partial \rho}{\partial t} = 0$$

# MAGNETIC FIELD of A STEADY CURRENT BIOT – SAVARD

$$\bar{B}(\bar{r}) = \frac{\mu_0}{4\pi}\int \frac{\bar{I} \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, dl' = \frac{\mu_0}{4\pi} I \int \frac{d\bar{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

NOTE $\mathscr{r}^2$ → SAME AS $\left.\begin{array}{l}\text{COULOMB}\\\text{GRAVITY}\end{array}\right\}$ ?

$\int$ ATION → ALONG CURRENT PATH

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{NEWTONS}}{(\text{AMPERE})^2}$$

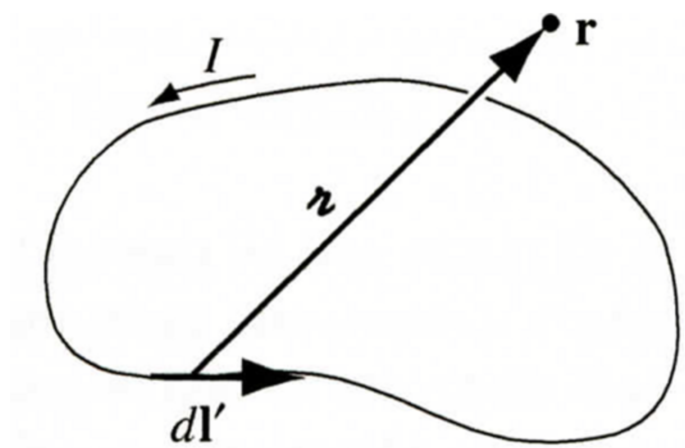
← PERMEABILITY OF FREE SPACE

→ EXACT NO. → DEFINES A

$\bar{B}$ NEWTONS/AMPERE – METER

↳ TESLA

$1\ T = 1\ N/A \cdot m$

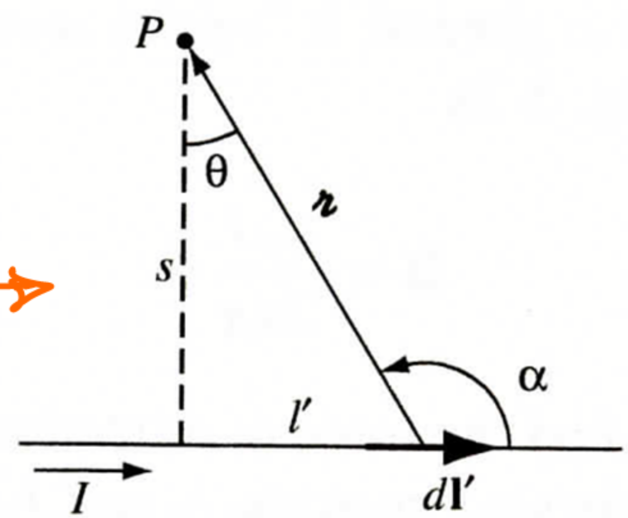


# MAGNETIC FIELDS FROM LONG STRAIGHT WIRE CARRYING A STEADY CURRENT $I$

$d\bar{l}' \times \hat{r}$ POINTS OUT OF PAGE

$$|d\bar{l}' \times \hat{r}| = dl \sin\alpha = dl' \cos\theta$$

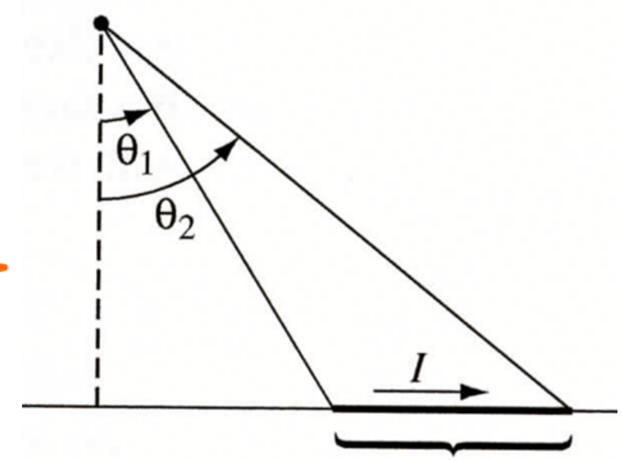
AND $\quad l' = s\tan\theta$




$$dl' = \frac{s}{\cos^2\theta}\, d\theta$$

AND $\quad s = r\cos\theta \rightarrow \dfrac{1}{r^2} = \dfrac{\cos^2\theta}{s^2}$

$$\bar{B} = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \left(\frac{\cos^2\theta}{s^2}\right)\left(\frac{s}{\cos^2\theta}\right) \cos\theta\, d\theta$$




$$\bar{B} = \frac{\mu_0 I}{4\pi s} \int_{\theta_1}^{\theta_2} \cos\theta\, d\theta = \frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right)$$

$$\vec{B} = \frac{\mu_0 I}{4\pi s}\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta = \frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right)$$

GIVES FIELD OF STRAIGHT SEGMENT

↳ CANNOT HAVE STEADY CURRENT

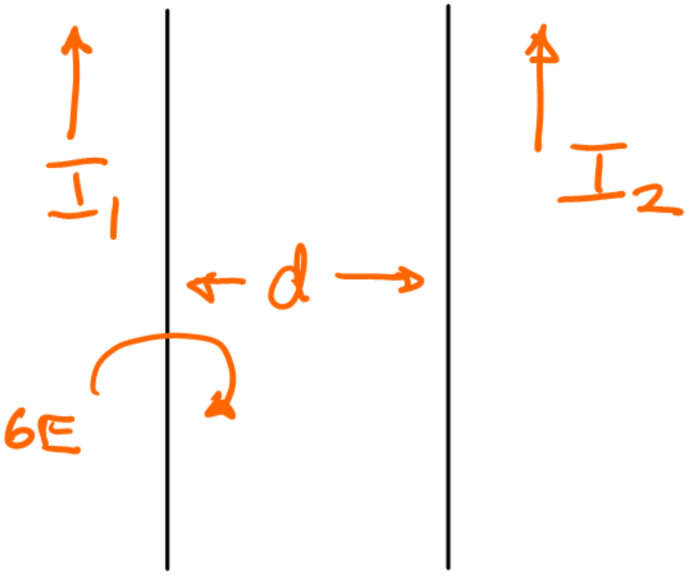
$$\int_{-\infty}^{+\infty} \longrightarrow \int_{-\pi/2}^{+\pi/2} d\theta$$

$$B = \frac{\mu_0 I}{2\pi s}$$


$s$ DEPENDENCE SAME AS ELECTROSTATICS

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

- FORCE BETWEEN TWO LONG STRAIGHT WIRES




- FIELD AT (2) DUE TO (1)

$$B = \frac{\mu_0 I_1}{2\pi d}$$ INTO PAGE

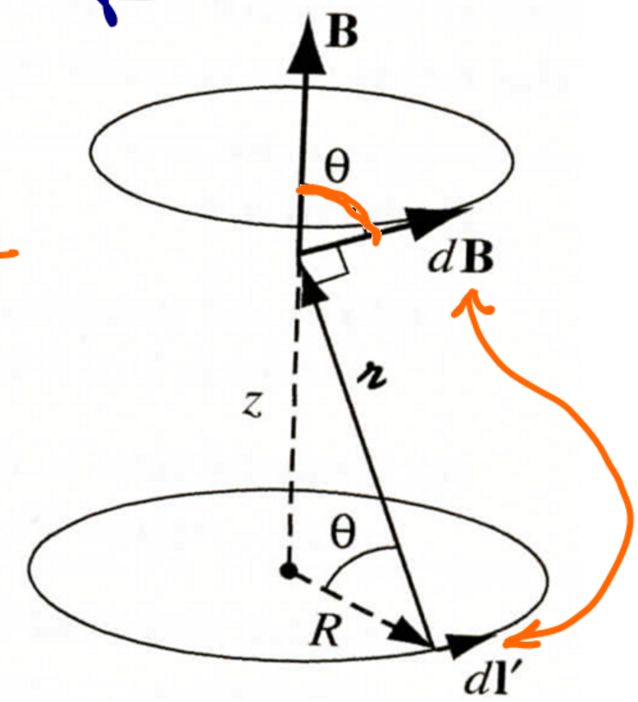
$$\bar{F}_{\substack{MAG\\ON\\(2)}} = \int I_2 (d\bar{\ell} \times \bar{B})\, d\ell = I_2 \left(\frac{\mu_0 I_1}{2\pi d}\right) \underbrace{\int_{-\infty}^{+\infty} d\ell}_{\infty}$$

$$\text{FORCE / UNIT LENGTH} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$$

$I_1 \uparrow \quad \downarrow I_2 \quad \rightarrow$ REPULSIVE

$I_1 \uparrow \quad \uparrow I_2 \quad \rightarrow$ ATTRACTIVE

MAGNETIC FIELD $z$ ABOVE CENTER of CIRCULAR LOOP $R$ CURRENT $I$




$d\vec{B}$ DUE TO $dI'$ NOT HORIZONTAL

$\int \rightarrow$ SWEEPS OUT A CONE

HORIZONTAL COMPONENTS CANCEL

VERTICAL $\quad B(z) = \dfrac{\mu_0}{4\pi} I \displaystyle\int \frac{dl'}{\mathfrak{r}^2} \cos\theta$

$dl', \mathfrak{r}^2$ PERP

$\cos\theta, \mathfrak{r}^2$ ARE CONSTANT $\quad \int dl' = 2\pi R$

$$B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{\mathfrak{r}^2}\right) 2\pi R = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$$

BIOT – SAVARD → SURFACE & VOLUME

SURFACE $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}\,') \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, da'$

VOLUME $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,') \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau'$

MOVING CHARGE? $\frac{\mu_0}{4\pi}\, q\, \frac{\vec{v} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$

WRONG → MOVING CHARGE IS NOT A STEADY CURRENT

SUPERPOSITION APPLIES TO $\vec{B}$

JUST LIKE $\vec{E}$

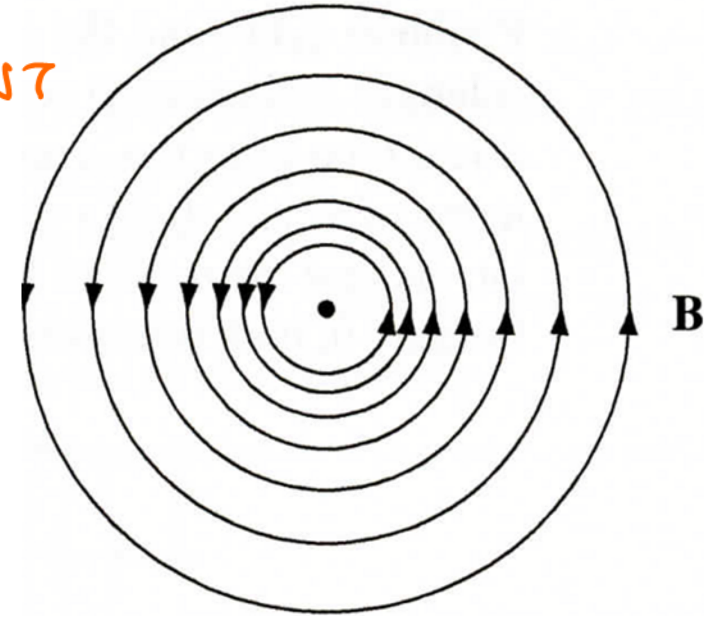
# DIVERGENCE & CURL OF $\vec{B}$

STRAIGHT LINE CURRENTS

INFINITE STRAIGHT WIRE

MUST HAVE $\vec{\nabla} \times \vec{B} \neq 0$

CURRENT OUT OF PAGE




USE CIRCULAR PATH RADIUS 'S' CENTRED ON WIRE

$$\oint \vec{B} \cdot d\vec{\ell} = \oint \frac{\mu_0 I}{2\pi s} d\ell = \frac{\mu_0 I}{2\pi s} \oint d\ell$$

$$= \mu_0 I$$

$\oint$ INDEPENDENT of $s$ $\rightarrow$ $\vec{B}$ DECREASES AT SAME RATE CIRCUMFERENCE INCREASES

COULD USE ANY CONTOUR FOR $\oint$

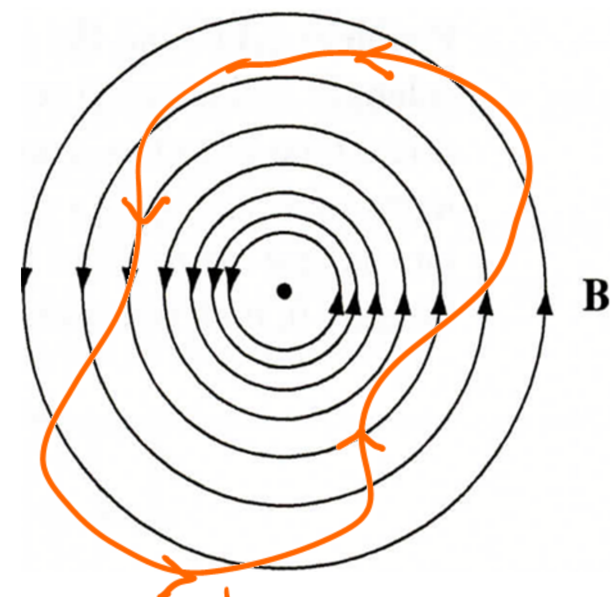
SEE INDEPENDENCE of PATH → CYLINDRICAL COORDS $(s, \phi, z)$

$$\vec{B} = \left(\frac{\mu_0 I}{2\pi s}\right)\hat{\phi} \quad ; \quad d\vec{l} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$

$$\oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{2\pi}\oint \frac{1}{s}\cdot s\,d\phi$$

$$= \frac{\mu_0 I}{2\pi}\int_0^{2\pi} d\phi$$

$$= \mu_0 I$$ AS BEFORE

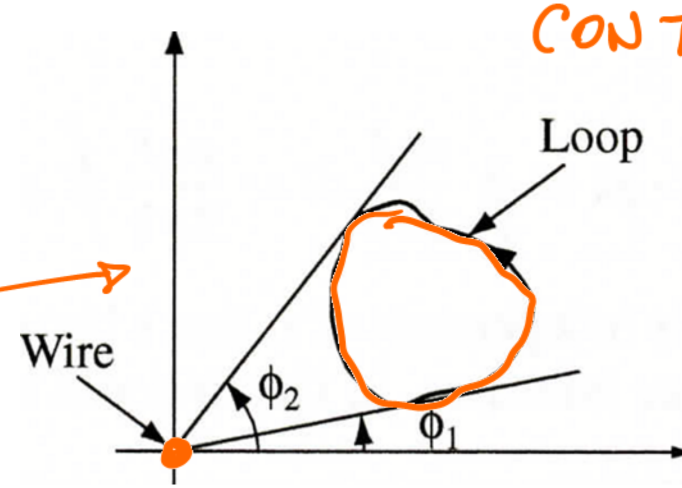



ANY OLD CONTOUR

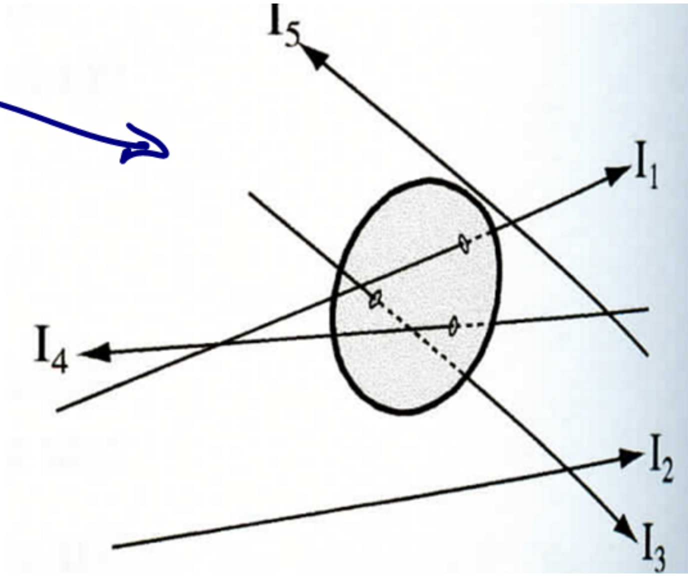
CONTOUR ENCLOSES WIRE

AND $\oint$ AROUND ONCE.

$$\int_{\phi_1}^{\phi_2} + \int_{\phi_2}^{\phi_1} = 0$$



BUNDLE of WIRES




EACH WIRE THAT PASSES THROUGH LOOP CONTRIBUTES

$$\mu_0 I$$

THOSE THAT DO NOT PASS THROUGH LOOP CONTRIBUTE NOTHING

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{ENCLOSE}$$

$$I_{ENCL} = \int \vec{J} \cdot \vec{a}$$

VOLUME CURRENT DENSITY

$$\int (\vec{\nabla} \times \vec{B}) \, d\vec{a} = \mu_0 \int \vec{J} \cdot d\vec{a}$$

FROM STOKES

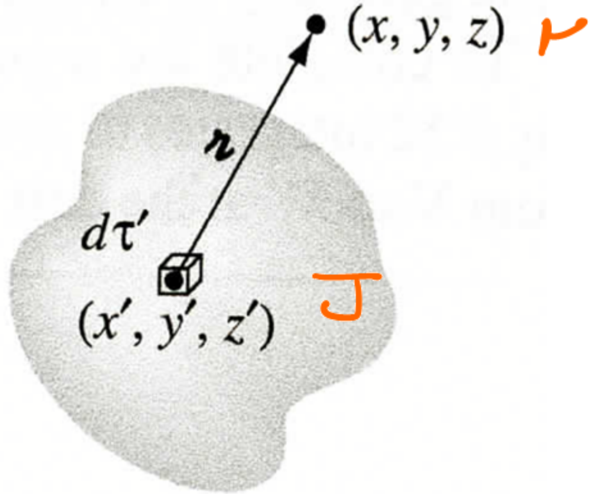
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

THIS ASSUMES A LONG STRAIGHT WIRE

→ DO IT GENERALLY ↓

# GENERAL DIVERGENCE & CURL of $\vec{B}$

BIOT & SAVARD for GENERAL VOLUME




$$B(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,') \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau'$$

$$\vec{B} \equiv \vec{B}(x, y, z)\ ;\ \vec{J} \equiv \vec{J}(x', y', z')$$

$$\mathscr{r} = (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z}$$

$$d\tau' = dx'\, dy'\, dz'$$

$\int$ IS OVER PRIMED COORDINATES

$\vec{\nabla} \times \vec{B} \rightarrow$ REFERS TO UNPRIMED COORDINATES

$$\bar{B}(\bar{r}) = \frac{\mu_0}{4\pi}\int \frac{\bar{J}(\bar{r}')\times\hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau'$$

$$\bar{\nabla}\cdot\bar{B} = \frac{\mu_0}{4\pi}\int \bar{\nabla}\cdot\left(\bar{J}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) d\tau'$$

GENERALLY $\bar{\nabla}\cdot(\bar{A}\times\bar{B}) = \bar{B}\cdot(\bar{\nabla}\times\bar{A}) - \bar{A}\cdot(\bar{\nabla}\times\bar{B})$

$$\bar{\nabla}\cdot\left(\bar{J}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}(\bar{\nabla}\times\bar{J}) - \bar{J}\cdot\left(\bar{\nabla}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)$$

ACTS ON UNPRIME COORDS

DOES NOT DEPEND ON UNPRIMED COORDINATES

$$\underbrace{\bar{\nabla}\times\bar{J} = 0 \qquad \left(\bar{\nabla}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = \bar{0}}_{\bar{\nabla}\cdot\bar{B} = \bar{0}}$$

AGAIN $\rightarrow$ $$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,') \times \hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau'$$

$$\vec{\nabla} \times \vec{B} = \frac{\mu_0}{4\pi}\int \vec{\nabla} \times \left(\vec{J} \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) d\tau'$$

$$\vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} - (\vec{A}\cdot\vec{\nabla})B + \vec{A}(\vec{\nabla}\cdot\vec{B}) - \vec{B}(\vec{\nabla}\cdot\vec{A})$$

$$\vec{\nabla} \times \left(\vec{J} \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\cdot\vec{\nabla}\right)\vec{J} - (\vec{J}\cdot\vec{\nabla})\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} + \vec{J}\left(\vec{\nabla}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) - \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}(\vec{\nabla}\cdot\vec{J})$$

TERMS IN $\vec{\nabla}\cdot\vec{J} \Rightarrow 0$ — $\vec{J} \neq \vec{J}(x, y, z)$

$$\vec{\nabla} \times \left(\vec{J} \times \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = \vec{J}\left(\vec{\nabla}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) - (\vec{J}\cdot\vec{\nabla})\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = 4\pi\,\delta^3(\vec{\mathscr{r}})$$

$\rightarrow$ MUST SHOW THIS IS ZERO

ASSUMING $\int (\vec{J}\cdot\vec{\nabla})\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \to 0$

$$\vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\int \vec{\nabla}\times\left(\vec{J}\times\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)d\tau' = \int \vec{J}\underbrace{\left(\vec{\nabla}\cdot\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)}_{\vec{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)=4\pi\delta^3(\vec{\mathscr{r}})} d\tau'$$

$$\vec{\nabla}\times\vec{B} = \frac{\mu_0}{4\pi}\int \vec{J}(\vec{r}\,')\,4\pi\,\delta^3(\vec{r}-\vec{r}\,')\,d\tau$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}(\vec{r}) + ?$$

WITHOUT ASSUMING STRAIGHT LINE CURRENT

STILL NEED TO SHOW THAT THIS $= 0$

$$-(\vec{J}\cdot\vec{\nabla})\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = 0 \quad ???$$

$$\vec{\mathscr{r}} = (x-x')\hat{x} + (y-y')\hat{y} - (z-z')\hat{z}$$

DEPENDS DIFFERENCE OF COORDINATES

$$\frac{\partial}{\partial x} f(x-x') = -\frac{\partial}{\partial x'} f(x-x')$$

$$\vec{\nabla}(r-r') = -\nabla'(r-r')$$

$$-(\vec{J}\cdot\vec{\nabla})\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = (\vec{J}\cdot\vec{\nabla}')\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$$

LOOK AT x-COMPONET

$$(\vec{J}\cdot\nabla')\left(\frac{x-x'}{\mathscr{r}^3}\right)$$

$$(\bar{J}\cdot\bar{\nabla}')\left(\frac{x-x'}{r^3}\right)$$

USE $\bar{\nabla}\cdot(f\bar{A}) = f(\bar{\nabla}\cdot\bar{A}) + \bar{A}\cdot(\bar{\nabla}f)$

$$\bar{J}\cdot\bar{\nabla}'\left(\frac{x-x}{r^3}\right) = \bar{\nabla}'\left(\frac{x-x'}{r^3}\right)\bar{J} - \left(\frac{x-x'}{r^3}\right)\bar{\nabla}'\cdot J$$

$\bar{\nabla}'\cdot\bar{J}=0$ FOR STEADY CURRENT

$$(\bar{J}\cdot\nabla')\left(\frac{x-x'}{r^3}\right) = \bar{\nabla}'\left(\frac{x-x'}{r^3}\cdot\bar{J}\right)$$

$$-\underbrace{(\bar{J}\cdot\bar{\nabla})\left(\frac{x-x'}{r^3}\right)}_{\text{WANT TO SHOW}=0} = \underbrace{\bar{\nabla}'\left(\frac{x-x'}{r^3}\cdot J\right)}_{\int \text{THIS} \rightarrow \text{SEE IF IT IS ZERO}}$$

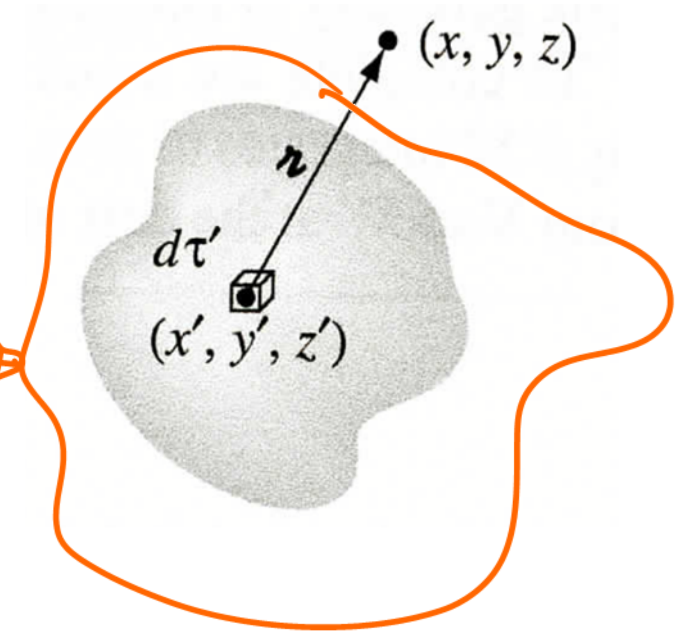
LOOK MA NO PRIME

$$\int \bar{\nabla}' \left(\frac{x-x'}{\mathscr{r}^3}\right) \bar{J}\, d\tau' = \oint \left(\frac{x-x'}{\mathscr{r}^3}\right) \bar{J}\cdot d\bar{a}'$$

DIVERGENCE THEOREM

$$\bar{J} \equiv \bar{J}(x', y', z')$$

$$\oint_S \left(\frac{x-x'}{\mathscr{r}^3}\right) \bar{J}\cdot d\bar{a}'$$




ALL CURRENT DENSITY IS IN VOLUME

↳ SURFACE $\int = 0$

$$\bar{\nabla}\times\bar{B} = \mu_0 \bar{J}(\bar{r})$$ GENERALLY