AMPÈRE'S LAW

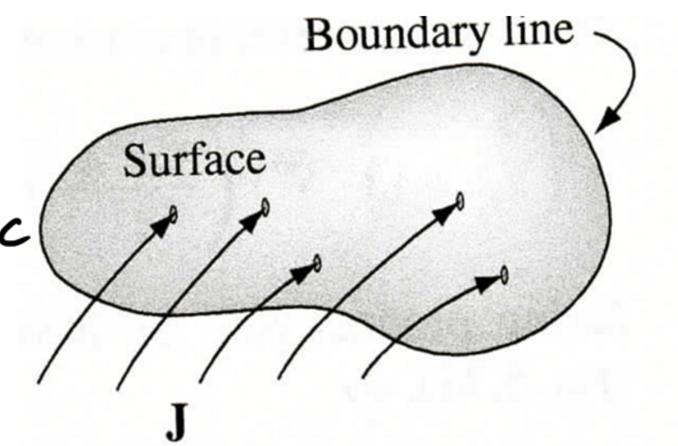
$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$$ ← AMPERE'S LAW

CAN CONVERT TO INTEGRAL FORM → STOKES'

$$\int_S (\bar{\nabla} \times \bar{v}) \cdot d\bar{a} = \oint_P \bar{v} \cdot d\bar{\ell}$$

$$\int (\bar{\nabla} \times \bar{B}) \cdot d\bar{a} = \oint_P \bar{B} \cdot d\bar{\ell} = \mu_0 \int \bar{J} \cdot d\bar{a}$$

$\int \bar{J} \cdot d\bar{a}$ → TOTAL CURRENT THRU SURFACE I_{enc}



$$\oint \bar{B} \cdot d\bar{\ell} = \mu_0 I_{enc}$$

$$\oint \bar{B} \cdot d\bar{l} = \mu_0 I_{enc}$$

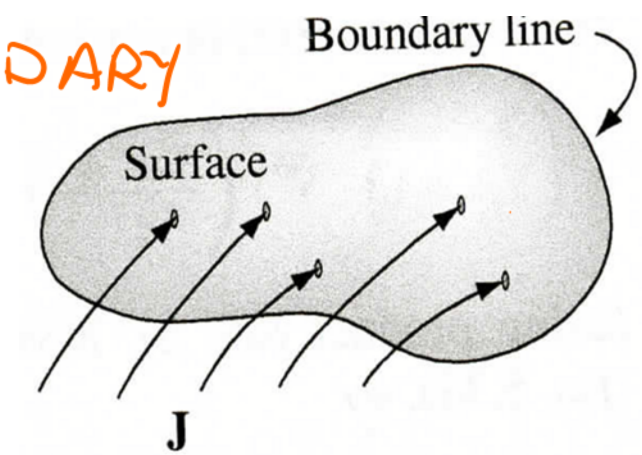
SHOWED THAT THIS WAS TRUE FOR STRAIGHT WIRE

HAVE GENERALIZED TO ARBITRARY CURRENT

WHICH WAY TO $\oint$ AROUND LOOP?

APPLY RIGHT HAND RULE ON BOUNDARY

THUMB GIVES +VE CURRENT



ELECTROSTATICS: COULOMB → GAUSS

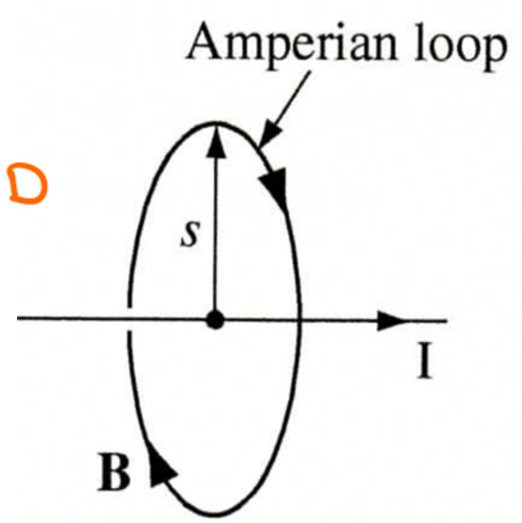
MAGNETOSTATICS: BIOT - SAVARD → AMPÈRE

USEFUL WHEN CURRENT HAS SOME SYMMETRY

EXAMPLE: FIND MAGNETIC FIELD DISTANCE S FROM LONG STRAIGHT WIRE CARRYING I

DIRECTION OF $\vec{B}$ → R.H. RULE

SYMMETRY → $\vec{B}$ CONSTANT AROUND LOOP



$$\oint \vec{B}\cdot d\vec{l} = B\oint dl = B\,2\pi s = \mu_0 I_{enc}$$
$$= \mu_0 I$$

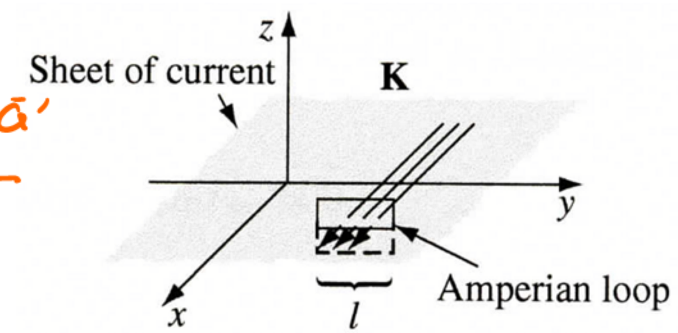
$$B = \frac{\mu_0 I}{2\pi s}$$

SAME AS BIOT-SAVART BUT EASIER

EXAMPLE: FIND THE MAGNETIC FIELD OF AN INFINITE UNIFORM SURFACE CURRENT $\vec{K} = K\hat{x}$ FLOWING OVER XY PLANE.

DIRECTION OF $\vec{B}$

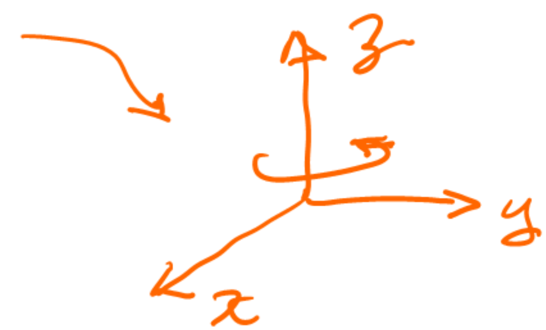
BIOT-SAVARD $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{K}(\vec{r}\,') \times \hat{\mathscr{r}}\, da'}{\mathscr{r}^2}$



SO $\vec{B}$ IS PERP TO CURRENT → NO x COMPONENT

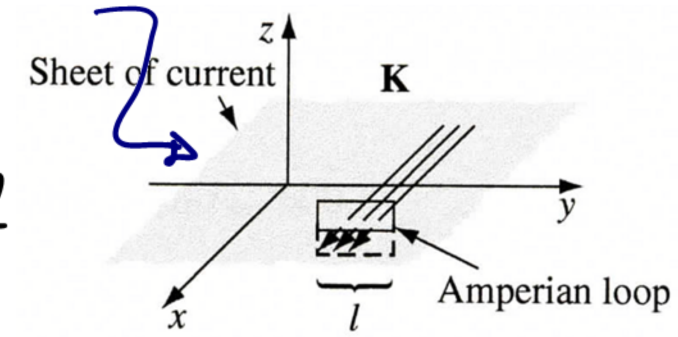
z- COMPONENT — IF FIELD POINTED AWAY FROM PLANE → REVERSING CURRENT WOULD MAKE FIELD POINT INTO PLANE

z- COMPONENT CANNOT DEPEND ON DIRECTION OF CURRENT IN XY PLANE



$\vec{B}$ ONLY HAS y- COMPONENT

AMPERIAN LOOP IN PLANE



$$\oint \bar{B}\cdot d\bar{l} = 2Bl = \mu_0 I_{en} = \mu_0 K l$$

$$B = \frac{\mu_0}{2} K$$

HEIGHT OF LOOP $\rightarrow 0$

$$\bar{B} = \begin{cases} +(\mu_0/2)K\hat{y} & \text{for } z < 0 \\ -(\mu_0/2)K\hat{y} & \text{for } z > 0 \end{cases}$$

FIELD IS INDEPENDENT OF DISTANCE FROM PLANE

Cf $\rightarrow$ ELECTROSTATIC CASE

EXAMPLE: MAGNETIC FIELD OF VERY LONG SOLENOID

n CLOSELY WOUND TURNS PER UNIT LENGTH, ON A CYLINDER OF RADIUS R. EACH TURN CARRIES A STEADY CURRENT I

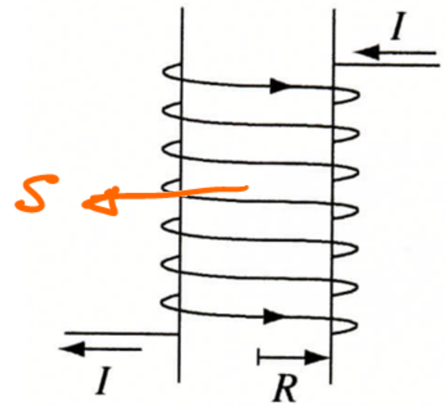
CLOSELY WOUND TURNS → EACH IS CIRCULAR

WHAT IS THE DIRECTION OF $\overline{B}$

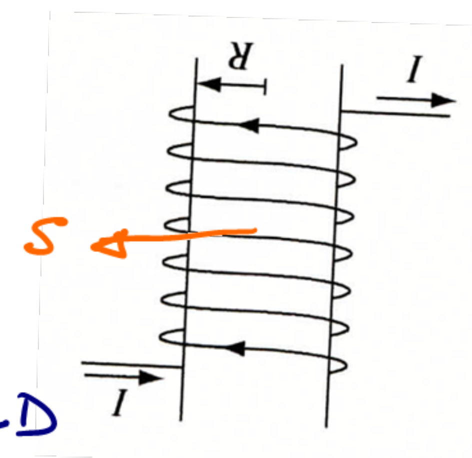
COULD IT HAVE A RADIAL COMPONENT?

– IF B_s +VE, REVERSING CURRENT → B_s –VE

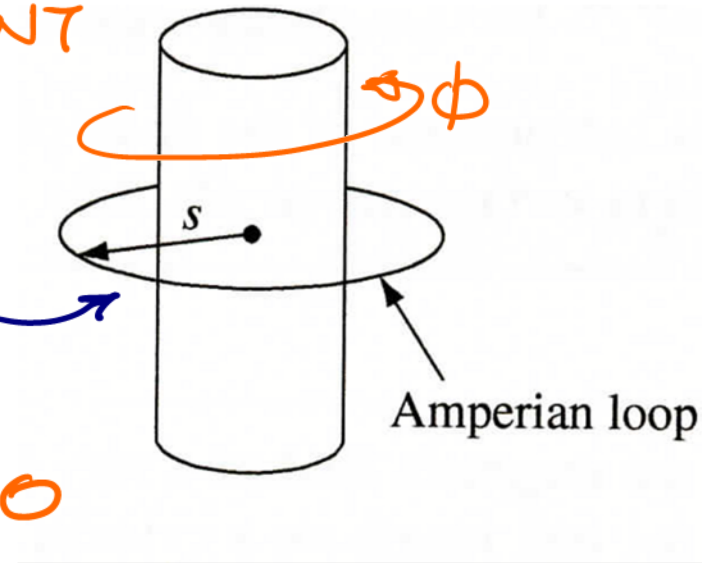
↳ EQUIVALENT TO TURNING THE SOLEND UPSIDE DOWN



CAN'T CHANGE MAGNETIC FIELD



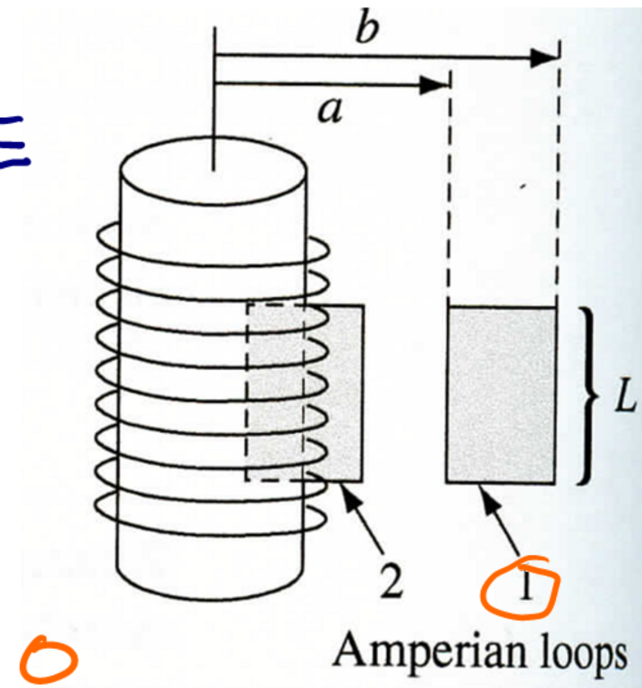
COULD THERE BE A ϕ COMPONENT OF B?



B_ϕ WOULD BE CONSTANT AROUND

$$\oint \vec{B} \cdot d\vec{\ell} = B_\phi \cdot 2\pi s = \mu_0 \underbrace{I_{enc}}_{=0} = 0$$

- MAGNET FIELD $\|^{el}$ to AXIS
- RH RULE $\uparrow \vec{B}$ INSIDE $\downarrow \vec{B}$ OUTSIDE
- GOES TO ZERO FAR FROM AXIS



LOOP 1 - OUTSIDE SOLENOID

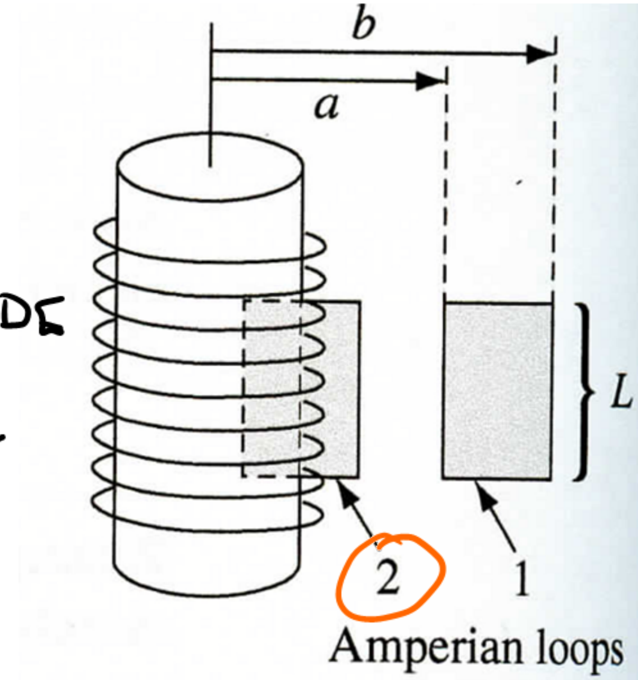
$$\oint \vec{B} \cdot d\vec{\ell} = \left[B(a) - B(b)\right] \cdot L = \mu_0 \underbrace{I_{enc}}_{=0}$$

SO $B(a) = B(b)$

FIELD DOES NOT DEPEND ON S,

BUT $s \to \infty$ $B \to 0$ $\Rightarrow$ $B = 0$ EVERYWHERE

LOOP 2 → $\frac{1}{2}$ INSIDE, $\frac{1}{2}$ OUTSIDE



AMPÈRE:

$$\int \bar{B}\cdot d\bar{\ell} = BL_{\text{INSIDE}} + BL_{\text{OUTSIDE}}$$

$$= \mu_0 I_{enc} + \mu_0 I_{enc}$$

$$\ \ \underset{nIL}{\shortparallel} \qquad\qquad \underset{0}{\shortparallel}$$

$$= \mu_0 n I L$$

$$B = \begin{cases} \mu_0 I n\ \hat{z} & \text{INSIDE} \\ 0 & \text{OUTSIDE} \end{cases}$$

FIELD IS UNIFORM

MS → SOLENOID

ES → $\parallel^{\ell}$ PLATE CAPACITOR

AMPÈRE → ANALOG OF GAUSS IN ELECTROSTATICS

ALWAYS TRUE → ONLY USEFUL IF SYMMETRY ALLOWS

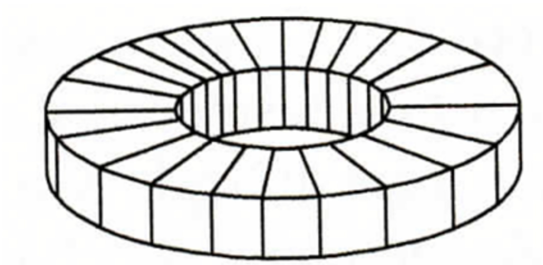
$$\oint \bar{B} \cdot d\bar{\ell} \rightarrow B \oint d\bar{\ell}$$

- INFINITE STRAIGHT LINES
- INFINITE PLANES
- INFINITE SOLENOIDS
- TOROIDS

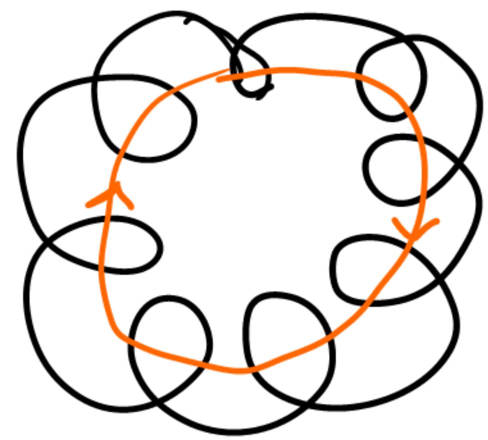
TOROID: A TOROIDAL COIL IS A CIRCULAR RING AROUND WHICH A LONG WIRE IS WRAPPED.

WINDING IS UNIFORM & TIGHT → EACH TURN IS A CLOSED LOOP

SHAPE OF COIL DOESN'T MATTER



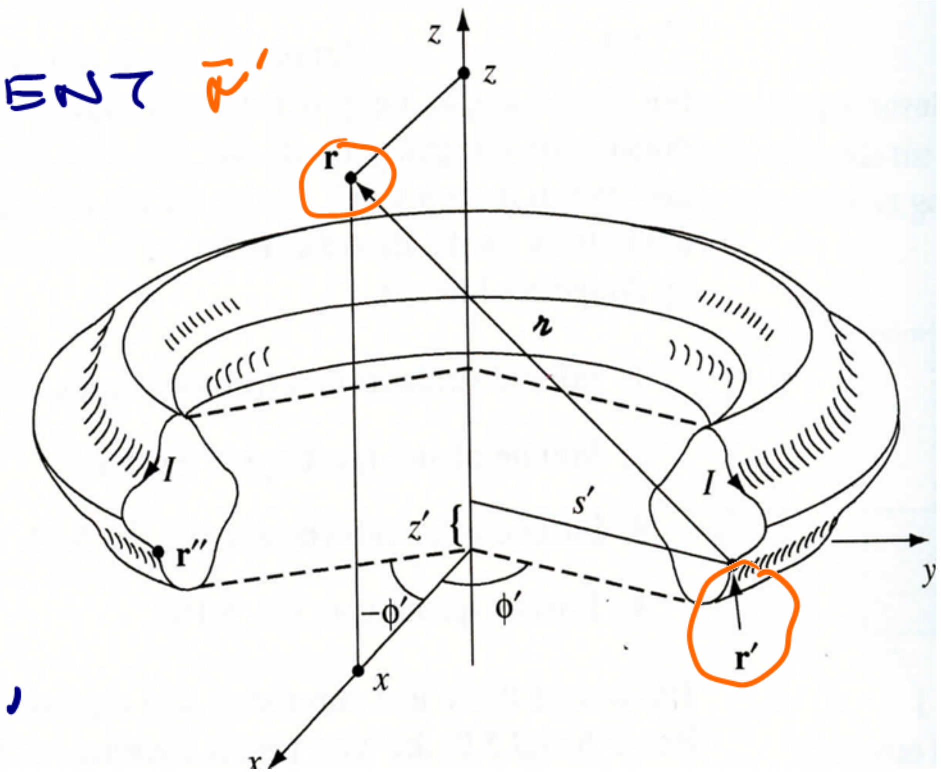
$\Rightarrow$



LOOKS LIKE A SOLENOID TURNED BACK ON ITSELF

↳ MAGNETIC FIELD IS CIRCUMFERENTIAL AT ALL POINTS INSIDE & OUTSIDE THE TOROID.

FIELD AT $\bar{r}$ DUE TO ELEMENT $\bar{r}'$



$$d\bar{B} = \frac{\mu_0}{4\pi} \frac{\bar{I} \times \bar{\mathscr{r}}}{\mathscr{r}^3} \, dl'$$

PUT $\bar{r}$ IN xy PLANE

$\Rightarrow \bar{r} = (x, 0, z)$

COORDINATES OF SOURCE $\bar{r}'$

$$\bar{r}' = (s'\cos\phi', s'\sin\phi', z')$$

$$\bar{\mathscr{r}} = (x - s'\cos\phi', -s'\sin\phi', z - z')$$

CURRENT HAS NO ϕ - COMPONENT

ASSUME INDEPENDENT LOOPS

NO ϕ COMPONENT $\quad \bar{I} = I_s \hat{s} + I_z \hat{z}$ CYLINDRICAL

IN CARTESIAN COORDS $\quad \bar{I} = (I_s \cos\phi', I_s \sin\phi', I_z)$

$$\bar{I} \times \bar{\mathscr{r}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ I_s \cos\phi' & I_s \sin\phi' & I_z \\ (x - s'\cos\phi') & (-s'\sin\phi') & (z - z') \end{vmatrix}$$

$$= \hat{x} \begin{vmatrix} I \sin\phi' & I_z \\ -s'\sin\phi' & z-z' \end{vmatrix} - \hat{y} \begin{vmatrix} I_s \cos\phi' & I_z \\ x - s'\cos\phi' & z-z' \end{vmatrix} + \hat{z} \begin{vmatrix} I\cos\phi' & I\sin\phi' \\ x - s'\cos\phi' & -s'\sin\phi' \end{vmatrix}$$

$$= \hat{x}\left(I_s \sin\phi (z - z') + I_z\, s'\sin\phi\right)$$

$$- \hat{y}\left(I_s \cos\phi' (z-z') - I_z (x - s'\cos\phi')\right)$$

$$+ \hat{z}\left(I_s \cos\phi' (-s'\sin\phi') - I_s \sin\phi' (x - s'\cos\phi')\right)$$

$$= \hat{x}\left(\sin\phi'\left(I_s(z-z')\right) + s' I_z\right)$$

$$+\hat{y}\left(I_z(x - s'\cos\phi') - I_s\cos\phi'(z-z')\right)$$

$$+\hat{z}\left(-I_s\cos\phi' \cdot s'\sin\phi' + I_s\sin\phi' \cdot s'\cos\phi' - I_s x \cdot \sin\phi'\right)$$

$$= \left[\cancel{\sin\phi'\left(I_s(z-z') + s' I_z\right)}\right]\hat{x}$$

$$+ \left[I_z(x - s'\cos\phi') - I_s\cos\phi'(z-z')\right]\hat{y}$$

$$+ \left[\cancel{-I_s x \sin\phi'}\right]\hat{z}$$

SYMMETRICAL ELEMENTS $\bar{r}' \leftrightarrow \bar{r}''$

SAME s', r, dl', I_s, I_z, ϕ' -ve

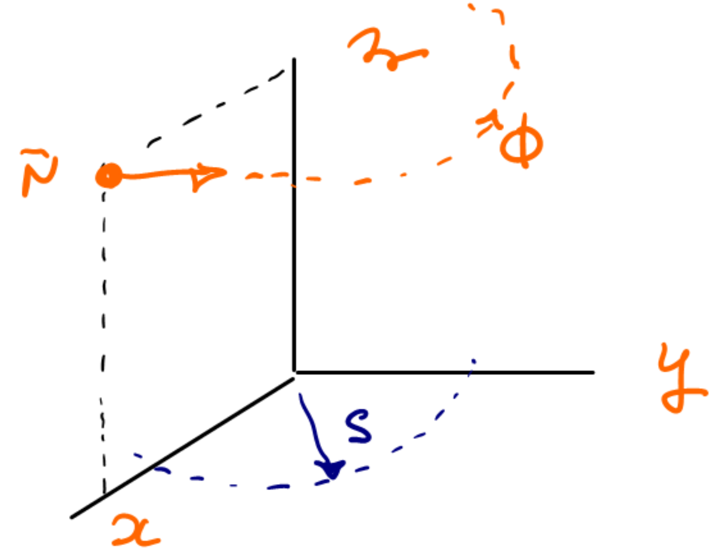
$\sin\phi' \rightarrow -\sin\phi'$ $\leftarrow$ THESE TERMS CANCEL

ONLY HAVE $\hat{y}$ TERM

SINCE PUT $\bar{r}$ on ($x-z$) PLANE

$\bar{r}$ POINTS IN y-DIRECTION

SO, IN GENERAL FIELD $\hat{\phi}$



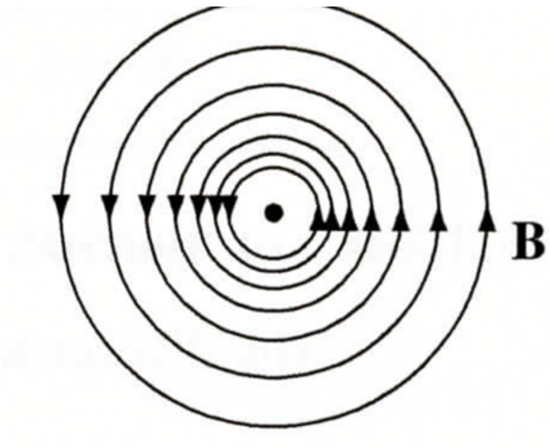
APPLY AMPÈRE'S LAW TO CIRCLE RAIDIUS S

$$B\,2\pi s = \mu_0 I_{enc} \rightarrow NI$$

$N \rightarrow$ TOTAL NUMBER OF TURNS

$$\bar{B}(\bar{r}) = \begin{cases} \dfrac{\mu_0 NI}{2\pi s}\,\hat{\phi} & \text{INSIDE TOROID} \\ \bar{0} & \text{OUTSIDE TOROID} \end{cases}$$

MAGNETOSTATICS $\Longleftrightarrow$ ELECTROSTATICS



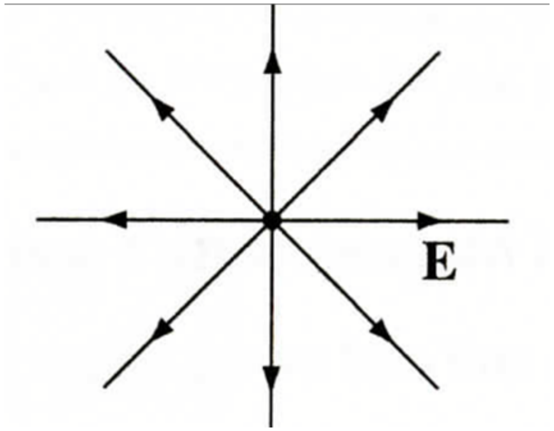
LONG WIRE

$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} \quad \text{AMPÈRE}$$

WITH $\vec{B} \to 0$ @ ∞

DETERMINE $\vec{B}$ from $\vec{J}$

BIOT – SAVARD
+
SUPERPOSITION



POINT CHARGE

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \quad \text{GAUSS}$$
$$\vec{\nabla}\times\vec{E} = \vec{0}$$

WITH $\vec{E} \to 0$ @ ∞

DETERMINE $\vec{E}$ from ρ

COULOMB + SUPERPOSITION

MAXWELL'S STATIC EQUATIONS

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = \vec{0} \qquad \vec{\nabla}\times\vec{B} = \mu_0 J$$

$$\vec{F} = Q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

CONCISE FORMULATION of ELECTROSTATICS MAGNETOSTATICS

- $\vec{E}$ DIVERGES FROM CHARGE
- $\vec{B}$ CURLS AROUND CURRENT
- $\vec{E}$ LINES START & END ON CHARGES
- $\vec{B}$ LINES CLOSED LOOPS OR $\rightarrow \infty$
  - $\hookrightarrow \vec{\nabla}\cdot\vec{B} = 0$
  - $\hookrightarrow$ NO MAGNETIC CHARGES

GENERALLY $\vec{E} \gg \vec{B}$

$\hookrightarrow$ CAN MAKE LARGE FROM LARGE CURRENTS

$\vec{E} \sim 0 \rightarrow$ WIRES NEUTRAL