ELECTRODYNAMICS

UP UNTIL NOW → ELECTROSTATICS
→ MAGNETOSTATICS

SITUATION OF CHARGES & CURRENTS MOVING AND ACCELERATING, MORE REAL & INTERESTING

→ LEADS TO PRACTICAL DEVICES
ELECTROMAGNETIC WAVES
SPECIAL RELATIVITY

→ WE HAVE TO START ON SIMPLER THINGS

ELECTROMOTIVE FORCE:

OHM'S LAW ← NOT A REAL LAW of NATURE

TO MAKE CURRENT FLOW → SOMETHING HAS TO PUSH CHARGES

HOW FAST THEY MOVE GIVEN A CERTAIN FORCE
→ DEPENDS ON MATERIAL CHARGES ARE IN:

USUALLY CURRENT DENSITY $\propto$ FORCE PER UNIT CHARGE

$$\bar{J} = \sigma \bar{f}$$

$$\rho = \frac{1}{\sigma}$$

RESISTIVITY

CONDUCTIVITY
EMPIRICAL — DEPENDS ON MATERIAL

Material	Resistivity	Material	Resistivity
Conductors:		*Semiconductors:*	
Silver	1.59×10^{-8}	Sea water	0.2
Copper	1.68×10^{-8}	Germanium	0.46
Gold	2.21×10^{-8}	Diamond	2.7
Aluminum	2.65×10^{-8}	Silicon	2500
Iron	9.61×10^{-8}	*Insulators:*	
Mercury	9.61×10^{-7}	Water (pure)	8.3×10^{3}
Nichrome	1.08×10^{-6}	Glass	$10^{9} - 10^{14}$
Manganese	1.44×10^{-6}	Rubber	$10^{13} - 10^{15}$
Graphite	1.6×10^{-5}	Teflon	$10^{22} - 10^{24}$

TABLE 7.1 Resistivities, in ohm-meters (all values are for 1 atm, 20° C). Data from *Handbook of Chemistry and Physics,* 91st ed. (Boca Raton, Fla.: CRC Press, 2010) and other references.

INSULATORS CONDUCT A LITTLE BIT

METALS CONDUCT $\sim 10^{20}$ MORE EFFICENTLY

FOR MANY PURPOSES ASSUME

$\sigma_{\text{INSULATOR}} = 0$

$\sigma_{\text{CONDUCTOR}} = \infty$

THIS IS ONLY APPROXIMATE — AS YOU KNOW FROM YOUR TOASTER

THERE REALLY ARE SOME SITUATIONS WHEN SOME METALS HAVE $\sigma = \infty$

eg NIOBIUM, LEAD $\sim$ FEW DEGREES K

SUPERCONDUCTIVITY ← LOTS OF INTERESTING EFFECTS

USUALLY FORCE DRIVING CHARGES TO PRODUCE CURRENT

↳ ELECTROMAGNETIC FORCE

$\bar{f} \to \bar{E} + \bar{v} \times \bar{B}$ ← LORENTZ

$$\bar{J} = \sigma(\bar{E} + \bar{v} \times \bar{B})$$

IGNORE IF $\bar{v}$ ~ SMALL

$$\bar{J} = \sigma \bar{E}$$

← OHM'S LAW

$\bar{E} = \bar{0}$ INSIDE CONDUCTOR ??

↳ ONLY FOR STATIC CHARGES $\bar{J} = \bar{0}$

FOR PERFECT CONDUCTORS $\bar{E} = J/\sigma = 0$

EVEN IF CURRENT IS FLOWING

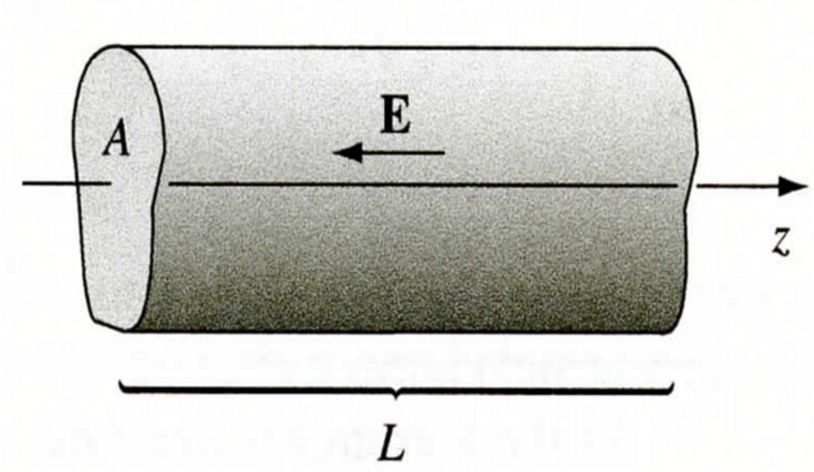
$\sigma \to \infty$, $\bar{E}$ IS VERY SMALL

WIRES IN ELECTRIC CIRCUITS → EQUIPOTENTIALS

RESISTORS  POOR CONDUCTORS

EXAMPLE: CYLINDRICAL RESISTOR X-SECT A
LENGTH L CONDUCTIVITY σ
→ CONSTANT ARBITRARY X-SECTN
FOR V BETWEEN ENDS → CURRENT?

ELECTRIC FIELD IN WIRE — UNIFORM
↳ J ALSO UNIFORM



$$I = \underbrace{J}_{\text{CURRENT DENSITY}} \overbrace{A}^{\text{AREA}}$$

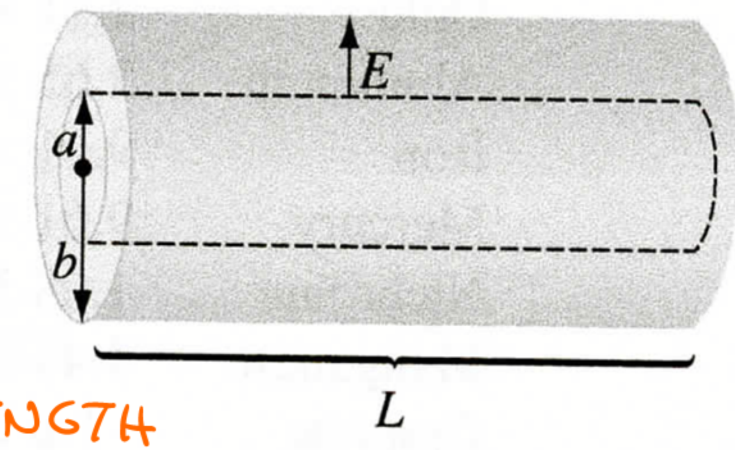
$$= \sigma E A$$

$$= \underbrace{\frac{\sigma A}{L}}_{R} \cdot V$$ ← OHM'S LAW

$I = V \cdot R$

EXAMPLE: TWO LONG CO-AXIAL METAL CYLINDERS SEPARATED BY MATERIAL CONDUCTIVITY σ

CURRENT BETWEEN CYLINDERS IN LENGTH L



FIELD BETWEEN CYLINDERS

$$\bar{E} = \frac{\lambda}{2\pi\varepsilon_0 s}\cdot\hat{s}$$

CHARGE PER UNIT LENGTH ON INNER CYLINDER

$$I = \int \bar{J}\cdot d\bar{a} = \sigma\int \bar{E}\cdot d\bar{a} = \sigma\,\underbrace{\frac{\lambda}{2\pi\varepsilon_0 s}}_{E}\cdot\hat{s}\cdot\underbrace{2\pi s L}_{\int da} = \frac{\sigma\cdot\lambda\cdot L}{\varepsilon_0}$$

POTENTIAL DIFF $\quad V = -\int_a^b \bar{E}\cdot d\bar{l} = \frac{-\lambda}{2\pi\varepsilon_0}\int_a^b \frac{ds}{s} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$

$$\frac{I}{V} = \frac{\sigma\lambda L}{\varepsilon_0}\cdot\frac{2\pi\varepsilon_0}{\lambda\ln\left(\frac{b}{a}\right)} \longrightarrow I = \frac{2\pi\sigma L}{\ln(b/a)}\cdot V$$

IN THESE TWO EXAMPLES:

$$V = \frac{L}{\sigma A} \cdot I \quad — (1)$$

$$V = \frac{\ln(b/a)}{2\pi\sigma L} \cdot I \quad — (2)$$

BOTH $V \propto \frac{1}{\sigma} \cdot I \propto \rho I \longrightarrow V = R I$

RESISIVITY RESISTANCE

R is A FUNCTION OF GEOMETRY & MATERIAL

$[R] = \text{VOLTS} \cdot (\text{AMPERE})^{-1} = \text{OHM} \rightarrow \Omega$

RECALL → FOR A STEADY CURRENT IN A WIRE CHARGE CANNOT PILEUP SOMEWHERE

CONTINUITY EQUATION $\bar{\nabla} \cdot \bar{J} = -\frac{\partial \rho}{\partial t}$ ← MUST BE ZERO

OHM'S LAW $\bar{J} = \sigma \bar{E}$

$$\bar{\nabla} \cdot \bar{J} = \sigma \bar{\nabla} \cdot E = 0 \rightarrow \bar{\nabla} \cdot \bar{E} = 0$$

$\bar{\nabla} \cdot \bar{E} = 0$ → ANY UNBALANCE CHARGE IS ON SURFACE

→ $\nabla^2 V = 0$ LAPLACE HOLDS IN HOMOGENEOUS OHMIC MATERIAL WITH A STEADY CURRENT

→ CAN UTILIZE TECHNIQUES FOR CALCULATING POTENTIAL

$\bar{E}$ FIELD IS UNIFORM IN A WIRE

WITHIN CYLINDER V OBEY'S LAPLACE

BOUNDARY CONDITIONS → ONE END $V = 0$

OTHER END $V = V_0$

ON CYLINDRICAL SURFACE $\bar{J} \cdot \hat{n} = 0$

OR ELSE CHARGE WOULD LEAK OUT

$\bar{E} \cdot \hat{n} = 0$

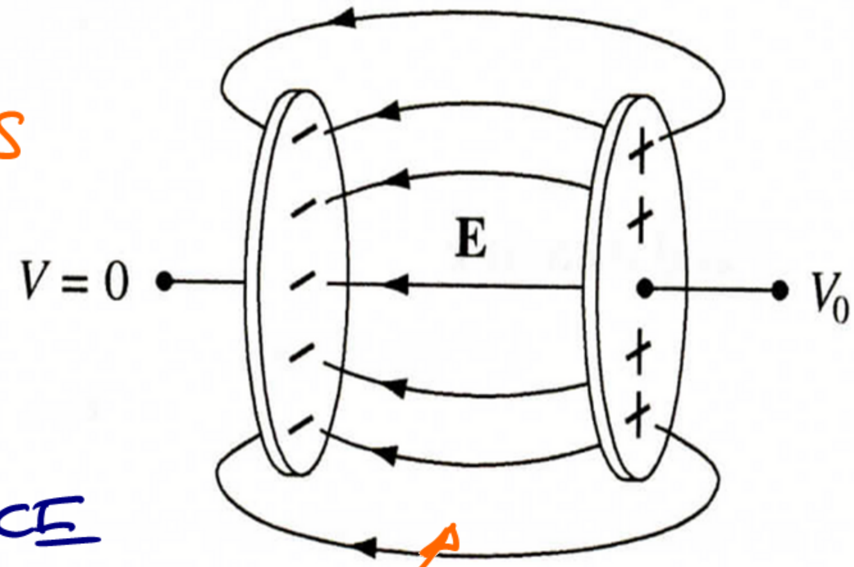
$$\frac{\partial V}{\partial n} = 0$$

ONE POTENTIAL MEETING THESE BOUNDARIES

$$V(z) = \frac{V_0 z}{L} \Rightarrow \bar{E} = -\bar{\nabla} V = -\frac{V_0}{L} \hat{z}$$

UNIFORM

IN THIS CASE FIELD SPILLS OUT → NON UNIFORM



REMOVED CONDUCTING MATERIAL

IN CONDUCTOR CHARGES ARRANGES OVER SURFACE OF WIRE IN ORDER TO HAVE UNIFORM $\bar{E}$ FIELD INSIDE.

WHY DOES OHM'S (NOT A) LAW WORK?

$\vec{E}$ → FORCE $q\vec{E}$ CAUSES ACCELERATION

NOT STEADY MOTION

↳ STEADY CURRENT

FREQUENT ATOMIC COLLISIONS

↳ EFFECTIVE −VE ACCELERATION

↳ AVERAGE VELOCITY CONSTANT.

λ → DISTANCE BETWEEN COLLISIONS ← MEAN FREE PATH

t → TIME BETWEEN COLLISIONS

$$\lambda = \frac{a}{2}t^2 \rightarrow t = \left(\frac{2\lambda}{a}\right)^{1/2}$$

AVERAGE VELOCITY $\frac{1}{2}at = \left(\frac{\lambda a}{2}\right)^{1/2}$

$$v_{AVE} = \left(\frac{\lambda a}{2}\right)^{1/2} \rightarrow \text{VELOCITY (CURRENT)} \propto \sqrt{a} \propto \vec{E}$$

CHARGES ACTUALLY MOVING WITH LARGE RANDOM THERMAL VELOCITIES

→ TIME BETWEEN COLLISIONS MUCH SHORTER THAN WOULD EXPECT FROM $\vec{E}$

$$t = \frac{\lambda}{v_{THERMAL}} \rightarrow v_{AVE} = \frac{1}{2}at = \frac{a\lambda}{2v_{THERMAL}}$$

n MOLECULES / UNIT VOLUME, f FREE ELECTRONS PER MOLECULE — MASS m, CHARGE q

$$\vec{J} = nfq\bar{v}_{av} = \frac{nfq\lambda}{v_{THERMAL}} \cdot \frac{\vec{E}}{m} = \left(\frac{nf\lambda q^2}{2m\,v_{THERMAL}}\right)\vec{E}$$

$$\bar{J} = \underbrace{\left(\frac{n f \lambda q^2}{2m\, v_{THERM}}\right)}_{\sigma} \bar{E}$$

$$I \propto E$$

$\sigma \propto$ DENSITY OF MOVING q

$$\sigma \propto \frac{1}{v_{THERM}} \propto \frac{1}{TEMP}$$

EVERY TIME THERE IS A COLLISION

ENERGY → ATOMS → THERMAL MOTION → HEAT

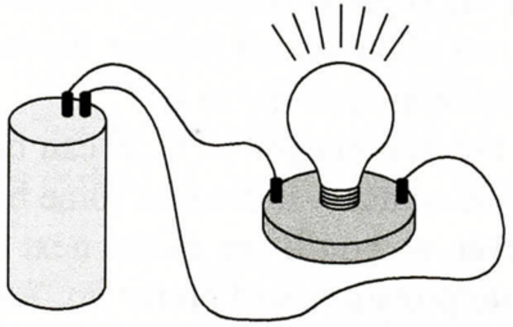
WORK DONE / UNIT CHARGE $= V$

CHARGE FLOWING / UNIT TIME $= I$

POWER $\underbrace{P = VI = I^2R}_{\text{JOULE HEATING LAW}}$

I AMPS
R OHMS $\}$ JOULES / SEC $=$ WATT

ELECTROMOTIVE FORCE:

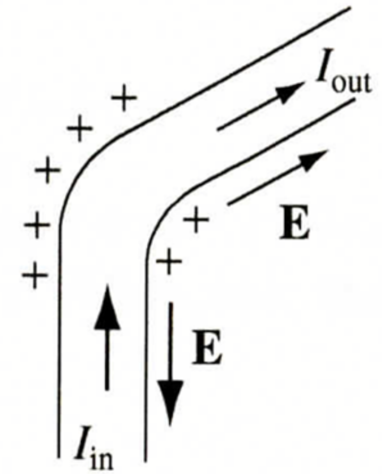


THINK ABOUT LIGHT BULB CONNECTED TO BATTERY

- WHY IS CURRENT SAME ALL AROUND CIRCUIT?
→ MIGHT EXPECT LARGEST CURRET AT BATTERY?
↳ WHAT IS PUSHING CURRENT AROUND CIRCUIT
↳ HOW DOES IT PRODUCE SAME CURRENT IN EACH SEGMENT OF LOOP?

→ CHARGES MOVE VERY SLOWLY IN WIRE — WALKING PACE
→ WHY DOES LIGHT COME INSTANTLY?
→ HOW DO ALL THE CHARGES "KNOW" TO START MOVING AT THE SAME TIME?
→ IF CURRENT WAS NOT SAME EVERYWHERE, CHARGE WOULD PILE UP SOMEWHERE → ELECTRIC FIELD OF ACCUMULATING CHARGE WOULD ACT TO EVEN OUT FLOW

SUPPOSE CURRENT INTO BEND GREATER THAN CURRENT OUT



↳ CHARGE ACCUMULATES AT BEND

↳ FIELD IS AWAY FROM BEND

↳ THIS OPPOSES CURRENT IN

PROMOTES CURRENT OUT

THIS PROCESS ACTS TO ESTABLISH EQUILIBRIUM

↳ CURRENTS IN & OUT → EQUAL

TWO FORCES ACTING:

SOURCE $\bar{f}_s$ → FOR EXAMPLE BATTERY

ELECTROSTATIC FORCE $\bar{E}$

↳ SMOOTHS OUT FLOW

→ COMMUNICATES INFLUENCE OF SOURCE TO DISTANT PARTS OF CIRCUIT.

$$\bar{f} = \bar{f}_s + \bar{E}$$

NET EFFECT IS $\oint$ OF THIS AROUND CIRCUIT.

$$\mathcal{E} \equiv \oint \bar{f} \cdot d\bar{l} = \oint \bar{f}_s \cdot d\bar{l} + \underbrace{\oint \bar{E} \cdot d\bar{l}}_{0}$$

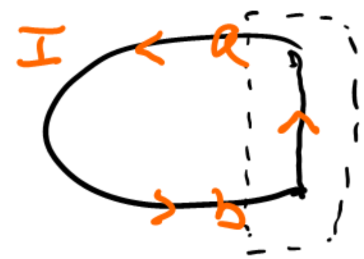
ELECTROMOTIVE FORCE

ACTUALLY NOT A FORCE — INTEGRAL OF FORCE / UNIT CHARGE

- IDEAL SOURCE OF EMF eg

NET FORCE ON CHARGES ZERO

$$\bar{E} = -\bar{f}_s$$



BATTERY
ZERO RESISTANCE

POTENTIAL BETWEEN BATTERY TERMINALS

$$V = -\int_a^b \bar{E} \cdot d\bar{l} = \int_a^b \bar{f}_s \cdot d\bar{l} = \oint \bar{f}_s \cdot d\bar{l} = \mathcal{E}$$

BATTERY ESTABLISHES $V \Rightarrow \mathcal{E}$

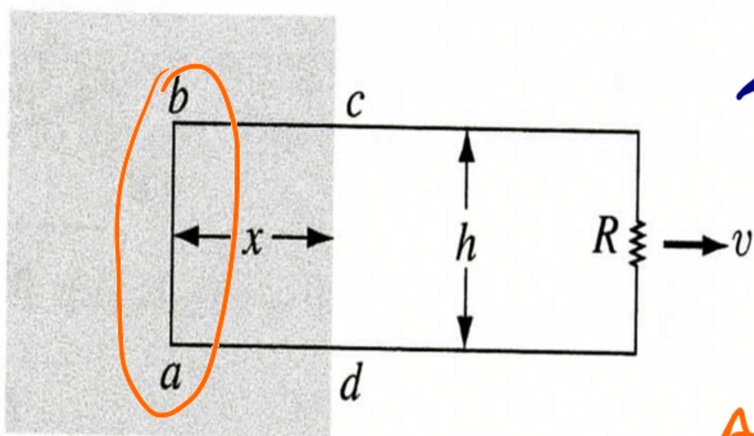
DRIVES CURRENT AROUND CIRCUIT.

MOTIONAL EMF

CAN PRODUCE EMF WITH MANY DIFFERENT DEVICES, BATTERIES THERMOCOUPLE etc . . . . .

MOST COMMON /USEFUL → GENERATOR

GENERATOR USES MOTIONAL EMF → WIRE MOVING THRU $\bar{B}$



SHADED REGION → UNIFORM $\bar{B}$ INTO PAGE

IF LOOP IS PULLED TO RIGHT AT v

CHARGES IN ab EXPERIENCE A MAGNETIC FORCE WHOSE VERTICAL COMPONENT qvB DRIVES CURRENT AROUND LOOP.

EMF $\mathcal{E} = \oint \bar{f}_{MAG} \cdot d\bar{\ell} = \underbrace{vB}_{f}\underbrace{h}_{d\ell}$

HORIZONTAL bc ad CONTRIBUTE NOTHING.

$$\mathcal{E} = \oint \bar{f}_{mag} \cdot d\bar{\ell} = vBh$$

- $\int$ CARRIED OUT AT AN INSTANT IN TIME
- $d\bar{\ell}$ FOR SEGMENT ab POINTS VERTICALLY UP
- MAGNETIC FORCE $\bar{f}_{mag}$ PRODUCES EMF, BUT DOES NO WORK → WORK DONE BY PERSON PULLING LOOP
- FREE CHARGES IN ab HAVE VERTICAL VELOCITY $\bar{u}$ ALSO HORIZONTAL VELOCITY $\bar{v}$ ← FROM MOTION OF LOOP
- MAGNETIC FORCE HAS COMP TO LEFT $quB \leftarrow$

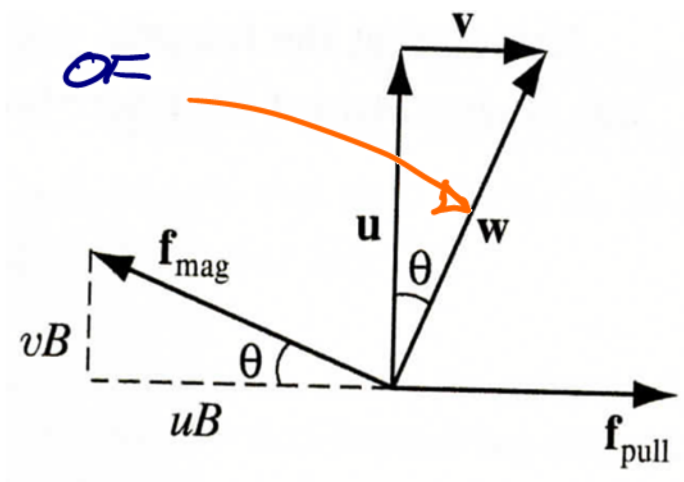
"USER" MUST EXERT FORCE TO RIGHT $\bar{f}_{pull} = uB \rightarrow$

STRUCTURE OF WIRE TRANSMITS THIS FORCE TO ALL CHARGES IN WIRE

CHARGES ACTULLY MOVE IN DIRECTION OF

RESULTANT VELOCITY $\overline{w}$

DISTANCE A CHARGE MOVES $\dfrac{h}{\cos\theta}$



WORK DONE PER UNIT CHARGE

$$\int \bar{f}_{pull} \cdot d\bar{l} = uB\left(\frac{h}{\cos\theta}\right)\sin\theta = vBh = \mathcal{E}$$

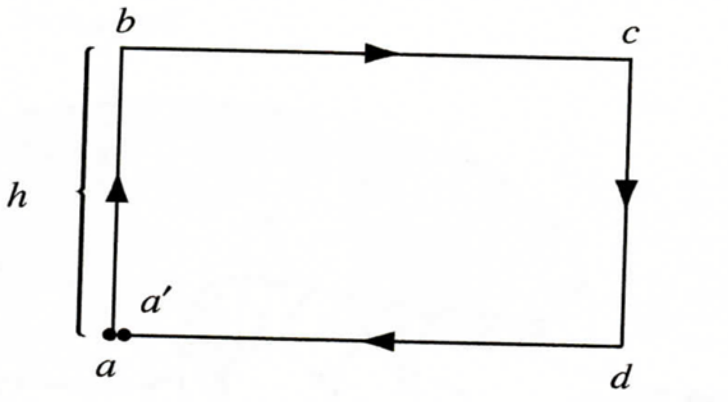
DOT PRODUCT

$$\underbrace{\text{WORK DONE PER UNIT CHARG} = \text{EMF}}$$

EVEN THO' $\int_s$ TAKEN ALONG

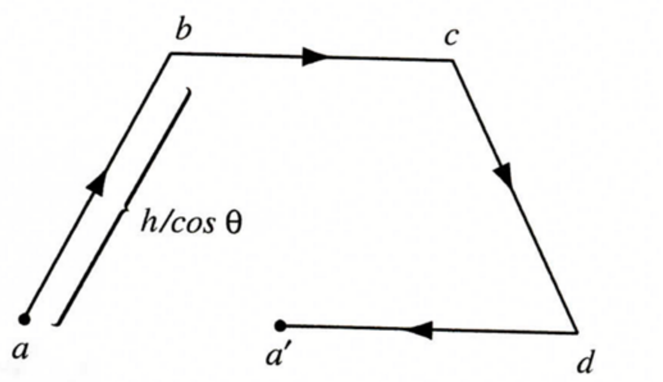
COMPLETELY DIFFERENT PATHS

AND DIFFERENT FORCES INVOLVED



(a) Integration path for computing $\mathcal{E}$ (follow the wire at one instant of time).

← EMF $\int$ AROUND LOOP AT INSTANT IN TIME



(b) Integration path for calculating work done (follow the charge around the loop).

← FOR WORK DONE $\int$ ALONG PATH OF CHARGE IN SPACE

$\bar{f}_{pull}$ CONTRIBUTES NOTHING TO EMF ← $\perp$ TO WIRE

$\bar{f}_{mag}$ CONTRIBUTES NOTHING TO WORK ← $\perp$ MOTION OF CHARGE

DEFINE FLUX THROUGH LOOP $\quad \Phi \equiv \int \vec{B} \cdot d\vec{a}$

FOR RECTANGULAR LOOP HERE $\quad \Phi = Bhx$

AS LOOP MOVES, FLUX DECREASES

$$\frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -\underbrace{Bhv}_{\text{EMF}}$$

$\frac{dx}{dt}$ -ve

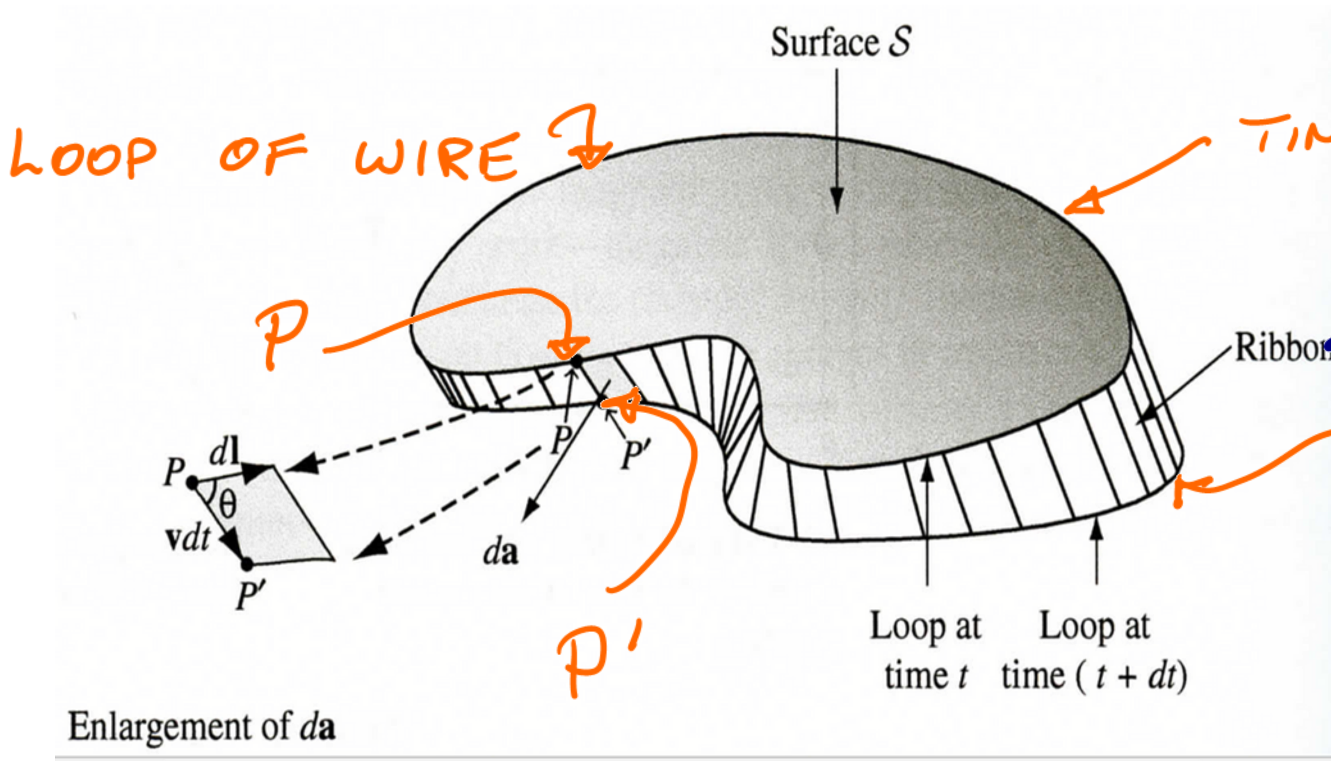
EMF = − RATE OF CHANGE OF FLUX

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

FLUX RULE FOR MOTIONAL EMF

APPLIES { NON RECTANGULAR LOOPS
MOVING IN ARBITRARY DIRECTION
THROUGH NON UNIFORM $\vec{B}$ FIELD

PROOF OF GENERALITY OF FLUX RULE



TIME t

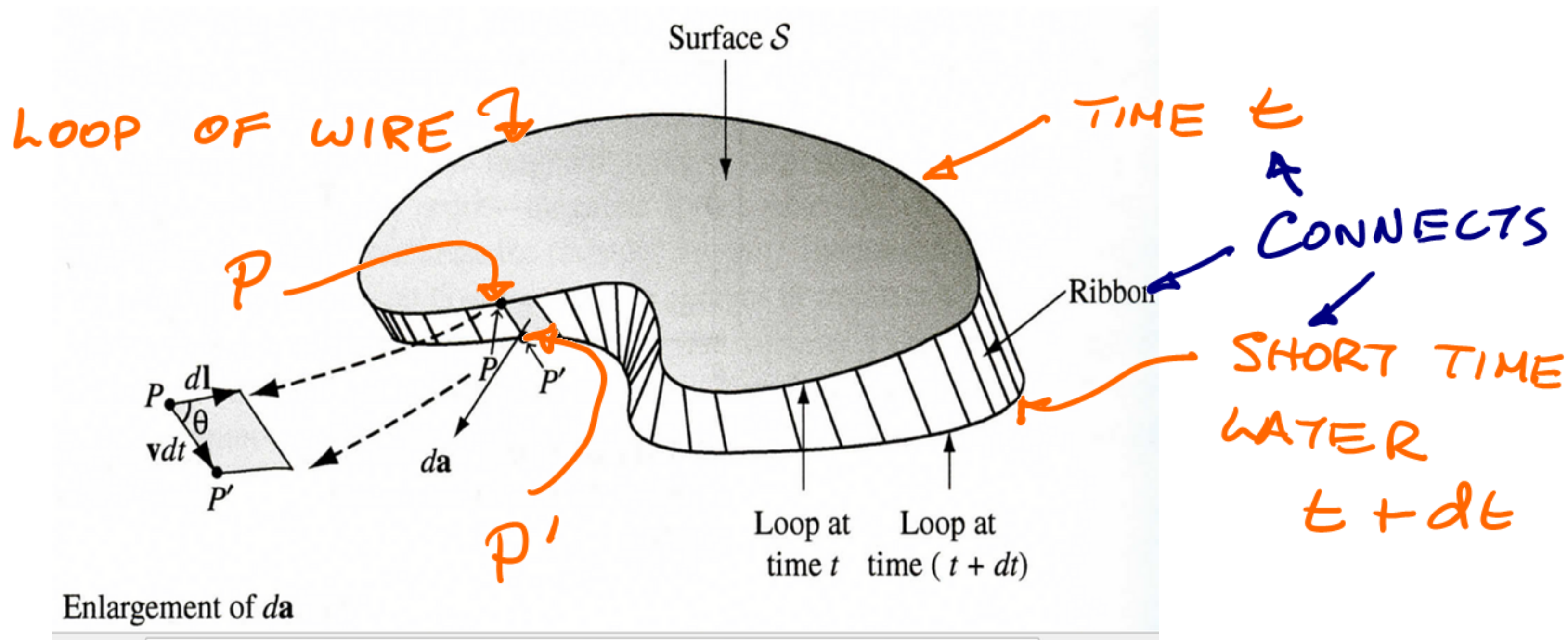
CONNECTS

SHORT TIME LATER $t+dt$

CHANGE IN FLUX

$$d\Phi = \Phi(t+dt) - \Phi(t)$$

$$= \Phi_{\text{RIBBON}} = \int_{\text{RIBBON}} \vec{B}\cdot d\vec{a}$$



IN TIME dt POINT $P \rightarrow P'$

VELOCITY OF WIRE $\bar{v}$, VELOCITY OF CHARGE $\bar{u}$

RESULTANT $\bar{w} = \bar{v} + \bar{u}$

INFITESIMAL AREA ON RIBBON $d\bar{a} = (\bar{v} \times d\bar{l})\, dt$

$$\frac{d\bar{\Phi}}{dt} = \oint \bar{B} \cdot (\bar{v} \times d\bar{l})$$

$\bar{w} = \bar{v} + \bar{u}$ AND $\bar{u}$ PARLLEL TO $d\bar{l}$ $\rightarrow$ $\frac{d\bar{\Phi}}{dt} = \oint \bar{B} \cdot (\bar{w} \times d\bar{l})$

$$\frac{d\bar{\Phi}}{dt} = \oint \bar{B} \cdot (\bar{\omega} \times d\bar{\ell})$$

$$\bar{B} \cdot (\bar{\omega} \times d\bar{\ell}) = -(\bar{\omega} \times \bar{B}) \cdot d\bar{\ell}$$

$$\frac{d\bar{\Phi}}{dt} = -\oint \underbrace{(\bar{\omega} \times \bar{B})}_{\text{MAGNETIC FORCE PER UNIT CHARGE}} \cdot d\bar{\ell}$$

$$\frac{d\bar{\Phi}}{dt} = -\underbrace{\int \bar{f}_{MAG} \cdot d\bar{\ell}}_{\text{EMF}}$$

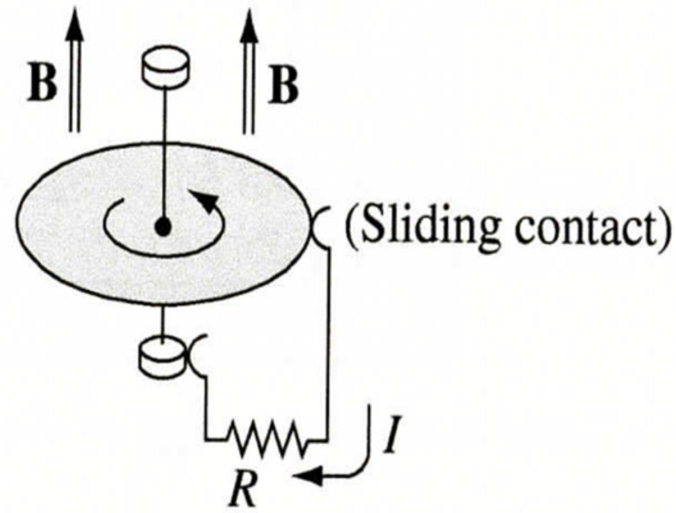
$$\mathcal{E} = -\frac{d\bar{\Phi}}{dt}$$

USE RHR TO GET DIRECTION OF CURRENT

ONLY MOTIONAL $\mathcal{E}$

EXAMPLE: ROTATING METAL DISK

ω ABOUT VERTICAL AXIS



AT A DISTANCE s FROM AXIS

$$v = \omega s$$

FORCE PER UNIT CHARGE

$$\vec{f}_{mag} = \vec{v} \times \vec{B} = \omega s B \hat{s}$$

$$\text{EMF} = \int_0^a f_{mag}\, ds = \omega B \int_0^a s\, ds = \frac{\omega B a^2}{2} = \mathcal{E}$$

$$I = \frac{\mathcal{E}}{R} = \frac{\omega a^2 B}{2R}$$

RESISTOR

CAN'T CALCULATE EMF FROM FLUX RULE

FLUX RULE ASSUMES DEFINITE PATH FOR CURRENT

→ NOT TRUE HERE

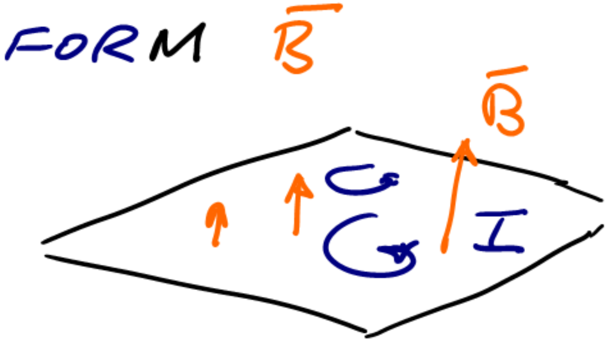
→ WHAT WOULD FLUX THRU CIRCUIT MEAN?

EDDY CURRENTS

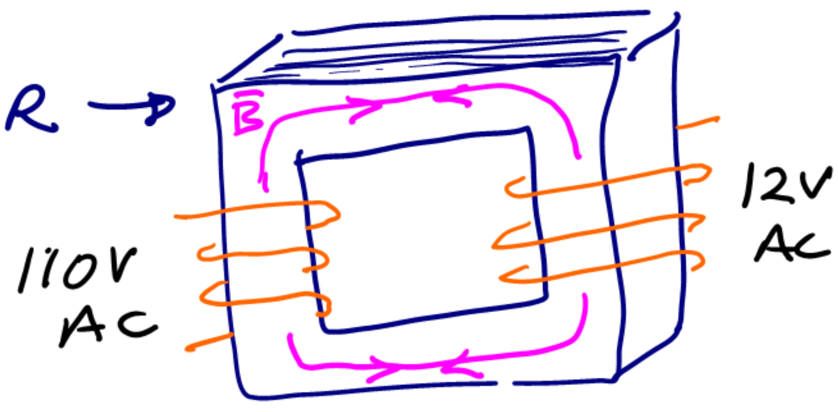
MOVE CONDUCTING SHEET IN NON UNIFORM $\bar{B}$

→ CURRENTS INDUCED

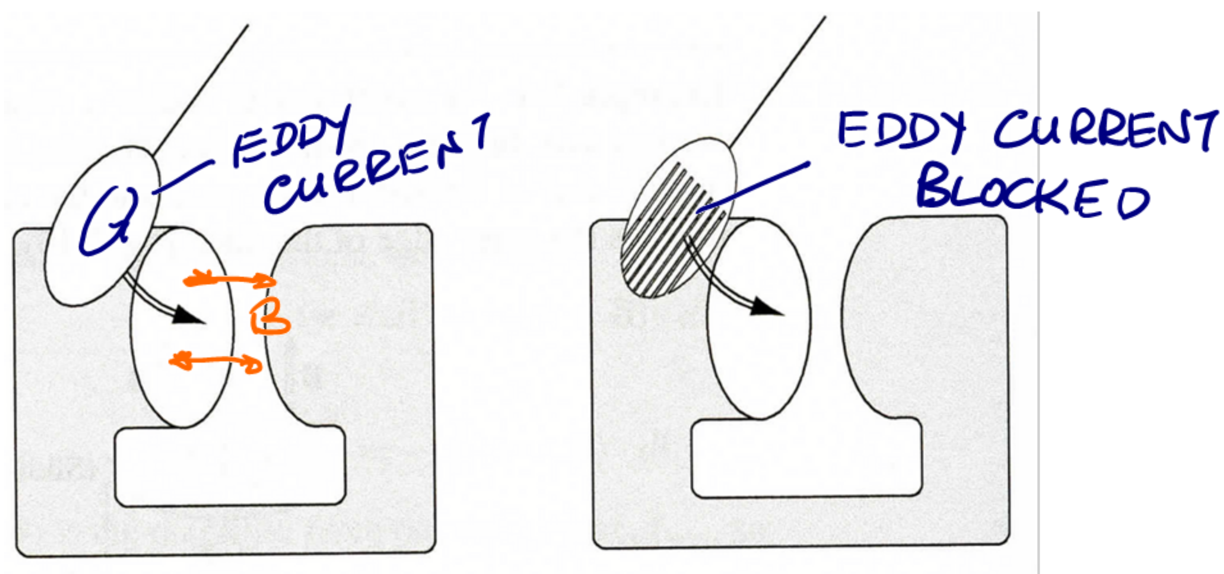
→ FEEL GENERATION OF EMF AS RESISTANCE TO MOTION



TRANSFORMER →



LAMINATED TO REDUCE EDDY CURRENTS



(a) SLOW

(b) FAST