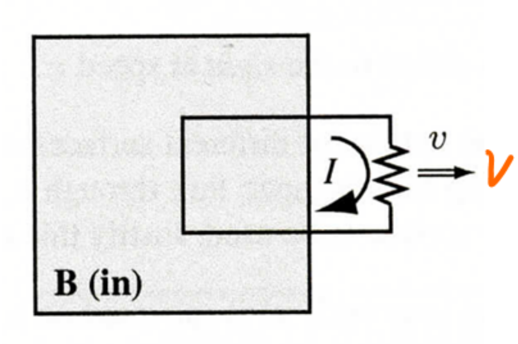
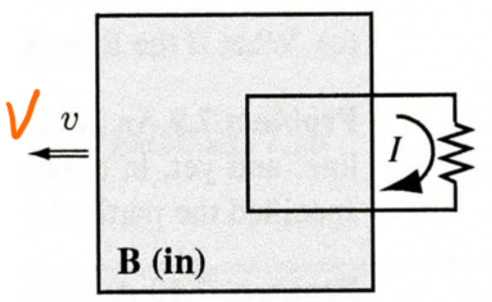
ELECTROMAGNETIC INDUCTION

FARADAYS EXPERIMENTS → 1831

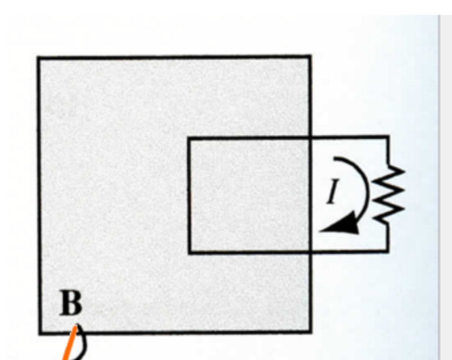


(a)

PULLED LOOP OF WIRE THROUGH MAGNETIC FIELD → CURRENT FLOWS



MOVED MAGNET TO RIGHT, HOLDING LOOP STATIONARY → CURRENT FLOWS



(c)

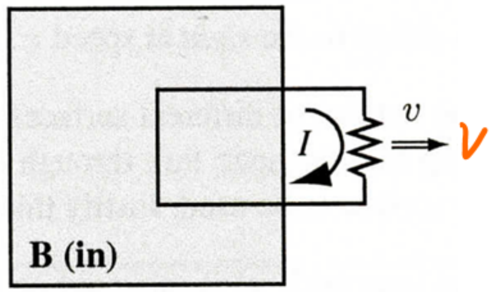
CHANGING $\vec{B}$ FIELD

LOOP & MAGNET AT REST

VARY STRENGTH OF $\vec{B}$

CURRNT FLOWS

MOTIONAL EMF $\quad \mathcal{E} = -\dfrac{d\Phi}{dt}$



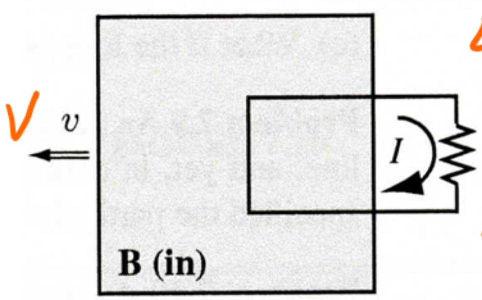
(a)

IF LOOP MOVES → CHARGES MOVE → $\bar{f}_{MAG}$

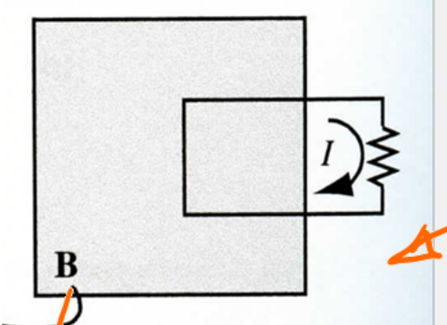
IF LOOP STATIONARY → CHARGES STATIONARY

STATIONARY CHARGE → NO $\bar{f}_{mag}$

ELECTRIC FIELD CAN MOVE STATIONARY CHARGES



CHANGING MAGNETIC FIELD INDUCES AN ELECTRIC FIELD



(c)

CHANGING $\bar{B}$ FIELD

INDUCED ELECTRIC FIELD PRODUCES EMF

$$\mathcal{E} = \oint \bar{E} \cdot d\bar{\ell} = -\frac{d\Phi}{dt}$$

$$\oint \bar{E} \cdot d\bar{\ell} = -\int \frac{\partial \bar{B}}{\partial t} \cdot d\bar{a}$$

IF $\frac{\partial \bar{B}}{\partial t} = 0$

$$\oint \bar{E} \cdot d\bar{\ell} = 0$$

ELECTROSTATICS

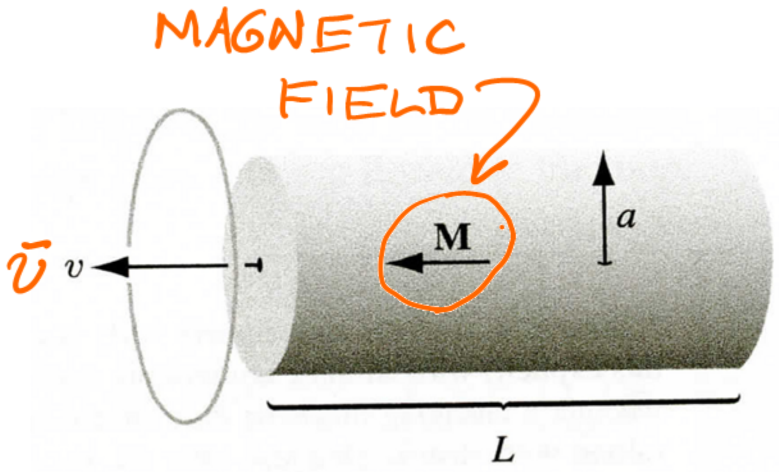
THIS IS FARADAY'S LAW

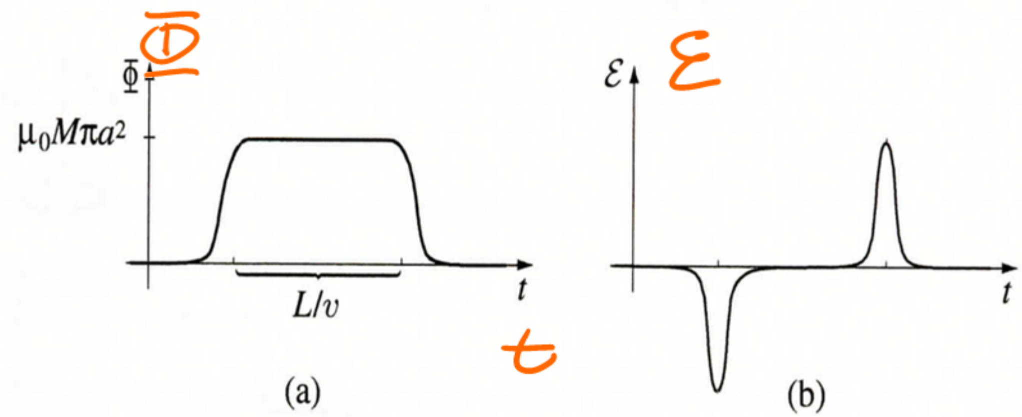
AS USUAL CAN USE STOKES

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

WHEN EVER A MAGNETIC FLUX THROUGH A LOOP CHANGES $\Big\}$ $\mathcal{E} = -\frac{d\Phi}{dt}$

EXAMPLE: CYLINDRICAL MAGNET PASSES THRU WIRE LOOP AT CONSTANT VELOCITY





MAGETIC FIELD IS SAME AS LONG SOLENOID $\bar{K}_b = M\hat{\phi}$

SURFACE CURRENT

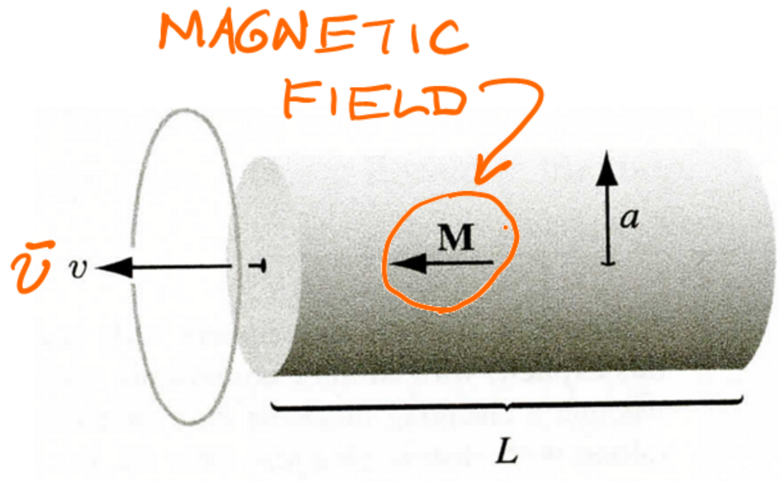
FIELD INSIDE $\bar{B} = \mu_0 \bar{M}$ SPREADS OUT AT ENDS

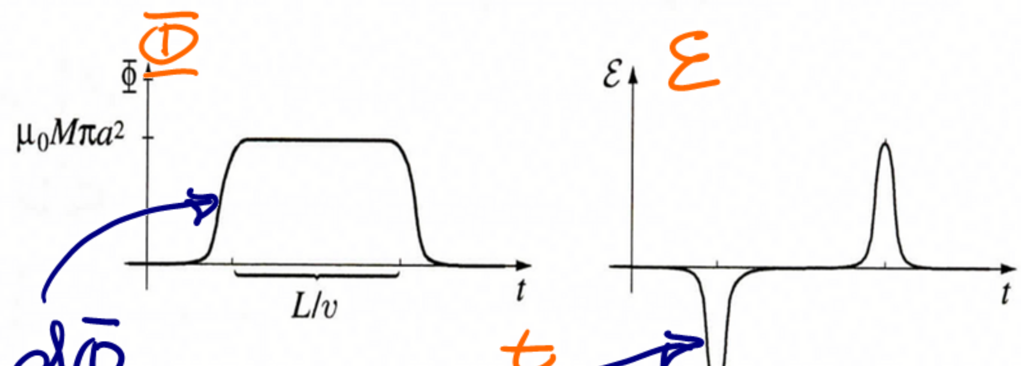
FLUX THRU RING ZERO $\rightarrow$ MAGNET FAR AWAY

BUILDS UP TO $\mu_0 \bar{M} \pi a^2$ AS MAGNET ENTES

STAYS CONSTANT

DROPS BACK TO ZERO AS MAGNET PASSES THRU





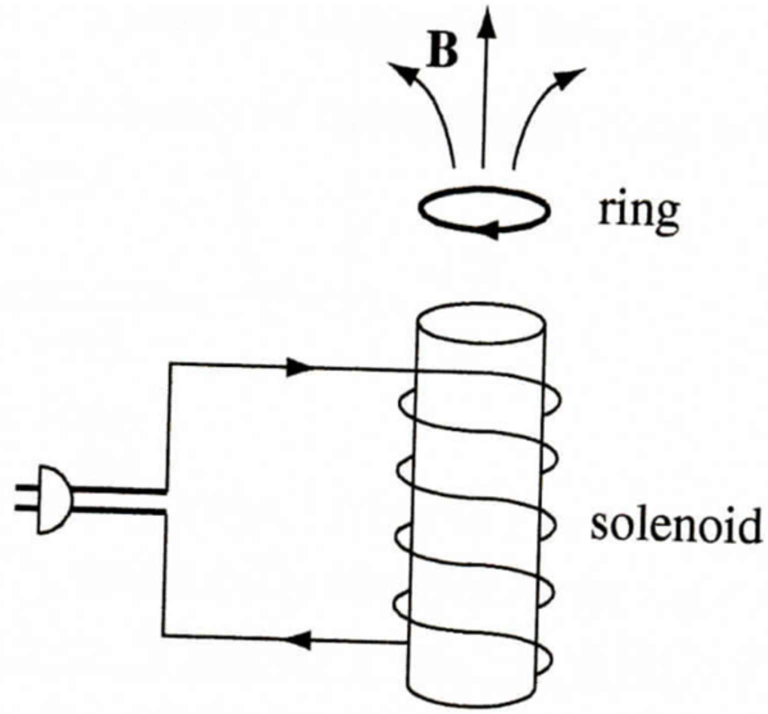
(a) (b)

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

INDUCED CURRENT FLOWS IN DIRECTION TO OPPOSE CHANGE IN FLUX

↳ CURRENT CHANGES IN ORDER TO KEEP FLUX CONSTANT

LENZ'S LAW



BEFORE CURRENT SWITCH ON → FLUX IN LOOP → ZERO

TURN ON CURRENT → FLUX THRU LOOP INDUCES $\mathcal{E}$

EMF IN DIRECTION THAT ITS FIELD IS OPPOSITE TO FIELD PRODUCED BY SOLENOID

CURRENT IN LOOP OPPOSITE CURRENT IN SOLENOID

OPPOSITE CURRENTS REPEL

THE INDUCED ELECTRIC FIELD

FARADAY'S LAW GENERALIZES THE ELECTROSTATIC RELATION $\bar{\nabla} \times \bar{E} = \bar{0}$ → TIME DEPENDENT REGIME

$$\bar{\nabla} \cdot \bar{E} = \rho/\varepsilon_0$$ ← STILL HOLDS

IF $\bar{E}$ IS PURELY DUE TO CHANGING $\bar{B}$

$$\bar{\nabla} \cdot \bar{E} = 0 \quad , \quad \bar{\nabla} \times \bar{E} = -\partial \bar{B}/\partial t$$

THIS IS MATHEMATICALLY IDENTICAL TO:

$$\bar{\nabla} \cdot \bar{B} = 0 \quad , \quad \bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$$

FARADAY INDUCED $\bar{E}$ DETERMINED BY $-\partial \bar{B}/\partial t$ IN EXACTLY SAME WAY THAT $\bar{B}$ IS INDUCED BY $\bar{J}$

FARADAY INDUCED $\bar{E}$ DETERMINED BY $-\partial\bar{B}/\partial t$

IN EXACTLY SAME WAY THAT $\bar{B}$ IS INDUCED

BY $\bar{J}$

ANALOG OF BIOT–SAVART $(\bar{J} \to \bar{B})$

$$\bar{E} = -\frac{1}{4\pi}\int \frac{(\partial\bar{B}/\partial t)\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau = -\frac{1}{4\pi}\frac{\partial}{\partial t}\int \frac{\bar{B}\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,d\tau$$

IN SYMMETRICAL SITUATIONS CAN USE ALL

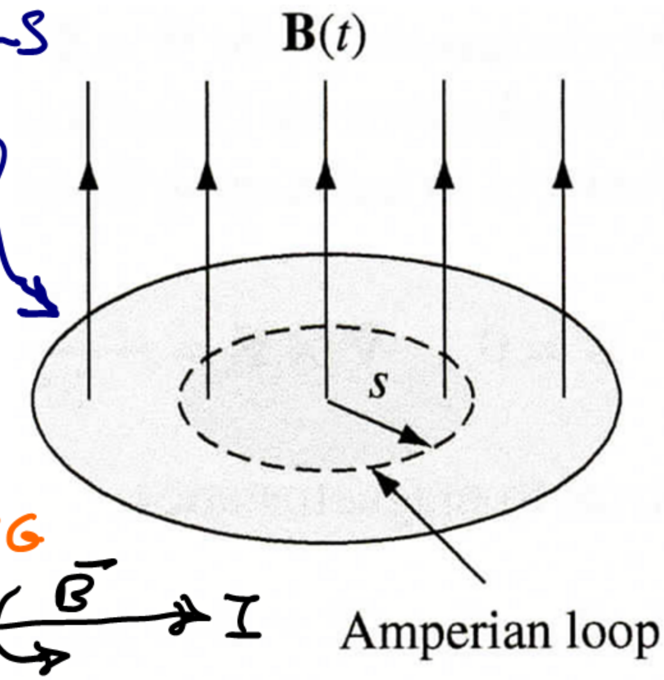
METHODS OF AMPÈRE $\oint \bar{B}\cdot d\bar{l} = \mu_0 I_{enc}$

BUT NOW FARADAY'S LAW

$$\oint \bar{E}\cdot d\bar{l} = -\frac{d\Phi}{dt}$$

THEN AN AMPERIAN LOOP PLAYS ROLE OF $\mu_0 I_{enc}$ IN AMPERE

EXAMPLE: UNIFORM $\bar{B}$ ↑ UP FILLS CIRCULAR SHADED REGION



$\bar{B}$ CHANGES IN TIME → INDUCED $\bar{E}$?

$\bar{E}$ POINTS IN CIRCUMFERENTIAL

↳ JUST LIKE $\bar{B}$ INSIDE LONG STRAIGHT WIRE CARRYING I

APPLY FARADAY AROUND AMPERIAN LOOP

$$\oint \bar{E} \cdot d\bar{\ell} = E(2\pi s) = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(\pi s^2 B(t)\right)$$

$$= -\pi s^2 \frac{dB}{dt}$$

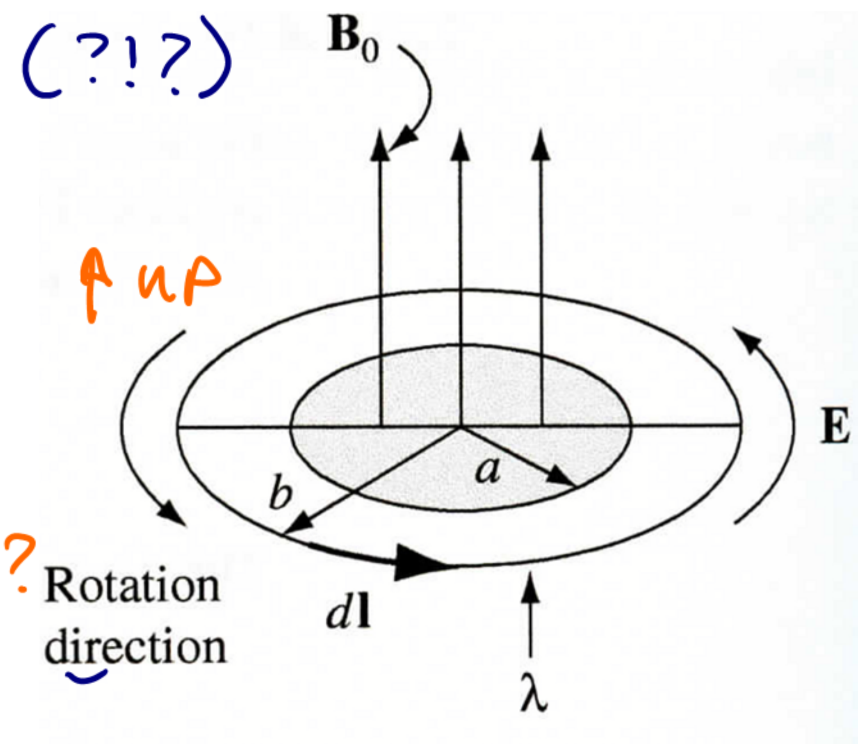
$$\bar{E} = -\frac{s}{2}\frac{dB}{dt}\hat{\phi}$$

IF $\bar{B}$ INCREASING, $\bar{E}$ CLOCKWISE — RHR

EXAMPLE: LINE CHARGE λ GLUED (?!?) ONTO RIM OF WHEEL RADIUS b OUT TO RADIUS a UNIFORM $\vec{B}_0$ ↑ UP

WHEEL FREE TO ROTATE.

$\vec{B}_0$ TURNED OFF → WHAT HAPPENS?



CHANGING B INDUCES $\vec{E}$ CURLING AROUND AXIS OF WHEEL

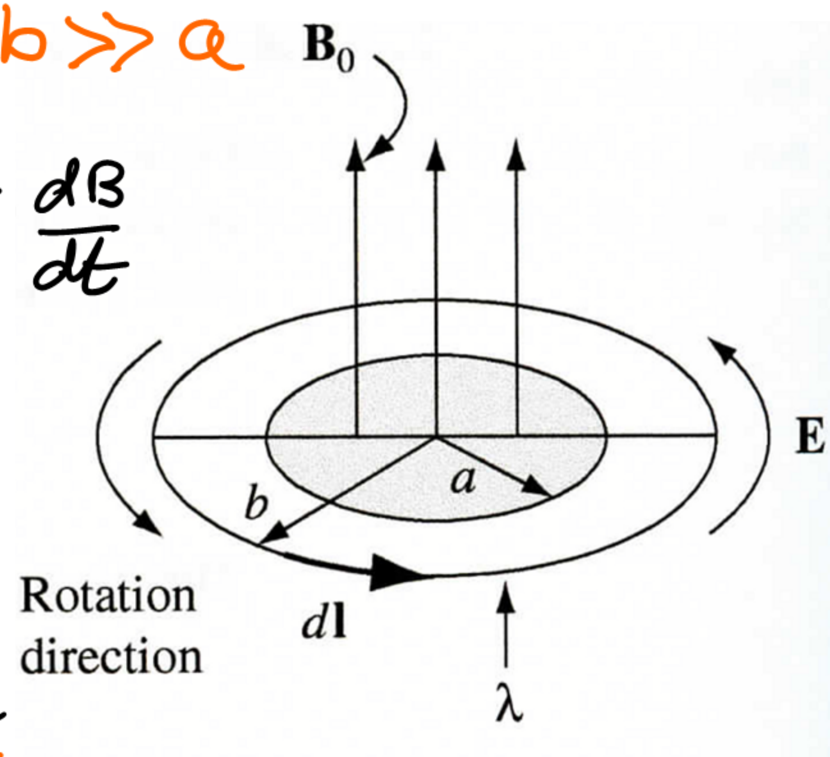
EXERTS FORCE ON CHARGES AT RIM

↳ WHEEL STARTS TO TURN

LENZ → ROTATION IS IN DIRECTION TO RESTORE $\vec{B}$

↳ COUNTER CLOCKWISE SEEN FROM ABOVE ← RHR

FARADAY APPLIED TO LOOP RADIUS $b \gg a$



$$\oint \vec{E} \cdot d\vec{l} = E(2\pi b) = -\frac{d\Phi}{dt} = -\pi a^2 \frac{dB}{dt}$$

$$\vec{E} = -\frac{a^2}{2b} \frac{dB}{dt} \hat{\phi}$$

TORQUE ON SEGMENT $d\vec{l}$ $(\vec{r} \times \vec{F})$

$$= \underset{\vec{r}}{b} \, \underbrace{\lambda E \, dl}_{F}$$

TOTAL TORQUE $N = b\lambda \left(-\frac{a^2}{2b} \right) \oint dl = -b\lambda \pi a^2 \frac{dB}{dt}$

ANGULAR MOMENTUM IMPARTED = TORQUE × TIME IT ACTS

$$\rightarrow = \int N \, dt = -\lambda \pi a^2 b \int \frac{dB}{dt} \cdot dt = -\lambda \pi a^2 b \int_{B_0}^{0} dB$$

$$= \int N\,dt = -\lambda \pi a^2 b \int \frac{dB}{dt} \cdot dt = -\lambda \pi a^2 b \int_{B_0}^{0} dB$$

$$\text{ANG MOM} = -\lambda \pi a^2 b B_0$$

INDEPENDENT OF RATE B_0 CHANGES

ELECTRIC FIELD DOES THE ROTATING

$\vec{B}$ = ZERO AT AMPERIAN LOOP RADIUS b

SWITCH OFF $\vec{B}$ $\longrightarrow$ $\vec{E}$ APPEARS

& DOES ROTATING.

QUASISTATIC APPROXIMATION

APPROXIMATION IN APPLICATION OF FARADAY'S LAW

→ ELECTROMAGNETIC INDUCTION ONLY
HAPPENS WHEN $\vec{B}$ CHANGING

→ BUT WOULD LIKE TO USE ELECTROSTATIC RELATIONS
↳ AMPÈRE, BIOT-SAVART

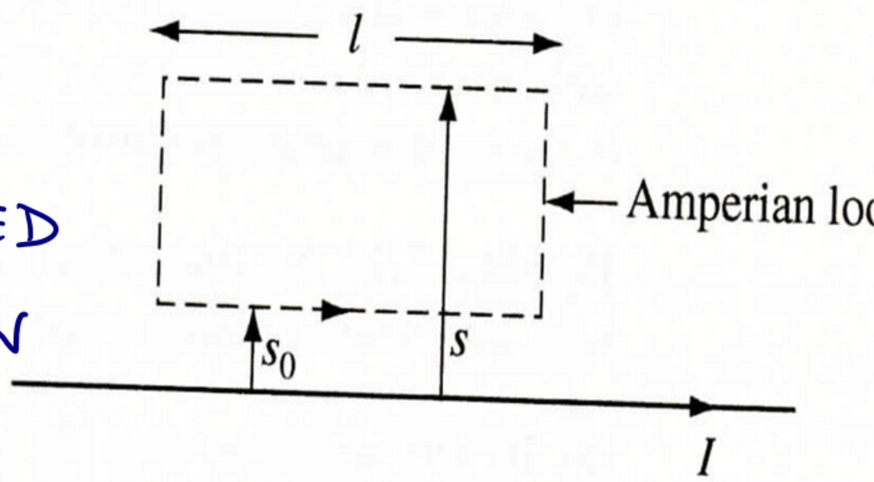
→ WORKS IF $\frac{dB}{dt}$ NOT TOO LARGE

→ OR NOT TOO FAR FROM SOURCE

BREAKS DOWN FOR ELECTROMAGETIC RADIATION

→ OR SOURCE IS $\sim ct$ AWAY
↳ VELOCITY OF LIGHT

EXAMPLE: ∞ly LONG STRAIGHT WIRE CARRIES A SLOWLY VARYING CURR $I(t)$. DETERMINE INDUCED ELECTRIC FIELD AS A FUNCTION OF DISTANCE s FROM WIRE



QUASISTATIC APPROXIMATION

$\bar{B} = \mu_0 I / 2\pi s$

LIKE $\bar{B}$ FOR SOLENOID, $\bar{E}$ RUNS $\parallel^{el}$ TO AXIS

APPLYING FARADAY AROUND AMPERIAN LOOP

$$\oint \bar{E}\cdot d\bar{l} = E(s_0) - E(s) = -\frac{d}{dt}\int \bar{B}\cdot d\bar{a}$$

VERTICAL ARMS CONTRIBUTE NOTHING

$$= -\frac{\mu_0 l}{2\pi}\frac{dI}{dt}\int_{s_0}^{s}\frac{1}{s'}\,ds'$$

$$= -\frac{\mu_0 I}{2\pi}\frac{dI}{dt}\left(\ln s - \ln s_0\right)$$

$$= -\frac{\mu_0 I}{2\pi}\frac{dI}{dt}\left(\ln s - \ln s_0\right)$$

$$\bar{E}(s) = \left[\frac{\mu_0}{2\pi}\frac{dI}{dt}\ln s + K\right]\hat{z}$$

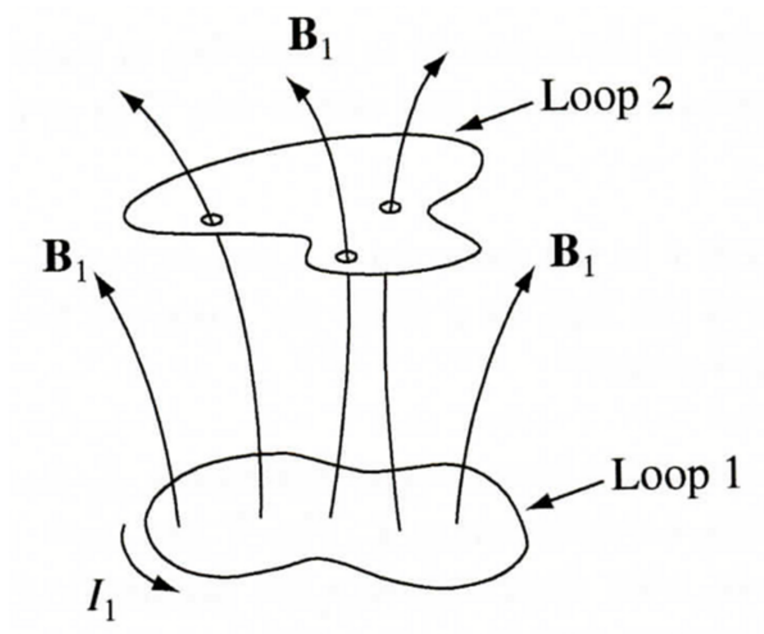
DIVERGES AS $S \to \infty$

$\bar{B}$ AT SOME LARGE DISTANCE DOES NOT DEPEND ON CURRENT NOW → BUT ON CURRENT AT SOME EARIER TIME

EFFECT OF CURRENT PROPAGATES AT VELOCITY OF LIGHT → C

IF τ is CHARACTERISTIC TIME FOR I TO CHANCE → THEN QUASISTATIC APROXIMATION ONLY HOLDS FOR $S \ll c\tau$

INDUCTANCE



TWO LOOPS OF WIRE, STEADY CURRENT I_1 IN LOOP 1 PRODUCES MAGNETIC FIELD $\bar{B}_1$

SOME OF THIS FIELD PASSES THRU LOOP 2

Φ_2 IS FLUX OF $\bar{B}_1$ THRU LOOP 2

$\bar{B}_1$ GENERALLY DIFFICULT TO CALCULATE

HOWEVER – BIOT-SAVART $\bar{B}_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\bar{l}_1 \times \hat{r}}{r^2}$

$\bar{B}_1 \propto I_1 \implies$ AND $\Phi_2 \propto I_1$

$$\Phi_2 \propto I_1$$

$$\Phi_2 = \int \bar{B}_1 \cdot d\bar{a}_2$$

MUTUAL INDUCTANCE

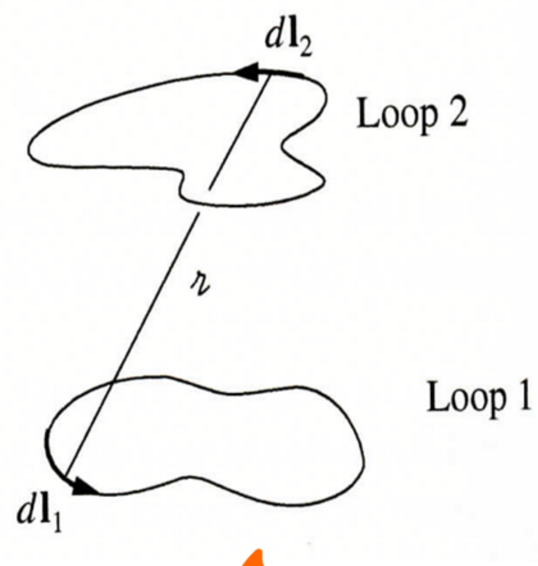
$$\Phi_2 = M_{21} \, I_1$$

$$\Phi_2 = \int \bar{B}_1 \, d\bar{a}_2 = \int (\bar{\nabla} \times \bar{A}_1) \, da_2$$

BUT →

$$\underbrace{\int (\bar{\nabla} \times \bar{A}_1) \, d\bar{a}_2 = \oint \bar{A}_1 \, d\bar{\ell}_2}_{\text{STOKES}}$$

$$\bar{A}_1 = \frac{\mu_0}{4\pi} \bar{I}_1 \int \frac{d\ell_1}{r} \rightarrow \Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\bar{\ell}_1}{r} \right) d\bar{\ell}_2$$

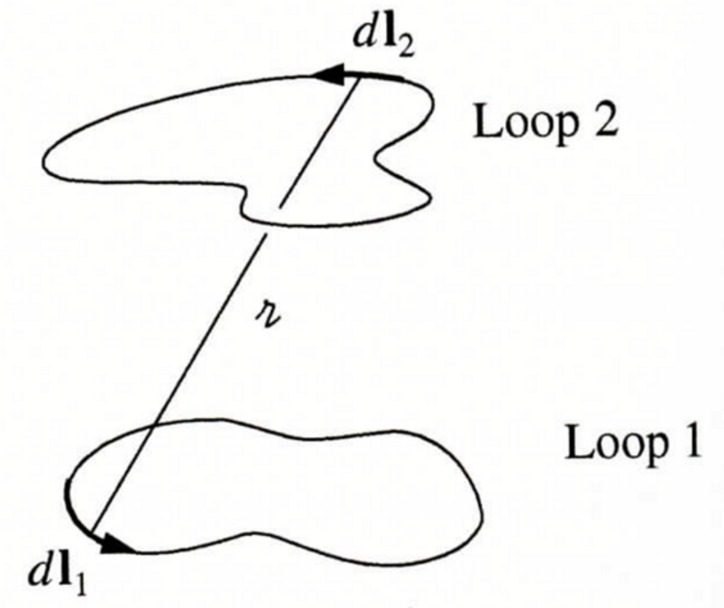


DOUBLE $\oint$

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\bar{\ell}_1 \cdot d\bar{\ell}_2}{r}$$

NEUMANN FORMULA

$$M_{21} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\bar{l}_1 \cdot d\bar{l}_2}{r}$$



$M_{21} \rightarrow$ PURELY GEOMETRICAL

UNCHANGED IF SWITCH LOOPS 1 & 2

$M_{21} = M_{12}$

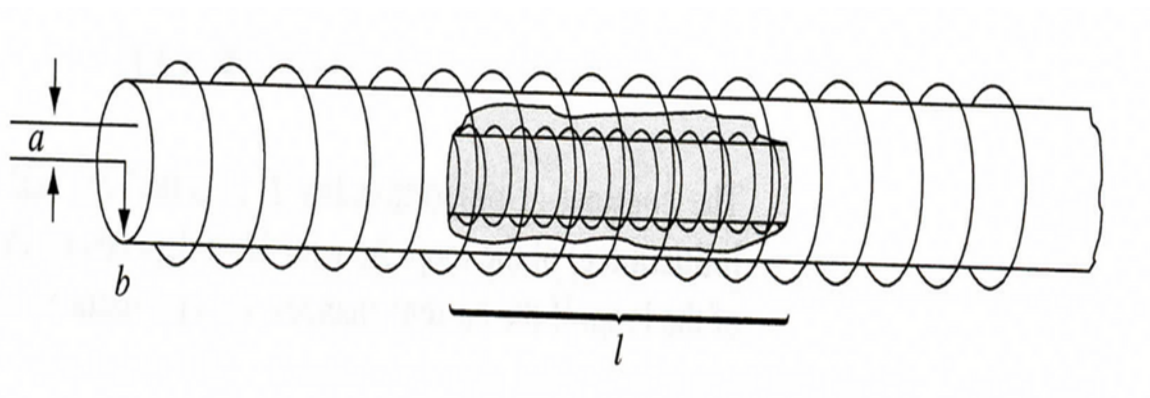
WHATEVER SHAPES & POSITIONS OF LOOPS

FLUX THRU 2 FOR CURRENT I IN LOOP 1

IS IDENTICAL TO FLUX THRU 1 FOR I IN LOOP 2

$M_{21} = M_{12} \Rightarrow = M$

PAUSE FOR EXAMPLE:



SHORT SOLENOID, LENGTH l RADIUS a, n_1 TURNS PER UNIT LENGTH. ON AXIS OF VERY LONG SOLENOID, RADIUS b, n_2 TURNS PER UNIT LENGTH

– INNER SOLENOID SHORT — COMPLEX FIELD, & PUTS DIFFERENT FLUX THRU EACH TURN OF LONG SOLENOID ⟶ HORRIBLE CALCULATION

⟹ USE PROPERTY OF MUTUAL INDUCTANCE

PUT I THRU LONG SOLENOID

INSIDE LONG SOLENOID → FIELD CONSTANT

$$= B = \mu_0 n_2 I$$

SO FLUX THRU SINGLE LOOP OF SHORT SOLENOID IS:

$$B\pi a^2 = \mu_0 n_2 I \pi a^2$$

NO. OF TURNS IN SHORT SOLENOID ℓn_1

SO FLUX THRU SHORT SOLENOID

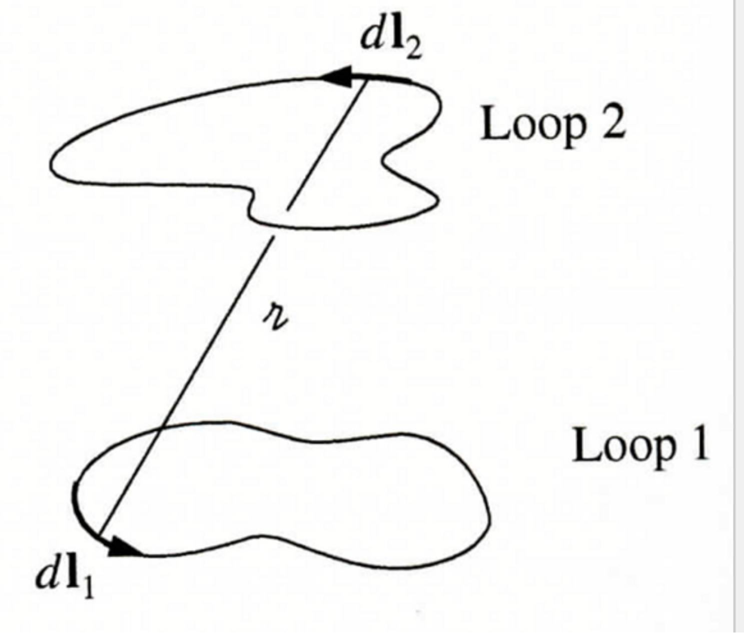
$$\Phi = \mu_0 \pi a^2 n_1 n_2 I \cdot \ell$$

THIS IS ALSO FLUX A CURRENT I IN SHORT SOLENOID WOULD PRODUCE IN LONG SOLENOID

→ THIS IS WHAT WE SET OUT TO CALCULATE

AND → $$M = \mu_0 \pi a^2 n_1 n_2 \ell$$ ← MUTUAL INDUCTANCE

SELF INDUCTANCE



GOING BACK TO 2 WIRE LOOPS

— VARY CURRENT IN LOOP1

FLUX THRU LOOP2 WILL VARY

FARADAY SAYS THIS FLUX VARYING IN LOOP2 → INDUCES AN EMF IN LOOP2

$$\mathcal{E}_2 = -\frac{d\Phi_2}{dt} = -M\frac{dI_1}{dt}$$

→ EVERY TIME YOU CHANGE CURRENT IN LOOP1

↳ A CURRENT FLOWS IN LOOP2

↳ INFLUENCE THRU EMPTY SPACE

↳ BUT ......

CHANGING CURRENT IN LOOP1 NOT ONLY INDUCES CURRENT IN LOOP2 → BUT IN LOOP1 ITSELF

$$\Phi = LI$$

SELF INDUCTANCE

AND EMF IS INDUCED BY THIS CURRENT

$$\mathcal{E} = -L\, dI/dt$$

VOLT-SECOND PER AMPERE

HENRY

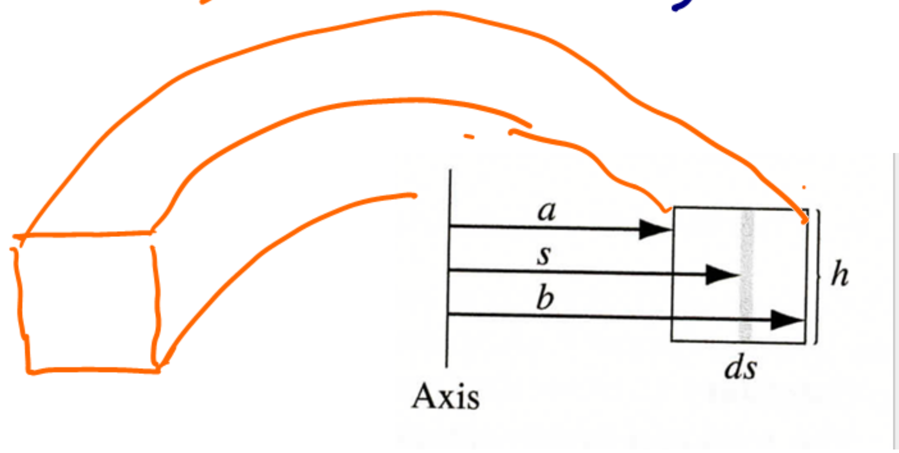
MINUS SIGN SHOWS EMF OPPOSES ANY CHANGE IN CURRENT — BACK EMF

INDUCTANCE IN ELECTRICAL CIRCUIT PLAYS ROLE ANALOGOUS TO MASS IN MECHANICAL SYSTEM

→ INERTIA — OPPOSES CHANGE OF STATE

EXAMPLE: FIND SELF INDUCTANCE OF TOROID

RECTANGULAR X-SECTN (INNER RADIUS a, OUTER RADIUS b, HEIGHT h) CARRIES N TURNS



X-SECTN THRU TOROID

FIELD INSIDE TOROID $B = \dfrac{\mu_0 N I}{2\pi s}$

FLUX THRU SINGLE TURN $\displaystyle\int \bar{B}\cdot d\bar{a} = \frac{\mu_0 N I}{2\pi} h \int_a^b \frac{1}{s'}\, ds'$

$$= \frac{\mu_0 N I h}{2\pi} \ln\left(\frac{a}{b}\right)$$

TOTAL FLUX N TIMES THIS

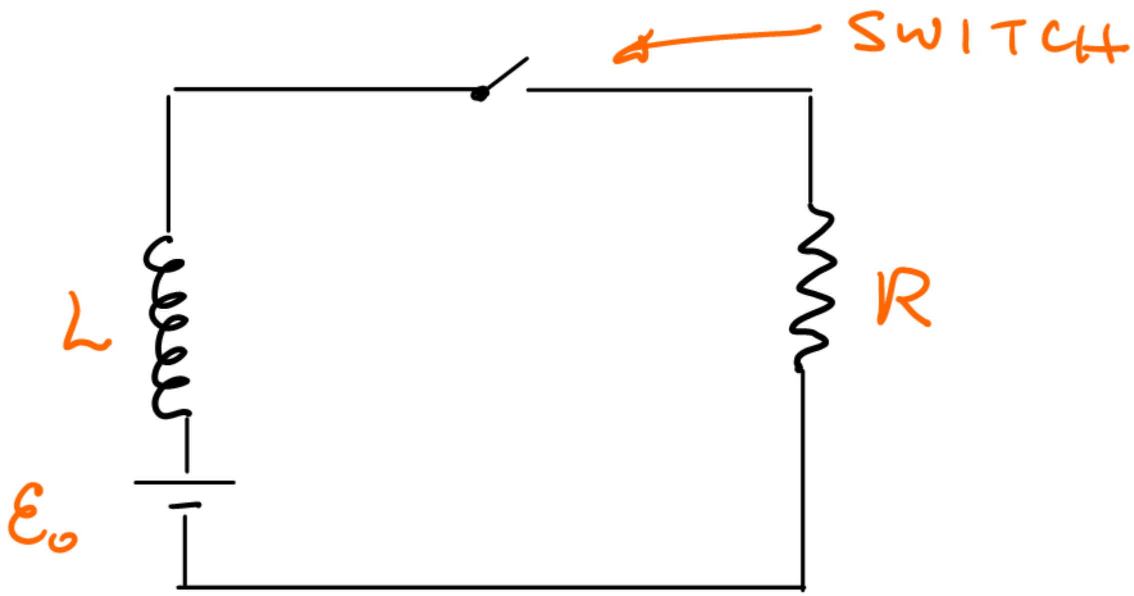
TOTAL FLUX $= \dfrac{\mu_0 N^2 I h}{2\pi} \ln\left(\dfrac{a}{b}\right)$

TOTAL FLUX $= \frac{\mu_0 N^2 I h}{2\pi} \ln\left(\frac{a}{b}\right)$

SELF INDUCTANCE $= LI$

$\hookrightarrow$ $L = \frac{\mu_0 N^2 I h}{2\pi} \ln\left(\frac{a}{b}\right)$

L IS INTRINSICALLY +VE QUANTITY



CURRENT I FLOWING AROUND LOOP

→ SOMEONE CUTS WIRE $I \to 0$ FAST

→ dI/dt → VERY LARGE → SPARK WHEN UNPLUGGING TOASTER

→ INDUCTION ALSO OPPOSES CURRENT FLOWING WHEN CLOSE SWITCH ABOVE

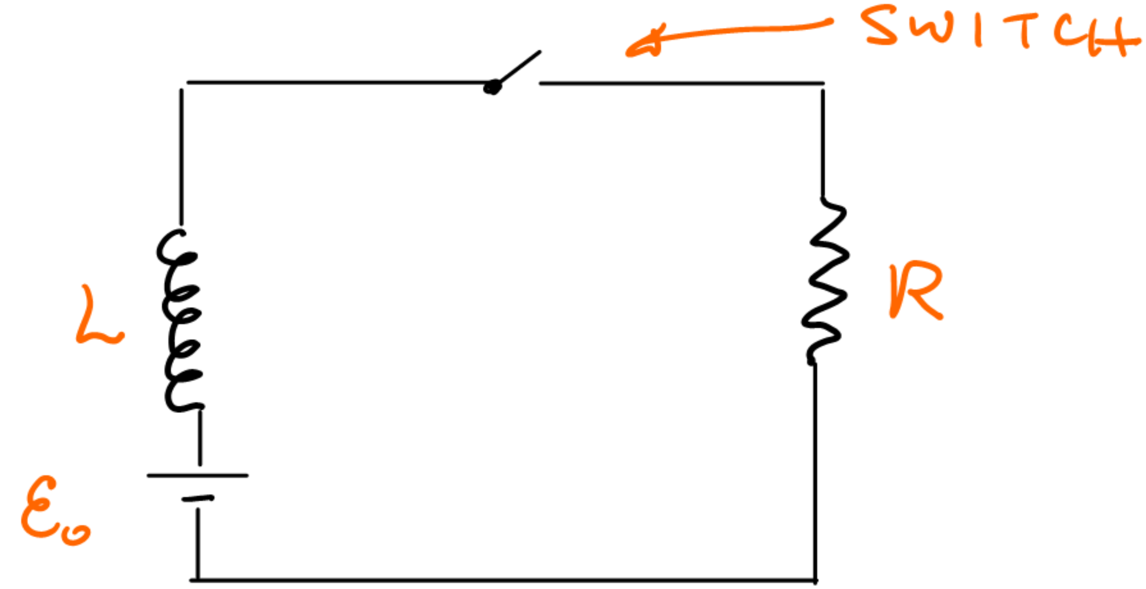
? WHAT CURRENT FLOWS AS A FUNCTION OF TIME?

$$\text{EMF} = \mathcal{E}_0 = -L\frac{dI}{dt} = IR$$

BATTERY

$V = IR$

DIFFERENTIAL EQUATION



GENERAL SOLUTION

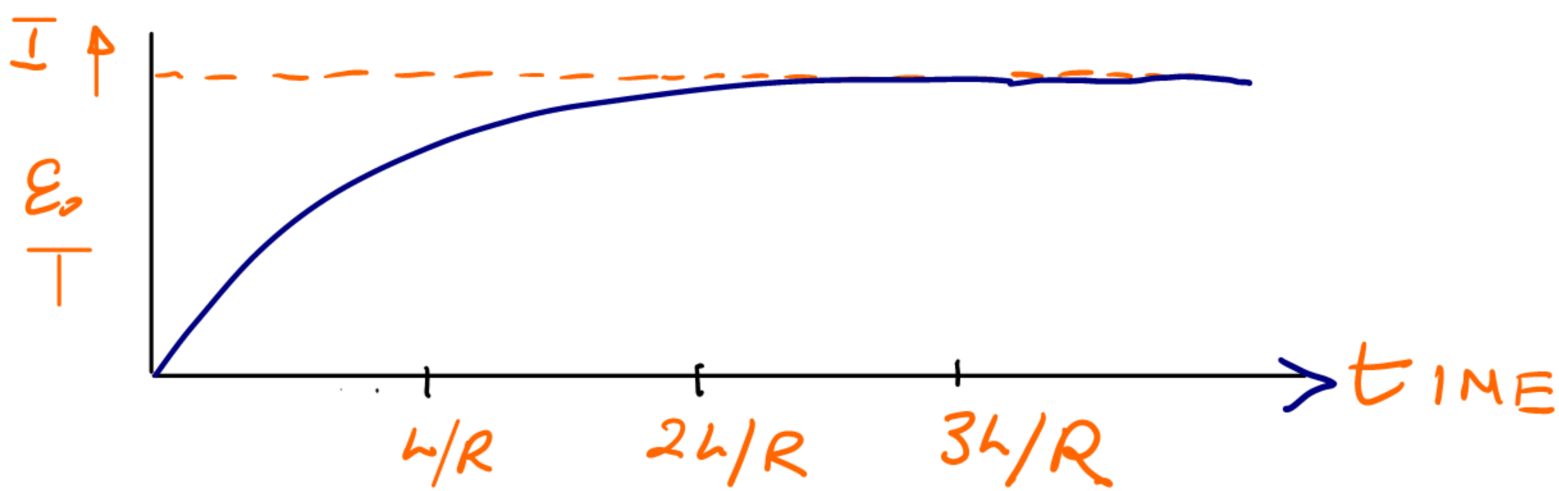
$$I(t) = \frac{\mathcal{E}_0}{R} + k e^{-(R/L)\cdot t}$$

CLOSE SWITH @ $t=0$, $I(0)=0 \rightarrow 0 = \frac{\mathcal{E}_0}{R} + k$

$$k = -\mathcal{E}_0/R$$

$$I(t) = \frac{\mathcal{E}_0}{R}\left[1 - e^{-(R/L)\cdot t}\right]$$

TIME CONSTANT



ENERGY IN MAGNETIC FIELDS

THERE ARE TWO KINDS OF ENERGY ASSOCIATED WITH CURRENT FLOWING IN A CIRCUIT

1) DISSIPATIVE ENERGY → ASSOCIATED WITH OHM'S LAW

- CONVERTED INTO HEAT IN RESISTORS
- THIS IS IRRETRIEVABLY LOST
- INCREASES WITH TIME THE CURRENT FLOW

2) WORK DONE AGAINST BACK EMF IN ESTABLISHING A MAGNETIC FIELD

- RECOVERABLE WHEN CURRENT IS TURNED OFF AND MAGNETIC FIELD COLLAPSES
- ENERGY LATENT IN CIRCUIT → ENERGY STORED IN MAGNETIC FIELD

- WORK DONE ON A UNIT CHARGE AGAINST BACK EMF IN ONE TRIP AROUND THE CIRCUIT $\rightarrow \mathcal{E}$

- AMOUNT OF CHARGE PASSED PER UNIT TIME I

CURRENT

TOTAL WORK DONE PER UNIT TIME $= \dfrac{dW}{dt} = \underbrace{-\mathcal{E} I}_{\text{POWER}} = L I \dfrac{dI}{dt}$

SELF INDUCTANCE OF CIRCUIT

START FROM ZERO CURRE

$$W = \int \frac{dW}{dt} = \int_0^I L I \frac{dI}{dt} \cdot dt = \frac{L I^2}{2}$$

$$W = \frac{L I^2}{2}$$

$$W = \frac{LI^2}{2}$$

WORK DONE AGAINST BACK EMF TO ESTABLISH CURRENT I

DOES NOT DEPEND ON HOW LONG IT TAKES TO ESTABLISH CURRENT

CAN WRITE W IN WAY THAT GENERALIZES TO SURFACES & VOLUMES

FLUX THRU CIRCUIT $\Phi = LI$

$$\Phi = \int \bar{B} \cdot d\bar{a} = \int (\bar{\nabla} \times \bar{A})\, d\bar{a} = \oint \bar{A} \cdot d\bar{\ell}$$

AROUND PERIMETER OF CIRCUIT LOOP

$$\text{SO} \quad LI = \oint \bar{A} \cdot d\bar{\ell}$$

$$LI = \oint \vec{A} \cdot d\vec{l}$$

HAD $W = \frac{1}{2} LI^2$ SO...

$$W = \frac{1}{2} I \oint \vec{A} \cdot d\vec{l} = \frac{1}{2} \oint (\vec{A} \cdot \vec{I}) dl$$

GENERALIZ TO VOLUME CURRENT

$$W = \frac{1}{2} \int_V (\vec{A} \cdot \vec{J}) d\tau$$

$$W = \frac{1}{2\mu_0} \int \vec{A} \cdot (\vec{\nabla} \times \vec{B}) \, d\tau$$

PRODUCT RULE #6

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

$$\vec{A} \cdot (\vec{\nabla} \times \vec{B}) = \vec{B} \cdot \vec{B} - \vec{\nabla} \cdot (\vec{A} \times \vec{B})$$

$$W = \frac{1}{2\mu_0}\int \vec{A}\cdot(\vec{\nabla}\times\vec{B})\,d\tau = \frac{1}{2\mu_0}\left[\int_V B^2\,d\tau - \int \vec{\nabla}\cdot(\vec{A}\times\vec{B})\,d\tau\right]$$

DIVERGENCE THEOREM $\int (\vec{\nabla}\cdot\vec{v})\,d\tau = \oint_S \vec{v}\cdot d\vec{a}$

$$W = \frac{1}{2\mu_0}\left[\int_V B^2\,d\tau - \oint_S (\vec{A}\times\vec{B})\cdot d\vec{a}\right]$$

BOUNDING

INTEGRAL OVER ENTIRE VOLUME INCLUDING CURRENT

↳ CAN USE LARGER VOLUME J IS ZERO OUT THERE

LARGER V → SMALLER CONTRIB FROM SURFACE

ALL SPACE → SURFACE CONTRIB → 0

$$W = \frac{1}{2\mu_0}\int_{\text{ALL SPACE}} B^2\,d\tau$$

$$W = \frac{1}{2\mu_0} \int_{\text{ALL SPACE}} B^2 \, d\tau$$

ENERGY STORED IN MAGNETIC FIELD $\frac{B^2}{2\mu_0}$ PER UNIT VOLUME

COULD ALSO LOOK AT IT AS ENERGY STORED IN CURRENT DISTRIBUTION $\frac{1}{2}\bar{A}\cdot\bar{J}$ PER UNIT VOLUME

- SETTING UP MAGNETIC FIELD INVOLVES CHANGING MAGNETIC FIELD. ACCORDING TO FARADAY

$$\frac{\partial \bar{B}}{\partial t} \longrightarrow \bar{E}$$

THIS IS WHAT WORK IS DONE AGAINST

COMPARISON OF ENERGY IN $\bar{E}$ & $\bar{B}$

$$W_{ELEC} = \frac{1}{2}\int (V\rho)\, d\tau = \frac{\varepsilon_0}{2}\int E^2 d\tau$$

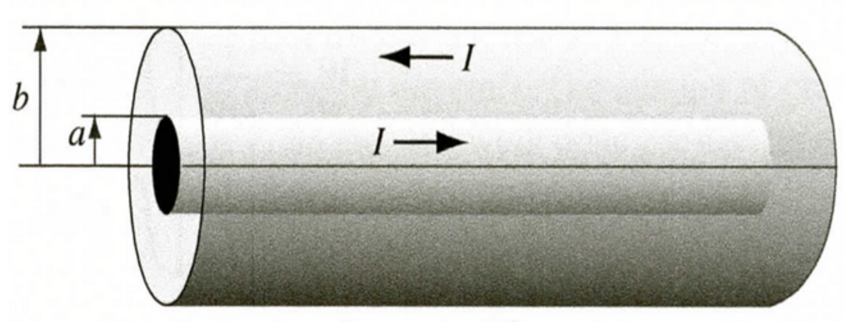
$$W_{MAG} = \frac{1}{2}\int (\bar{A}\cdot\bar{J})\, d\tau = \frac{1}{2\mu_0}\int B^2 d\tau$$

EXAMPLE: LONG COAXIAL CABLE

CARRIES CURRENT I $\rightleftarrows$

FIND MAGNETIC ENERGY

STORED IN LENGTH l



ACCORDING TO AMPÈRE, FIELD BETWEEN CYLINDERS

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$ → ZERO ELSEWHERE

ENERGY PER UNIT VOLUME

$$\frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi s}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 s^2}$$

ENERGY IN CYLINDRICAL SHELL, LENGTH l

THICKNESS ds $$= \left(\frac{\mu_0 I^2}{8\pi^2 s^2}\right) 2\pi l s \, ds$$

$$= \frac{\mu_0 I^2 l}{4\pi}\left(\frac{ds}{s}\right)$$

$$W_{a \to b} = \int \frac{\mu_0 I^2 l}{4\pi}\frac{ds}{s} = \frac{\mu_0 I^2 l}{4\pi}\ln\left(\frac{b}{a}\right)$$

CAN DERIVE INDUCTANCE FROM THIS

$$\underbrace{W = \frac{1}{2} L I^2} \Rightarrow L = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right)$$

CAN USE THIS AS DEFINITION OF INDUCTANCE → WORKS IN COMPLEX SITUATIONS