VECTOR FIELDS

ELECTRODYNAMIC IS :-

$$\bar{\nabla} \cdot \bar{E} = \rho / \varepsilon_0$$

$$\bar{\nabla} \times \bar{E} = -\partial \bar{B} / \partial t$$

$$\bar{\nabla} \cdot \bar{B} = 0$$

$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J} + \mu_0 \varepsilon_0 \, \partial \bar{E} / \partial t$$

$\bar{E} \rightarrow$ ELECTRIC FIELD VECTOR

$\bar{B} \rightarrow$ MAGNETIC FIELD VECTOR

$\rightarrow$ ALL VECTOR DERIVATIVES

IS A VECTOR FUNCTION DETERMINED BY ITS CURL & DIVERGENCE

IF $\bar{\nabla}\cdot\bar{F}$ ($\bar{E}$ OR $\bar{B}$) $= D$ ← SCALAR

$\bar{\nabla}\times\bar{F} = \bar{C}$ ← VECTOR

$(\bar{\nabla}\cdot\bar{C} = 0)$ ← $\bar{\nabla}\cdot(\bar{\nabla}\times F) = 0$

DOES THIS DETERMINE $\bar{F}$?

NOT REALLY → NEED BOUNARY CONDITIONS

→ $\bar{F} \to 0$ @ ∞

E.G. MANY FIELDS HAVE $\bar{\nabla}\cdot\bar{F}$, OR $\bar{\nabla}\times\bar{F} = 0$

HELMHOLTZ THEOREM → FIELD IS UNIQUELY DETERMINED BY $\bar{\nabla}\cdot F$ $\bar{\nabla}\times\bar{F}$

APPENDIX B

POTENTIALS

IF $\bar{\nabla} \times \bar{F} = 0$ EVERY WHERE

F CAN BE WRITTEN AS GRAIDENT OF SCALAR FIELD

$$\bar{\nabla} \times \bar{F} = 0 \implies \bar{F} = -\bar{\nabla} V$$

CONVENTION

THEOREM #1 $\quad \bar{\nabla} \times \bar{F} = 0$ IRROTATIONAL

FOLLOWING SATISFIED

(a) $\bar{\nabla} \times \bar{F} = \bar{0}$ EVERYWHERE

(b) $\int_{\bar{a}}^{\bar{b}} \bar{F} \cdot d\bar{\ell}$ INDEPENDENT OF PATH

(c) $\oint \bar{F} \cdot d\bar{\ell} = 0$ FOR ANY CLOSED LOOP

(d) $\bar{F} \implies \bar{F} = -\bar{\nabla} V$

SCALAR POTENTIAL

NOT UNIQUE

CAN ADD ANY CONSTANT

THEOREM #2 $\quad \bar{\nabla}\cdot\bar{F}=\bar{0}$ SOLENOIDAL

$$\bar{\nabla}\cdot\bar{F}=\bar{0} \longrightarrow \bar{F}=\bar{\nabla}\times\bar{A}$$

VECTOR POTENTIAL

(a) $\bar{\nabla}\cdot\bar{F}=\bar{0}$

(b) $\int \bar{F}\cdot d\bar{a}$ INDEPENDENT OF SURFACE FOR ANY BOUNARY LINE

(c) $\oint \bar{F}\cdot d\bar{a}=0$ FOR ANY CLOSED SURFACE

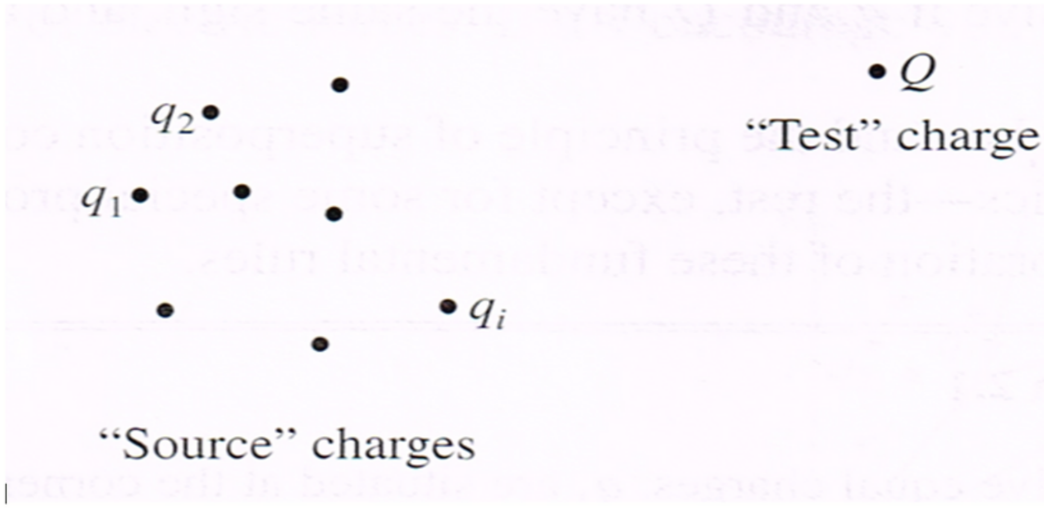
(d) $\bar{F}=\bar{\nabla}\times\bar{A}$ ← VECTOR POTENTIAL

VECTOR POTENTIAL NOT UNIQUE

$$\bar{A}\rightarrow\bar{A}+\bar{\nabla}c \qquad \bar{\nabla}\times(\bar{\nabla}c)=0$$

ANY FIELD $\Longrightarrow \bar{F}=-\bar{\nabla}V+\bar{\nabla}\times\bar{A}$

ELECTROSTATICS



HAVE SOME SOURCE CHARGES

WHAT FORCE DO THEY EXERT ON TEST CHARGE Q $\longrightarrow$ WHAT IS POSITION OF Q AS A FUNCTION OF TIME?

GENERALLY q/Q MOVING/ACCELERATING

COMPLEX $\Longrightarrow$ ELECTROSTATICS

PRINCIPLE OF SUPERPOSITION

EXPERIMENTALLY FIND THAT

$$F = \overline{F_1} + \overline{F_2} + \overline{F_3} + \cdots\cdots + \overline{F_n}$$

IF CAN FIND SOLUTION FOR <u>ONE</u> SOURCE CHARGE, THEN PROBLEM SOLVED

IF $F_{em} \propto q^2$ AND NOT q

$$(q_1 + q_2)^2 = q_1^2 + q_2^2$$

NATURE IS KIND!

EXPERIMENTAL → COULOMBS LAW

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q\,Q}{r^2}\hat{r}$$

VECTOR BETWEEN CHARGES

CHOICE OF UNITS

PERMETTIVITY OF FREE SPACE

ACTUALLY

$$F = k_1 \frac{q_1 q_2}{r^2}\hat{r}$$

DETERMINED BY UNITS CHOSEN FOR ELECTRIC CHARGE

OR CHOSEN ARBITRARILY TO DEFINE UNIT OF ELECTRIC CHARGE

DIMENSION IN CLASSICAL MECHANICS

MASS [M], LENGTH [L], TIME [T]

CLASSICAL MECHANICS DOES NOT INVOLVE ANY FUNDEMENTAL CONSTANT

E & M DOES! $\longrightarrow$ C VELOCITY OF LIGHT

PHYSICAL LAWS SCALE ONLY IF

$\frac{[L]}{[T]}$ CONSTANT $\longrightarrow$ SPECIAL RELATIVITY

MKS (SI) $\quad k = 1/4\pi\varepsilon_0$

GAUSSIAN (CGS) $\quad k = 1$

HEAVISIDE LORENTZ (CGS) $\quad k = \dfrac{1}{4\pi}$

NATURAL $(c = \hbar = 1)$ $\quad k = 1/4\pi$

UNITS OF ELECTRIC CHARGE

MKS (SI) $\quad \dfrac{e^2}{4\pi\hbar c\varepsilon_0} = \dfrac{1}{137.039}$

GAUSSIAN $\quad \dfrac{e^2}{\hbar c} = \dfrac{1}{137.039}$

HEAVISIDE LORENT $\quad \dfrac{e^2}{4\pi\hbar c} = \dfrac{1}{137.039}$

$\dfrac{1}{137.039}$ IS PHYSICS

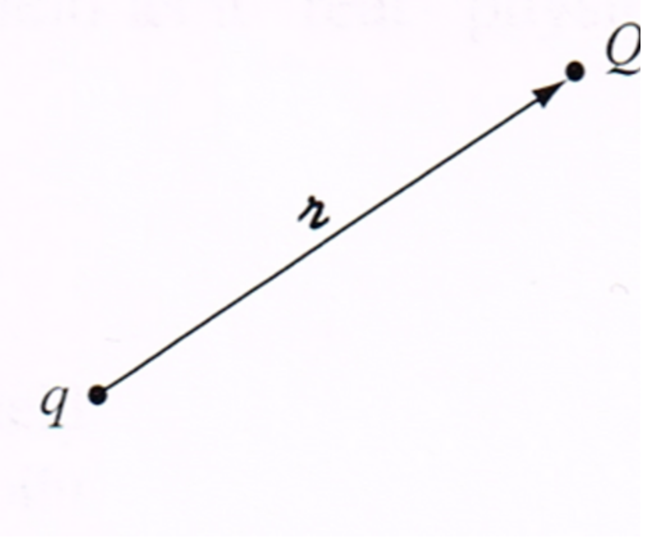
WE WILL USE SI

$$F = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{r^2} \hat{r}$$

NEWTONS

COULOMB

$8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$



- FORCE $q \leftrightarrow Q$
- REPULSIVE $q\ Q$ SAME SIGN
- ATTRACTIVE IF $q\ Q$ OPP SIGN

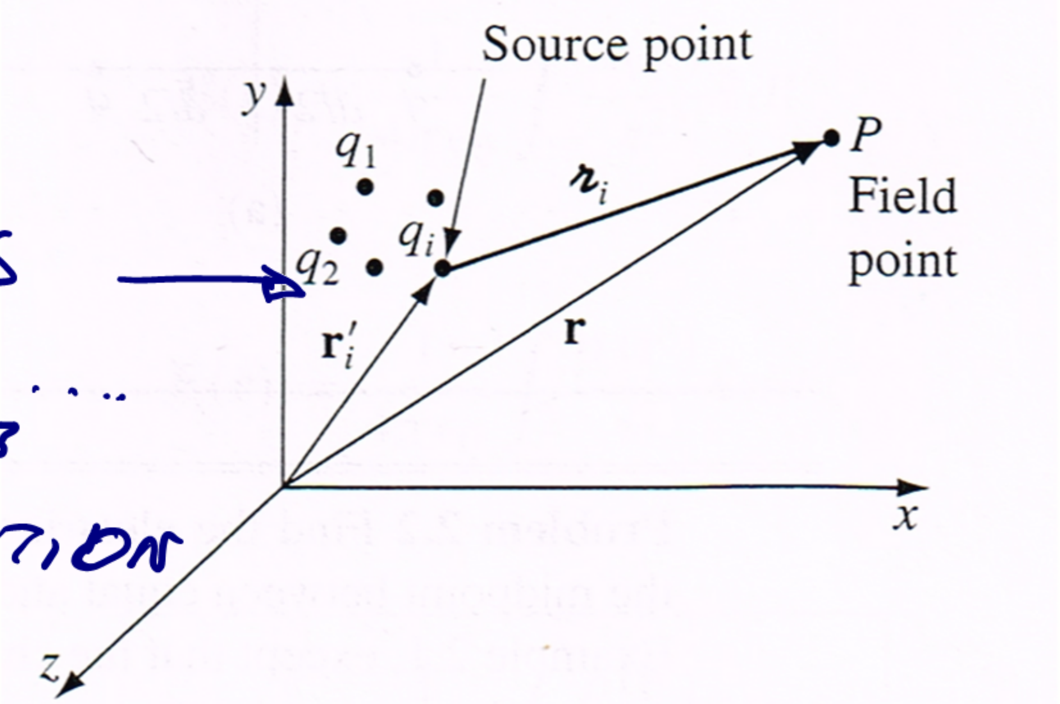
THAT'S ALL THE PHYSICS IN ELECTROSTATICS

ELECTRIC FIELD

SEVERAL POINT CHARGES

DISTANCES $\mathscr{r}_1, \mathscr{r}_2, \mathscr{r}_3 \ldots$

FROM Q. BY SUPERPOSITION



$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \cdots = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 Q}{\mathscr{r}_1^2}\hat{\mathscr{r}}_1 + \frac{q_2 Q}{\mathscr{r}_2^2}\hat{\mathscr{r}}_2 + \cdots\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{q_1}{\mathscr{r}_1^2}\hat{\mathscr{r}}_1 + \frac{q_2}{\mathscr{r}_2^2}\hat{\mathscr{r}}_2 + \cdots\right)$$

$$\bar{F} = Q\bar{E}$$

ELECTRIC FIELD OF SOURCE

$$E(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i}^{n} \frac{q_i}{\mathscr{r}_i^2}\hat{\mathscr{r}}_i$$

$$\bar{E}(\bar{r}) \equiv \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

ELECTRIC FIELD → JUST DEPENDS ON SOURCE q

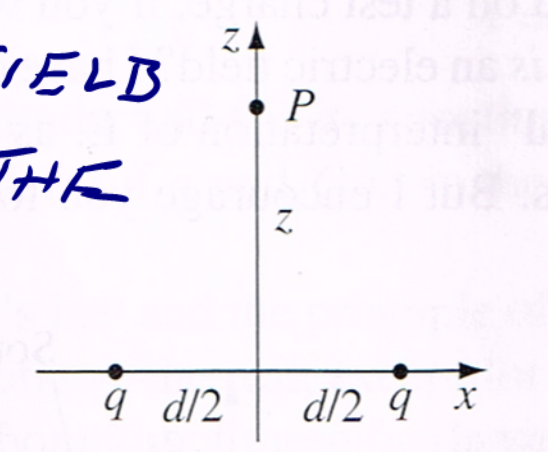
NOT TEST Q

VECTOR DEPENDING ON POINT IN SPACE

FIELD IS A REAL PHYSICAL ENTITY

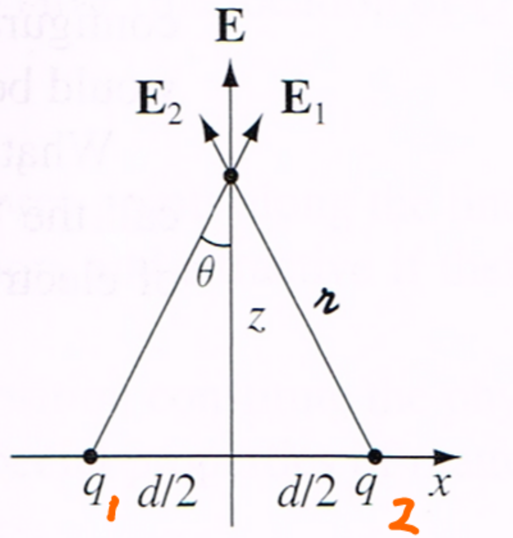
IN QUANTUM FIELD THEORY FIELDS ARE FUNDEMENTAL PARTICLES EXCITATIONS OF QUANTUM FIELD.

EXAMPLE: FIND THE ELECTRIC FIELD A DISTANCE z ABOVE THE MIDPOINT OF 2 EQUAL CHARGES SPACED BY d



$\bar{E}_1$ FIELD DUE TO q_1

$\bar{E}_2$ FIELD DUE TO q_2



$r = \left(z^2 + (d/2)^2\right)^{\frac{1}{2}}$ $\quad \cos\theta = z/r$

$$\bar{E} = \frac{1}{4\pi\varepsilon_0} \frac{2qz}{\left[z^2 + (d/2)^2\right]^{\frac{1}{2}}} \hat{z}$$

WHEN $z \gg d$ LOOKS LIKE SINGLE CHARGE $2q$

$d \to 0$

$$\bar{E} = \frac{1}{4\pi\varepsilon_0} \frac{2q}{z^2} \hat{z} \quad ✓$$

ONTINUOUS CHARGE DISTRIBUTIONS

HAVE ASSUMED DISCRETE CHARGES, FOR CHARGE DISTRIBUTED CONTINUOUSY OVER REGION

$$E(\vec{r}) \approx \frac{1}{4\pi\varepsilon_0}\int \frac{1}{\mathscr{r}^2}\,\hat{\mathscr{r}}\; dq$$

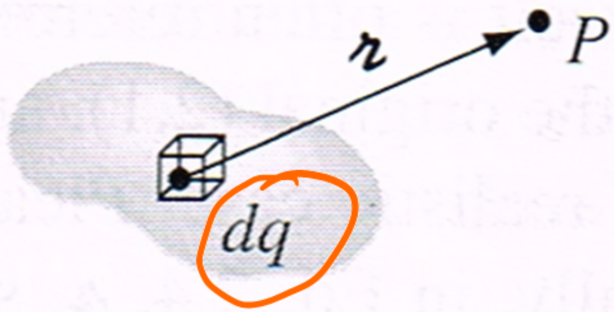
$\hookrightarrow$ LINE
SURFACE
VOLUME

$dq \rightarrow \lambda\, dl'$ LINE

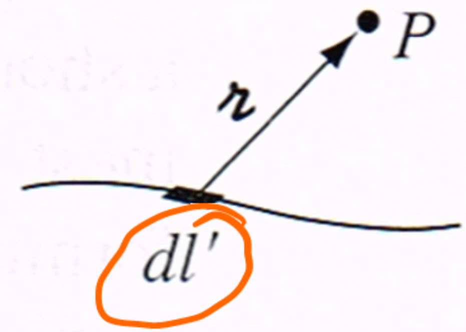
$\rightarrow \sigma\, da'$ SURFACE

$\rightarrow \rho\, d\tau'$ VOLUME

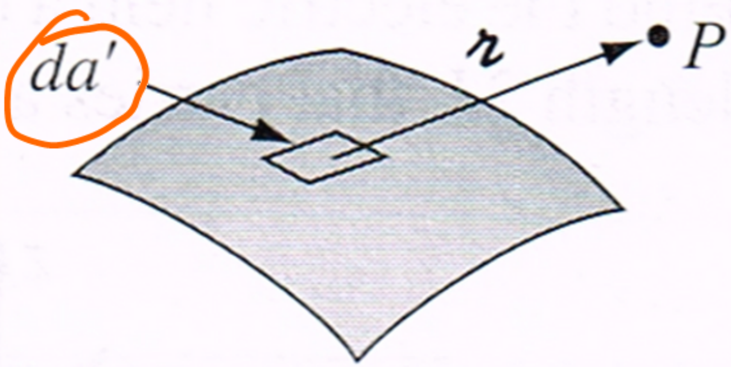
$\lambda, \sigma, \rho \rightarrow$ CHARGE DENSITY



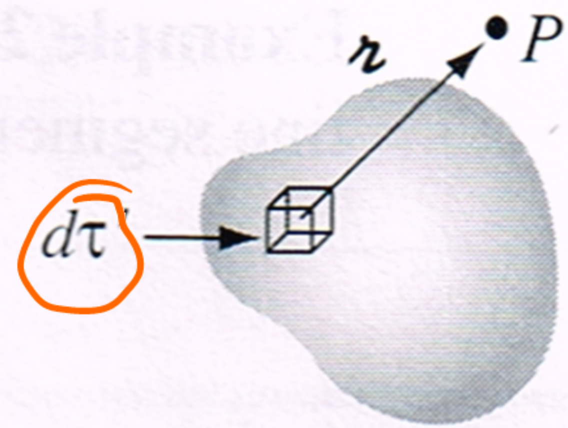
(a) Continuous distribution



(b) Line charge, λ



(c) Surface charge, σ



(d) Volume charge, ρ

COULOMB'S LAW

FOR A GENERAL VOLUME CHARGE DISTRIB

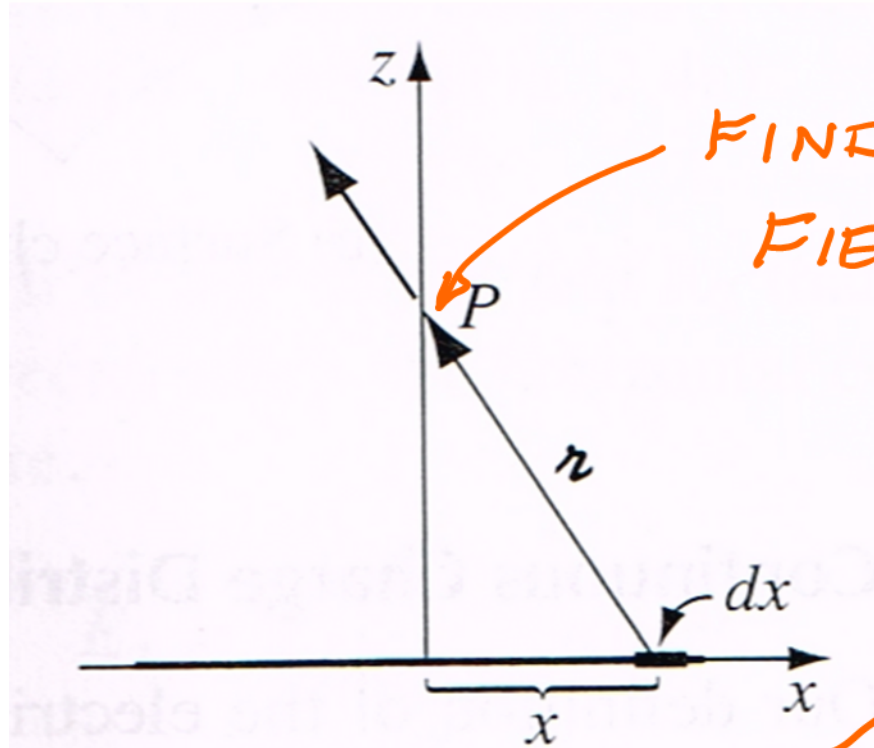
$$E(\bar{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\bar{r}')}{\eta^2}\hat{\eta}\, d\tau'$$

A POINT IN SPACE

VOLUME DENSITY OF CHARGE AT A POINT IN SPACE

DISTANCE FROM VOLUME ELEMENT TO POINT IN SPACE

EXAMPLE



FIND ELECTRIC FIELD HERE

CHARGE λ

$2L$

$$\hat{r} = z\hat{z} \quad ; \quad \bar{r}' = x\hat{x} \quad ; \quad dl' = dx$$

$$\bar{\mathscr{r}} = \bar{r} - \bar{r}' = z\hat{z} - x\hat{x}$$

$$\mathscr{r} = (z^2 + x^2)^{1/2} \quad ; \quad \hat{\mathscr{r}} = \frac{\bar{\mathscr{r}}}{\mathscr{r}} = \frac{z\hat{z} - x\hat{z}}{(z^2+x^2)^{1/2}}$$

$$\underline{E} = \frac{1}{4\pi\varepsilon_0}\int_{-L}^{+L} \frac{\lambda}{z^2+x^2} \cdot \frac{z\hat{z} - x\hat{x}}{(z^2+x^2)^{1/2}}\, dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left\{ z\hat{z}\int_{-L}^{+L} \frac{1}{(z^2+x^2)^{3/2}}\,dx - \hat{x}\int_{-L}^{+L} \frac{x}{(z^2+x^2)^{3/2}}\,dx \right.$$

$$\int \frac{dx}{(a+bx^2)^{m+1}} = \frac{1}{2ma}\frac{x}{(a+bx^2)^m} + \frac{2m-1}{2ma}\int \frac{dx}{(a+bx^2)^m}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left\{ z\hat{z}\left[\frac{x}{z^2}\frac{1}{(z^2+x^2)^{\frac{1}{2}}}\right]_{-L}^{+L} - \hat{x}\left[-\frac{1}{(z^2+x^2)^{\frac{1}{2}}}\right]_{-L}^{+L}\right.$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left\{\frac{2\lambda L}{z(z^2+L^2)}\,\hat{z}\right\}$$

FAR FROM LINE $z \gg L \rightarrow$ LOOKS LIKE POINT CHARGE

$$E \simeq \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z^2}$$

$q = 2\lambda L$

FOR $L \rightarrow \infty$ INFINITE LINE CHARGE

$$E \rightarrow \frac{1}{4\pi\varepsilon_0}\frac{2\lambda L}{z\sqrt{L^2}} \rightarrow \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{z}$$