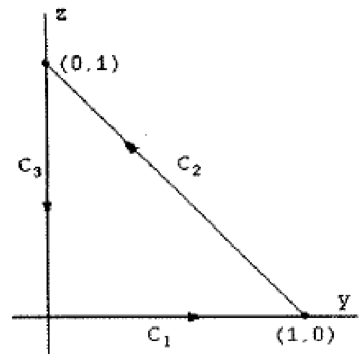
Mid Term Solutions

1)

$$\mathbf{F} = \mathbf{i}\frac{\partial}{\partial x}(x^2 + y^2 + z^2) + \mathbf{j}\frac{\partial}{\partial y}(x^2 + y^2 + z^2) + \mathbf{k}\frac{\partial}{\partial z}(x^2 + y^2 + z^2) =$$

$$= 2(\mathbf{i}x + \mathbf{j}y + \mathbf{k}z).$$



For the triangular path shown in the figure x = 0 so we have $\int_C \mathbf{F}\cdot\mathbf{t}ds = 2\int_C (ydy + zdz)$. On C_1 z = 0 so $\int_{C_1}\mathbf{F}\cdot\hat{\mathbf{t}}ds = 2\int_0^1 ydy = 1$. On C_3 y =0 so $\int_{C_3}\mathbf{F}\cdot\hat{\mathbf{t}}ds = 2\int_1^0 zdz = -1$. For the path C_2 we have $x = 1 - \frac{s}{\sqrt{2}}$ and $y = \frac{s}{\sqrt{2}}$. Thus $\int_{C_2}\mathbf{F}\cdot\hat{\mathbf{t}}ds =$

$$2\int_0^{\sqrt{2}}\left(1 - \frac{s}{\sqrt{2}}\right)\left(-\frac{ds}{\sqrt{2}}\right) + 2\int_0^{\sqrt{2}}\frac{s}{\sqrt{2}}\frac{ds}{\sqrt{2}} = 2\int_0^{\sqrt{2}}\left(\sqrt{2}s - 1\right)ds = 0.$$ Hence

$$\oint_C \mathbf{F}\cdot\hat{\mathbf{t}}ds = 0.$$

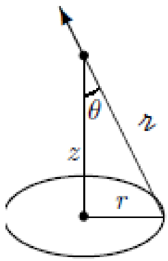
2)

13\. a. We know that $\nabla\cdot\mathbf{E} = \rho/\varepsilon_0$. Hence $\rho = \varepsilon_0\nabla\cdot\mathbf{E} = 3\varepsilon_0 g$, using the given field.

b. Because $\mathbf{E} = -\nabla\Phi$ we have $\frac{\partial\Phi}{\partial x} = -gx$, $\frac{\partial\Phi}{\partial y} = -gy$, $\frac{\partial\Phi}{\partial z} = -gz$ whence $\Phi = -\frac{g}{2}(x^2 + y^2 + z^2) + \text{const}$. Alternatively, we can

c. $\nabla^2\Phi = -\frac{g}{2}(2 + 2 + 2) = -3g = -\frac{\rho}{\varepsilon_0}$.

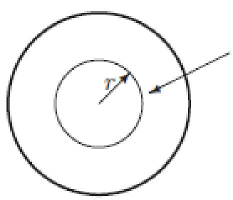
3)



"Horizontal" components cancel, leaving: $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\left\{\int \frac{\lambda dl}{\mathscr{r}^2}\cos\theta\right\}\hat{\mathbf{z}}$.
Here, $\mathscr{r}^2 = r^2 + z^2$, $\cos\theta = \frac{z}{\mathscr{r}}$ (both constants), while $\int dl = 2\pi r$. So

$$\boxed{\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{\lambda(2\pi r)z}{(r^2+z^2)^{3/2}}\,\hat{\mathbf{z}}.}$$

4)



$$\oint \mathbf{E}\cdot d\mathbf{a} = E\cdot 4\pi r^2 = \frac{1}{\epsilon_0}Q_{\text{enc}} = \frac{1}{\epsilon_0}\int \rho\, d\tau = \frac{1}{\epsilon_0}\int (k\bar{r})(\bar{r}^2\sin\theta\, d\bar{r}\, d\theta\, d\phi)$$
$$= \frac{1}{\epsilon_0}\, k\, 4\pi \int_0^r \bar{r}^3 d\bar{r} = \frac{4\pi k}{\epsilon_0}\frac{r^4}{4} = \frac{\pi k}{\epsilon_0}r^4.$$

$$\therefore \boxed{\mathbf{E} = \frac{1}{4\pi\epsilon_0}\pi k r^2\hat{\mathbf{r}}.}$$