PHY250 Solutions to Prob Set#4

1)

If a charged particle of mass m and charge q moves in a magnetic field B, the force on the particle is perpendicular to its velocity. The field thus does no work on the particle, and the particle's kinetic energy (and hence speed v) will remain constant. We can thus equate the magnetic force Bqv with the centripetal force mv^2/r, where r is the radius of curvature of the particle's path, to give

$$m = \frac{Bqr}{v}.$$

Now the particle's kinetic energy $E_k = mv^2/2$, so we can put

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2eV}{m}},$$

where V is the kinetic energy measured in electron-volts. Substituting into our expression for m, and rearranging to make m the subject, gives

$$m = \frac{B^2q^2r^2}{2eV}.$$

Taking $q = e$ (for singly charged ions), and substituting the given values,
▶ yields $m = 6.7 \times 10^{-27}$ kg (i.e. they must be helium ions).

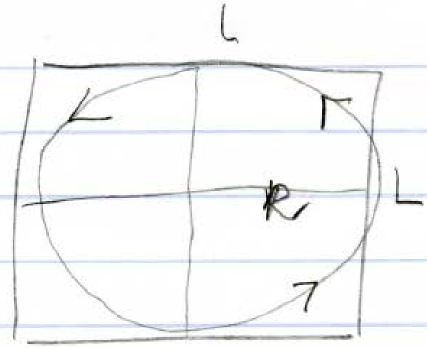
If an electric field E is superimposed such that the ions are undeflected, the electric force qE must balance the magnetic force Bqv, so that $E = vB$. Substitution of our value for m into the expression

$$v = \sqrt{\frac{2eV}{m}}$$

▶ gives $v = 2.2 \times 10^6\ \mathrm{m\,s^{-1}}$, so that $E = 4.9 \times 10^5\ \mathrm{V\,m^{-1}}$.

2)

2a)

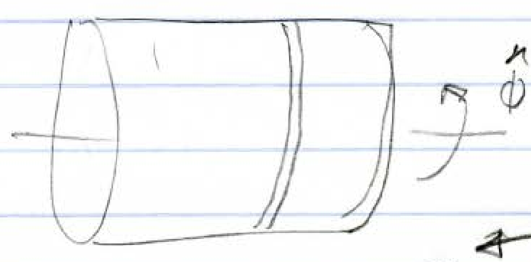


$R = L/2$

velocity = $\frac{\omega L}{2}$

charge density = $\sigma \times$ velocity = $\sigma \frac{\omega L}{2}$

b)



uniform

charge density = $\rho = Q/V$ — $\pi r^2 L$

$V = r\omega$

$|J| = \rho V = \frac{Q}{4\pi R^2 L} \cdot \omega r$

$\bar{J} = \frac{Q \omega r}{4\pi R^2 L} \hat{\phi}$

3)

(a) The total current in the wire is given by

$$I = \int \mathbf{j} \cdot \mathbf{dS} = 2\pi j_0 \int_0^R \cos\left(\frac{\pi r}{2R}\right) r\, dr. \tag{2.68}$$

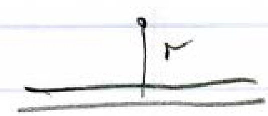
Doing the integral by parts gives

$$\begin{aligned} I &= 2\pi j_0 \left\{ \left[\frac{2rR}{\pi} \sin\left(\frac{\pi r}{2R}\right) \right]_0^R - \left(\frac{2R}{\pi}\right) \int_0^R \sin\left(\frac{\pi r}{2R}\right) dr \right\} \\ &= 2\pi j_0 \left[2R^2/\pi - (2R/\pi)^2 \right] \\ &= 4R^2 j_0 (1 - 2/\pi). \end{aligned} \tag{2.69}$$

3) b) Magnetic field outside wire is given by Ampère

$$B = \frac{\mu_0 I}{2\pi r}$$

some radius



$$= \frac{\mu_0}{2\pi r} \cdot 4R^2 j_0 (1 - 2\pi)$$

$$= \frac{2\mu_0 R^2}{\pi r} j_0 (1 - 2\pi)$$

c) the magnetic field at radius $r < R$ is only due to current with r

$$I(r) = 2\pi \int_0^r j(r')\, r'\, dr'$$

$$= 2\pi j_0 \int_0^r \cos\left(\frac{\pi r'}{2R}\right) r'\, dr'$$

$$= 2 j_0 \left\{ 2rR \left(\frac{\pi r}{2R}\right) + \left(\frac{4R^2}{\pi}\right)\left[\cos\left(\frac{\pi r}{2R}\right) - 1\right]\right.$$

3c) $$B(r) = \frac{\mu_0 I(r)}{2\pi r}$$

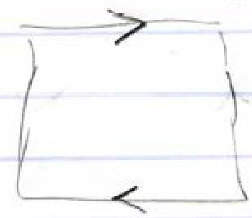
$$= \frac{\mu_0 \cancel{2} j_0}{\cancel{2}\pi r}\left\{ 2rR \sin\left(\frac{\pi r}{2R}\right) + \left(\frac{4R^2}{\pi}\right)\left[\cos\left(\frac{\pi r}{2R}\right) - 1\right]\right.$$

4)

4) the magnetic field acting on square loop is out of page. Magnitude is

$$B = \frac{\mu_0 I}{2\pi r} \leftarrow \text{distance from long wire}$$

Force on 2 vertical segments cancel



$\leftarrow$ equal + opp

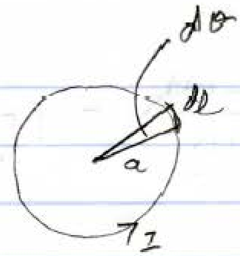
Net force on two horizontal segments

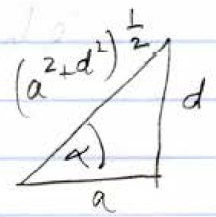
$$F_{BOTTOM} = \left(\frac{\mu_0 I}{2\pi a}\right) \cdot Ia = \frac{\mu_0 I^2}{2\pi} \text{ UP}$$

$$F_{TOP} = \frac{\mu_0 I}{2\pi \cdot 2a} \cdot Ia = \frac{\mu_0 I^2}{4\pi} \text{ DOWN}$$

$$\text{Net force} = \frac{1}{4\pi} \mu_0 I^2 \text{ UP}$$

5a)





From axial symmetry horizontal component of $\bar{B}$ cancels

Contribution to $\bar{B}$ from $d\bar{l}$ on the loop

$$d\bar{B}(d) = \frac{\mu_0 I \, d\bar{l}}{4\pi} = \frac{\mu_0 I a \, d\theta}{4\pi \, (a^2+d^2)}$$

Vertical component

$$d\bar{B}_v(d) = \frac{\mu_0 I a \, d\theta}{4\pi \, (a^2+d^2)} \cos\alpha$$

$$= \frac{\mu_0 I a \, d\theta}{4\pi \, (a^2+d^2)} \frac{a}{(a^2+d^2)^{\frac{1}{2}}}$$

from whole loop

$$B_v(d) = \frac{\mu_0 a^2 I}{4\pi (a^2+d^2)^{3/2}} \int_0^{2\pi} d\theta$$

$$= \frac{\mu_0 a^2 I}{2(a^2+d^2)^{3/2}}$$

TEXT Book Defines θ differently !

5) b) Magnetic field on axis due to one loop

$$B = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$$

field due to 2 loop at $z = +\frac{L}{2}, -\frac{L}{2}$

$$B(z) = \frac{\mu_0 I R^2}{2}\left\{\frac{1}{\left[\left(z+\frac{L}{2}\right)^2+R^2\right]^{3/2}} + \frac{1}{\left[\left(z-\frac{L}{2}\right)^2+R^2\right]^{3/2}}\right\}$$

$$\frac{d}{dz}\left\{\frac{1}{\left[(z+L/2)^2+R^2\right]^{3/2}}\right\} = -\frac{3}{\cancel{2}}\cdot\frac{\cancel{2}(z+L/2)}{\left[\left(z+\frac{L}{2}\right)^2+R^2\right]^{5/2}}$$

so total differential

$$\frac{dB}{dz} = -3\mu_0 I R^2 \cdot \left\{\frac{z+\frac{L}{2}}{\left[\left(z+\frac{L}{2}\right)^2+R^2\right]^{5/2}} + \frac{z-L/2}{\left[(z-L/2)^2+R^2\right]^{5/2}}\right\}$$

at $z = 0$

$$= -3\mu_0 I R^2\left\{\frac{\frac{L}{2}}{\left[L^2/4+R^2\right]^{5/2}} - \frac{\frac{L}{2}}{\left[L^2/4+R^2\right]^{5/2}}\right\}$$

$$= 0 \quad !$$

$$\frac{d^2B}{\partial z^2} = -3\mu_0 I R^2 \left\{ \frac{1}{[(z+L/2)^2+R^2]^{5/2}} - \frac{5(z+L/2)^2}{[\cdots]^{7/2}} + \frac{1}{[(z-L/2)^2+R^2]^{5/2}} - \frac{5(z-L/2)^2}{[(z-\frac{L}{2})^2+R^2]^{7/2}} \right\}$$

$$= -3\mu_0 I R^2 \left\{ \frac{[R^2-4(z+\frac{L}{2})^2]}{[(z+\frac{L}{2})^2+R^2]^{7/2}} + \frac{[R^2-4(z-L/2)^2]}{(z-\frac{L}{2})^2+R^2]^{7/2}} \right\}$$

$$\left.\frac{d^2B}{\partial z^2}\right|_{z=0} = -6\mu_0 I R^2 \left\{ \frac{R^2-L^2}{(L^2/4+R^2)^{\frac{7}{2}}} \right\}$$

$$\left.\frac{d^2B}{\partial z}\right|_{z=0} \rightarrow 0 \quad \text{for} \quad L=R$$

6)

6) Make up your own symbols

length of sides $= a$

diameter of wire $= d$

ρ_c — resistivity of copper

ρ_m — density of copper

i) Rate at which flux thru loop changes is

$$Bav \to \frac{d\Phi}{dt}$$

$$\text{emf } \mathcal{E} = -\frac{d\Phi}{dt} = -Bav$$

Resistance of the loop is given by

$$R = \frac{\rho_c (4a)}{\pi (d/2)^2} = \frac{16 \rho_e a}{\pi d^2}$$

so the current is $I = \mathcal{E}/R$

$$I = \frac{B\cancel{a} v \pi d^2}{16 \rho_e \cancel{a}}$$

Since magnetic field is into page, current in upper loop must flow to right in order that force acts upward → LENZ'S LAW

So current flows clockwise around loop

ii) Upper arm experiences force BIa UP↑

two vertical arms cancel out

lower arm experiences no force since it is not in magnetic field

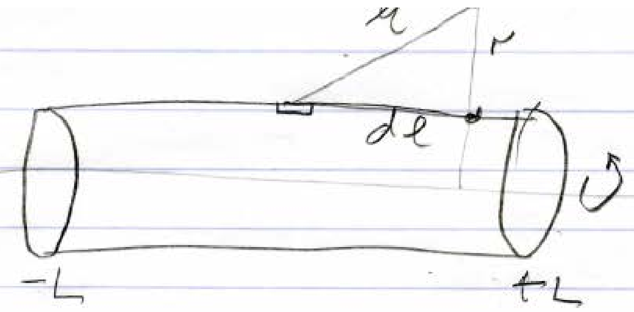
Total force $F = \dfrac{\pi B^2 v d^2 a}{16 \rho_e}$ UP↑

iii) If velocity has reached a steady value, upward force balances weight

$$\rho_m 4\pi a \left(\frac{d}{2}\right)^2 g = \pi \rho_m d^2 a g$$

So $V = \dfrac{16 \rho_e \rho_m g}{B^2} = \quad v = 17$ mm/s.

7)



$$da = R\, d\phi\, dl$$

$$r = \sqrt{r^2 + l^2}$$

$$A(\bar{r}) = \frac{\mu_0}{4\pi} \int_S \frac{\bar{K}(\bar{r}')\, da}{\sqrt{r^2 + l^2}}$$

$$= \frac{\mu_0}{4\pi} \int_S \frac{\bar{K}(\bar{r}')\, R\, d\phi\, dl}{\sqrt{r^2 + l^2}}$$

$$= \frac{\mu_0}{4\pi} R \int_0^{2\pi} d\phi \int_{-L}^{+L} \frac{dl}{\sqrt{r^2 + l^2}}$$

$$= \frac{\mu_0 R\, 2\pi}{4\pi} \int_{-L}^{+L} \frac{dl}{\sqrt{r^2 + l^2}}$$

outside cylinder

$$= \frac{\mu_0 R}{2} \left[\log \left(l + \sqrt{l^2 + r^2} \right) \right]_{-L}^{+L}$$

8) Field constant between plates and zero outside. Ampere around dashed curve $\oint \bar{B} \cdot d\bar{l} = \mu_0 I_{enc}$

$$B \cdot d = \mu_0 I \rightarrow B = \frac{\mu_0 I}{d}$$

Flux thru coil is flux density by area of coil $\Phi = B \cdot la = \frac{\mu_0 I}{d} \cdot la$

Self Inductance of coil is given by

$$\Phi = LI \Rightarrow L = \frac{\mu_0 la}{b}$$

Energy stored in a magnetic field is

$$W = \frac{LI^2}{2}$$

$$W = \frac{\mu_0 la}{2b} \cdot I^2$$

Current ~~constant~~ constant (battery)

(a) changing a by δa changes Self Inductance & hence stored energy

$$\delta W = \frac{\mu_0 I^2 l}{2b} \delta a$$

b) no emf — so there can be no change in flux thru coil, so the current will change as the self inductance changes

$$W = \frac{\mu_0 I^2 la}{2b} = \frac{\Phi^2 b}{2\mu_0 la}$$

$$\delta W = -\frac{\Phi^2 b}{2\mu_0 la} \delta a$$

$$= -\frac{\mu_0 I^2 l}{2b} \delta a$$

Since this is −ve, the change reduces the energy, so forces on plates must be REPULSIVE → outwards

$F = -\frac{dW}{da}$ → force/unit area $= \frac{\mu_0 I^2 l}{2b} \cdot \frac{1}{lb} = \frac{\mu_0 I^2}{2b^2}$