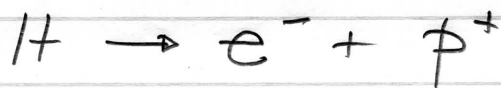
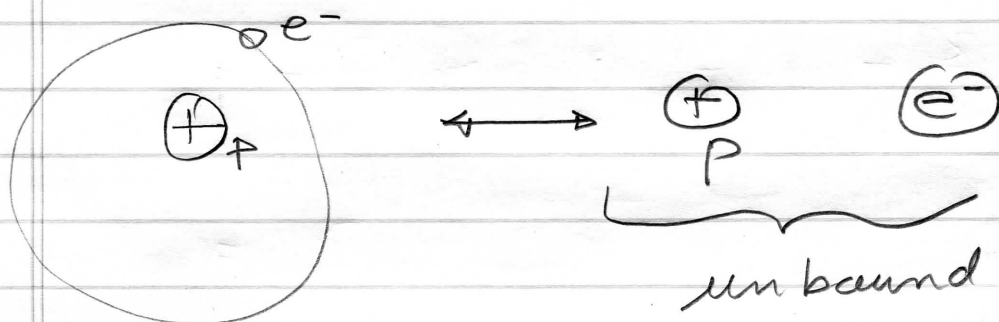


CHEMICAL EQUILIBRIUM

Consider process like



Dissociation or ionization of atomic hydrogen.



Gas of hydrogen at Temperature T

↳ collisions \rightarrow ionization if energy (τ)
is high enough.

↳ energy required for ionization = 13.6 eV

General Chemical reactions



we'll keep to $1 \rightarrow 2$ for
simplicity

Other examples



2 ammonia molecules \leftrightarrow 1 Nitrogen + 3 hydrogen.



STOICHIOMETRIC COEFFICIENTS

↳ number of molecules of i^{th}

Species taking part in chemical
reaction

All very nice, but let's go back

and think about IDEAL GAS

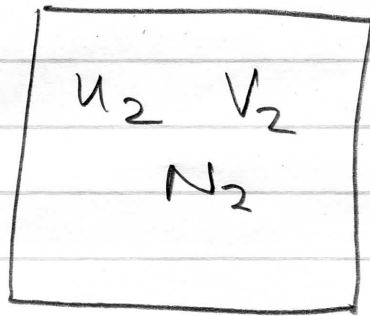
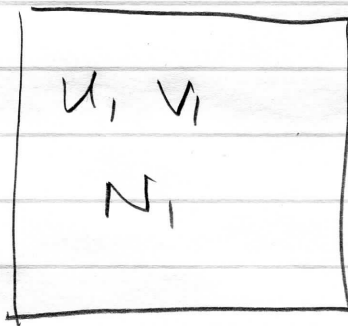
+

DIFFUSION

Volume \rightarrow SPLIT into 2 parts.

\rightarrow each volume contains an
IDEAL GAS \rightarrow SAME PARTICLES

(Note, it doesn't have to be an ideal gas — but we want to use Sakur — Tetrode to be specific.



KEEP THIS ON BOARD

Bring into THERMAL CONTACT

\rightarrow PARTITION is porous

Both ENERGY & PARTICLES can pass thru

SIMPLICITY \rightarrow keep barrier fixed

2 volumes are CONSTANT.

$U_1' V_1$	$U_2' V_2$
N_1'	N_2'

KEEP THIS ON BOARD

→ Energy exchange → Equalize T

→ Particles pass through porous barrier until ENTROPY MAXIMIZED

So →

$$U = \text{Total Energy} = U_1' + U_2' = U_1 + U_2$$

$$N = \text{Total number of particles } N_1' + N_2' = N_1 + N_2$$

After thermodynamic equilibrium is established

w. int ↓

Before thermal Diffusive Chemical contact

Postulate $\rightarrow S_{TOTAL}$ becomes MAXIMUM.

$$S_{TOTAL} = S_1(u_1', N_1') + S_2(u_2', N_2')$$

$$= S_1(u - u_2', N - N_2') + S_2(u_2', N_2')$$

extremum w.r.t. u_2'

$$\left(\frac{\partial S_1}{\partial u_1'} \right)_{N_1', V_1'} = \left(\frac{\partial S}{\partial u_2'} \right)_{N_2', V_2'}$$

\rightarrow Subject to $N_1' + N_2' = N$
 $u_1' + u_2' = u$

this gives (as usual)

$$\frac{1}{T_1} = \frac{1}{T_2} = \frac{1}{T} = \left(\frac{\partial S}{\partial u} \right)_{N, V}$$

Next maximize entropy w.r.t N_2'

$$\left(\frac{\partial S}{\partial N_2'} \right) = 0$$

$$\left(\frac{\partial S_1}{\partial N_1'}\right)_{u_1', v_1'} (-1) + \left(\frac{\partial S_2}{\partial N_2'}\right)_{u_2', v_2'} = 0$$

→ Comes from differentiating w.r.t N_2'

$$\left(\frac{\partial S_1}{\partial N_1'}\right) = \left(\frac{\partial N_1'}{\partial N_2'}\right) \cdot \left(\frac{\partial S}{\partial N_1}\right)$$

$$\hookrightarrow \frac{\partial(N - N_2')}{\partial N_2'}$$

constant

$$\frac{\partial N}{\partial N_2'} = 0 \qquad \frac{\partial N_2'}{\partial N_2'} = (-1)$$

$$\sum_0 \left(\frac{\partial S_1}{\partial N_1'}\right)_{u_1', v_1'} = \left(\frac{\partial S_2}{\partial N_2'}\right)_{u_2', v_2'}$$

$$u_1' = u - u_2'$$

$$N_1' = N - N_2'$$

Each of these is a quantity characterizing individual system.

Equilibrium wrt DIFFUSION when these quantities for system ① & system ②, equal

Call this quantity CHEMICAL POTENTIAL

$$\frac{\mu_1}{T_1} = - \left(\frac{\partial S_1}{\partial N_1} \right)_{U_1, V_1}, \quad \frac{\mu_2}{T} = - \left(\frac{\partial S_2}{\partial N_2} \right)_{U_2, V_2}$$

-ve in definition → A POTENTIAL

(cf $\vec{E} = -\nabla\phi$) potential

μ maximises S .

↳ particles flow from system with LARGER μ → SMALLER μ .

CHEMICAL POTENTIAL $\mu = -T \left(\frac{\partial S}{\partial N} \right)_{U, V}$

$$S = kN \left\{ \ln \left[\frac{V}{N} \left(\frac{U}{3N} \right)^{3/2} \left(\frac{4\pi m}{h^2} \right)^{3/2} \right] + \frac{5}{2} \right\}$$

$$\frac{\mu}{T} = - \left(\frac{\partial S}{\partial N} \right)_{V,U}$$

$$= - \frac{\partial}{\partial N} \left(kN \left\{ \ln \left[\frac{V}{N} \left(\frac{U}{3N} \right)^{3/2} \left(\frac{4\pi m}{h^2} \right)^{3/2} \right] \right\} + \frac{5}{2} kN \right)$$

$$= - \frac{\partial}{\partial N} \left(kN \left\{ \ln \left[\frac{V}{N} \left(\frac{U}{3N} \right)^{3/2} \left(\frac{4\pi m}{h^2} \right)^{3/2} \right] \right\} \right) - \frac{5}{2} k$$

Differentiate this as a product

$$= -k \cdot 1 \cdot \ln \left[\frac{V}{N} \left(\frac{U}{3N} \right)^{3/2} \left(\frac{4\pi m}{h^2} \right)^{3/2} \right]$$

$$\textcircled{A} \Rightarrow \left\{ \begin{array}{l} - \left\{ kN \cdot \frac{\partial}{\partial N} \left(\ln \left[\frac{V}{N} \left(\frac{U}{3N} \right)^{3/2} \left(\frac{4\pi m}{h^2} \right)^{3/2} \right] \right) \right\} \\ - \frac{5}{2} k \end{array} \right\}$$

lets differentiate the term \textcircled{A}

$$\begin{aligned}
 \textcircled{A} &= -kN \frac{\partial}{\partial N} \left\{ \cancel{\ln V} + \cancel{\ln \left(\frac{U}{3} \right)^{3/2}} - \ln \left(\frac{1}{N} \right)^{5/2} \right. \\
 &\qquad \qquad \qquad \left. + \left(\frac{4\pi m}{h^2} \right)^{3/2} \right\} - \frac{5}{2} k
 \end{aligned}$$

V, U constant

$$= -kN \frac{\partial}{\partial N} \left(\underbrace{\ln \left(\frac{1}{N} \right)^{5/2}}_{\left(\frac{1}{N^{5/2}} \right)} \right) - \frac{5}{2} k$$

$$= -kN \frac{\partial}{\partial N} \left(-\frac{5}{2} \ln(N) \right) - \frac{5}{2} k$$

$$= +kN \cdot \frac{5}{2} \frac{1}{N} - \frac{5}{2} k$$

$$= +\frac{5}{2} k - \frac{5}{2} k = 0$$

only first term left

$$\left(-\frac{\partial S}{\partial N} \right)_{U,V} = -k \ln \left[\frac{V}{N} \left(\frac{U}{3N} \right)^{3/2} \left(\frac{4\pi m}{h^2} \right)^{3/2} \right]$$

for an ideal gas $U = \frac{3}{2} NkT$

$$\mu = -kT \left[\ln \left\{ \frac{V}{N} \left(\frac{1}{3N} \cdot \frac{3}{2} kT \right)^{3/2} \left(\frac{4\pi m}{h^2} \right)^{3/2} \right\} \right]$$

and $\frac{V}{N} = \text{density } \rho$

$$\mu = -kT \left[\ln \left\{ \left(\frac{1}{2} kT \right)^{3/2} \left(\frac{4\pi m}{h^2} \right)^{3/2} \cdot \frac{1}{\rho} \right\} \right]$$

$$\mu = -kT \ln \left[\frac{N - \text{indep}}{\rho} \right]$$

$$[\] > 1 \quad \therefore \ln [\] > 0$$

this is the same sort of behaviour
as entropy.

As $\rho \uparrow$ $[\] \downarrow$ and $\ln [\] \downarrow$

But $\mu = -kT \ln [\]$ makes $\mu \uparrow$.

AT LARGER DENSITY, μ IS LARGER

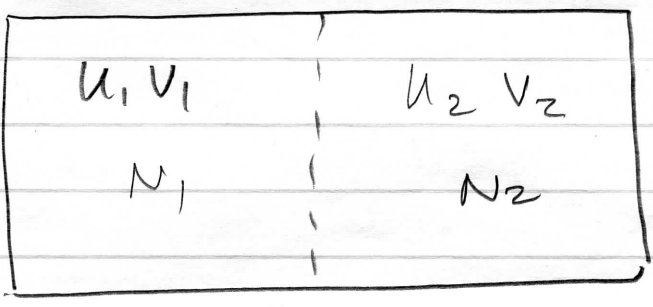
↳ PARTICLES flow from

LARGE $\rho \rightarrow$ SMALLER ρ

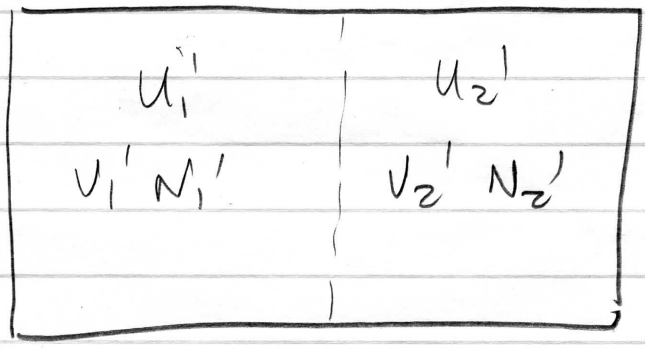
THIS IS DIFFUSION

↳ MAXIMIZES ENTROPY

go back to picture \rightarrow SHOULD STILL BE ON BOARD



IDEAL GAS



$$T_1 = T_2$$

$$\& \mu_1 = \mu_2$$

$$\frac{U_1'}{N_1'} = \frac{U_2'}{N_2'}$$



$$\frac{N_1'}{V_1} = \frac{N_2'}{V_2}$$

ENERGY
EQUIPARTITIONS

SAME
DENSITY

& KEPT
VOLUMES
CONSTANT.

Also have $N_1' + N_2' = N$

$$U_1' + U_2' = U$$

So have 4 equations for 4 unknown.

Can find U_1' U_2' N_1' N_2'

if know N, U, V_1, V_2

Clearly would have been better to

call this particular $\mu \rightarrow$ DIFFUSIVE POTENTIAL

\rightarrow "CHEMICAL" comes from use in chemistry.

the $-\mu$ generally needed to

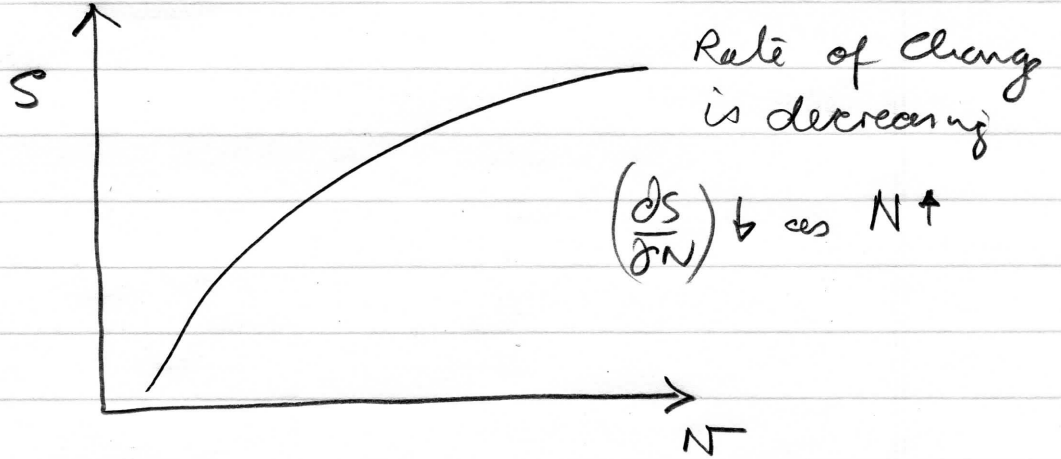
give μ property as $N \uparrow \rightarrow \mu \uparrow$

This is because in all "NORMAL SYSTEMS"

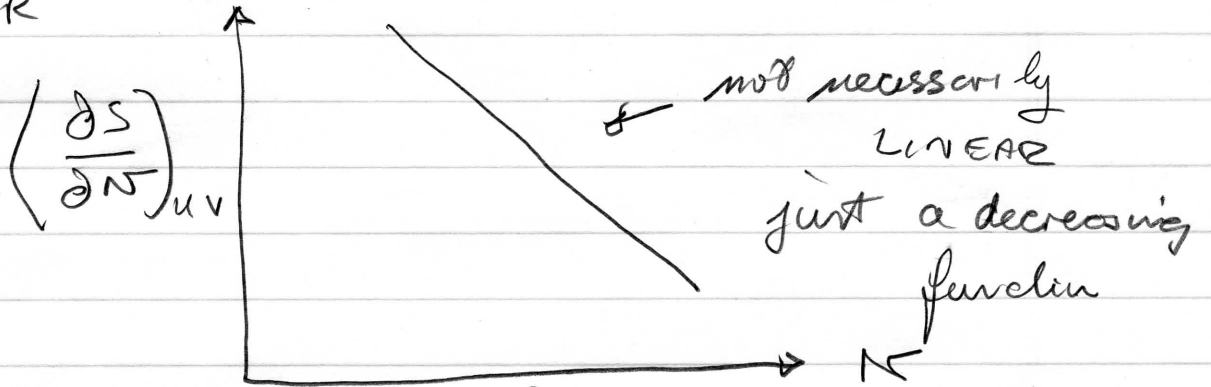
$\left(\frac{\partial S}{\partial N}\right)_{UV}$ has RATE of CHANGE which DECREASES with N

$\rightarrow \left(\frac{\partial^2 S}{\partial N^2}\right) < 0$ is -ve.

$\left(\frac{\partial^2 S}{\partial N^2}\right)$ -ve looks like



OR



could also write $\frac{\partial S}{\partial u} \sim u$

When 2 systems of

DIFFERENT

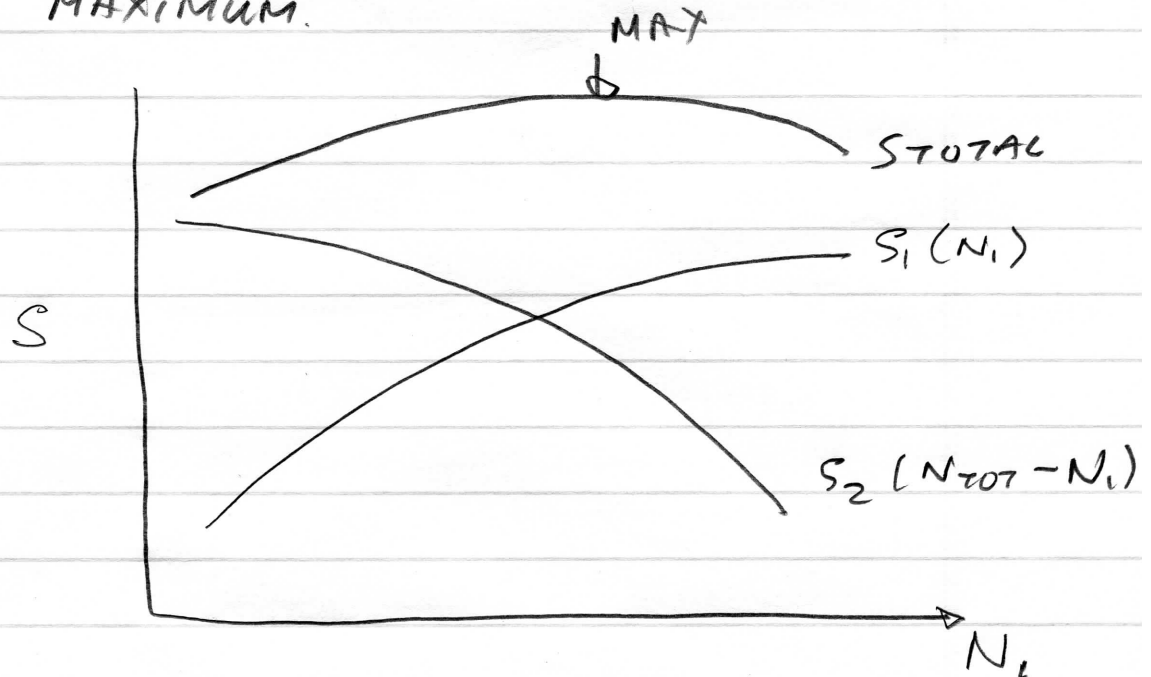
- TEMP
- PRESSURE
- DENSITY

put in contact, one entropy will

DECREASE, and the other entropy

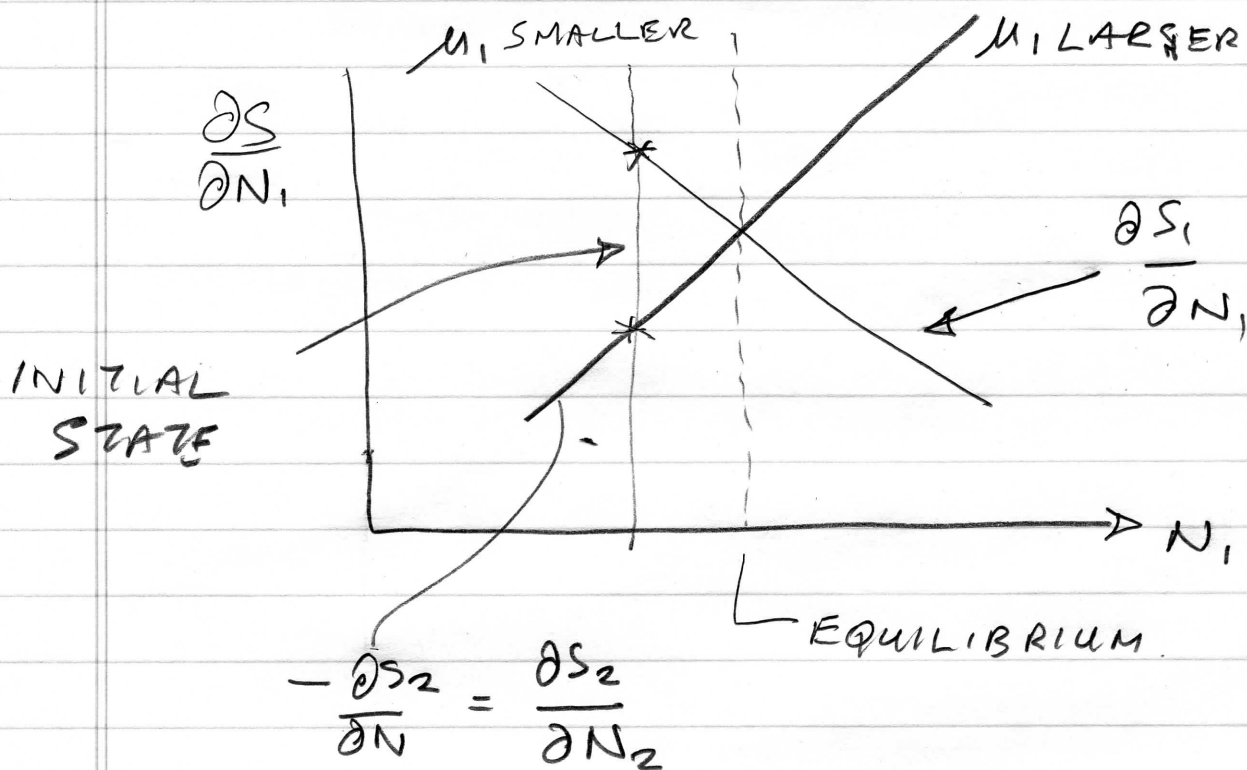
will INCREASE, until total Entropy

is MAXIMUM.



Must be that $\frac{\partial S_2}{\partial N_2} \downarrow$ as $N_2 \uparrow$
 or $N_1 \downarrow$

For DERIVATIVES will have something like



Suppose INITIAL (NON-EQUILIBRIUM) state

has value of N_1 at * in drawing

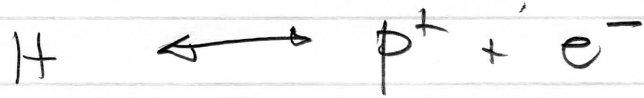
Since μ_1 is SMALLER, there will be a

flow of particles TOWARDS SYSTEM ①

N_1 increases towards equilibrium point

$\mu \rightarrow$ FLOW from Large μ
↓
Small μ

Now go back to dissociation of Hydrogen.



$$\begin{array}{l} \text{REST} \\ \text{ENERGY} \end{array} \rightarrow \begin{array}{l} mc^2 \\ \text{Atom} \end{array} = \begin{array}{l} mc^2 \\ p^+ \end{array} + \begin{array}{l} mc^2 \\ e^- \end{array} - I$$

$$I = \text{IONIZATION ENERGY } (= 13.6 \text{ eV})$$

$$1 \text{ eV} \approx 1.6 \times 10^{-19} \text{ J}$$

$$k_B \approx 1.4 \times 10^{-23} \text{ J/K}$$

$$k_B \cdot T \approx 13.6 \times 1.6 \times 10^{-19}$$

$$T \approx \frac{13.6 \times 1.6 \times 10^{-19}}{1.4 \times 10^{-23}} \text{ . K}$$

$$\sim 10^5 \text{ K}$$

Temp at which gas becomes

PLASMA of $p \ e^-$

(18)
11

For $kT \lesssim 10^4 \text{ K}$ don't expect an appreciable amount of ionized hydrogen.
↳ energy of collisions not high enough to strip off electrons

From Special Relativity

$$E_{\text{TOTAL}}^2 = m^2 c^4 + \vec{p}^2 c^2$$

for $\vec{p} = 0$ $E_{\text{TOT}}^2 = m^2 c^4$

for $\vec{p}^2 c^2 \ll m^2 c^4$ can expand as

$$\sqrt{1+x^2} \approx 1 + \frac{x^2}{2} + \dots \quad x \ll 1$$

$$U = \sqrt{p^2 c^2 + m^2 c^4} = m c^2 \sqrt{1 + \frac{p^2}{m^2 c^2}}$$

$$\vec{p} \ll m c, \quad \vec{v} \ll c$$

$$U = (p^2 c^2 + m^2 c^4)^{\frac{1}{2}}$$

$$= (m^2 c^4)^{\frac{1}{2}} \left(1 + \frac{p^2 c^2}{m^2 c^4} \right)^{\frac{1}{2}}$$

$$= mc^2 \left(1 + \frac{p^2}{m^2 c^2} \right)^{\frac{1}{2}}$$

$$\approx mc^2 \left(1 + \frac{1}{2} \frac{p^2}{m^2 c^2} \right)$$

$$\approx mc^2 + \frac{\vec{p}^2}{2m}$$

↑ kinetic energy

kinetic energy (w Total Energy)

of a non relativistic particle

$$= mc^2 + \frac{\vec{p}^2}{2m}$$

So for a gas of non-relativistic particles

$$U = Nmc^2 + \frac{1}{2m} \sum_{i=1}^N \vec{p}_i^2$$

↑
Total kinetic energy including
SELF ENERGY

When considered MULTIPLICITY absorbed

Nmc^2 into U , making this

explicit

$$\frac{1}{2m} \sum_{i=1}^N \vec{p}^2 = \underbrace{(U - Nmc^2)}$$

total energy of gas

with SELF ENERGY subtracted.

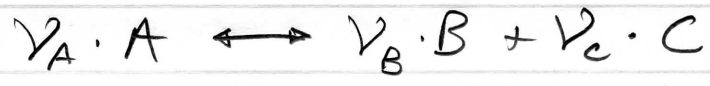
$$\frac{\Omega_U}{\sum \# \vec{p}^2}$$

↓
non relativistic $U_{KINETIC}$

- Rest energy does not affect MULTIPLICITY of a system of non-relativistic particles
- Only have to account for Rest Energy in CHEMICAL REACTIONS.

Suppose gas container with 3 gases

(A, B, C) → can undergo some reaction



(for $H \leftrightarrow p^+ + e^-$ all $\nu = 1$)

Total Entropy is

$$S_{TOTAL} = S_A (U_A^{kin}, N_A, V) + S_B (U_B^{kin}, N_B, V) + S_C (U_C^{kin}, N_C, V)$$

KEEP ON BOARD

$N_A, N_B, N_C \rightarrow$ same volume.

U_A^k, U_B^k, U_C^k

S_A depends on $U_A^k = \sum_{i=1}^{N_A} \frac{p_i^2}{2m_A} \quad \text{--- (1)}$

$N_A, N_B, N_C \rightarrow$ can change

\rightarrow take account of TOTAL ENERGY affected by dissociation of H

\hookrightarrow it fixes energy I

\uparrow
this is fixed for A, B, C

not $\sum_{A+B+C} U^k$

Write ① as \rightarrow

$$U_A^k = U_A - N_A m_A c^2$$

$$U_B^k = U_B - N_B m_B c^2$$

$$U_C^k = U_C - N_C m_C c^2$$

$$U_{TOT} = U_A + U_B + U_C$$

$$= U_A^k + U_B^k + U_C^k + N_A m_A c^2 + N_B m_B c^2 + N_C m_C c^2$$

N_A, N_B, N_C can change

↳ under constraint $\nu_A \cdot A \leftrightarrow \nu_B \cdot B + \nu_C \cdot C$



$$N_A = N_A^0 - \nu_A \xi$$

$$N_B = N_B^0 + \nu_B \xi$$

$$N_C = N_C^0 + \nu_C \xi$$

↑ initial concentrations

N_A decreases by ν_A

N_B & N_C increase by ν_B, ν_C

SHOULD
BE
ON
BOARD

$$S_{TOTAL} = S_A(\mu_A^k, N_A, V) + S_B(\mu_B^k, N_B, V) + S_C(\mu_C^k, N_C, V)$$

$$S_{TOTAL} = S_A \left(\underbrace{U_A - N_A^0 m_A c^2 + N V_A m_A c^2}_{U_A^K}, \underbrace{N_A^0 - V_A N}_{N_A} \right)$$

KEEP
ON
BOARD

$$+ S_B \left(U_B - N_B^0 m_B c^2 - N V_B m_B c^2, N_B^0 + V_B N \right)$$

$$+ S_C \left(\underbrace{U_{TOT} - U_A - U_B - N_0^0 m_C c^2 - N V_C m_C c^2}_{U_C^K}, \underbrace{N_C^0 + V_C N}_{N_C} \right)$$

$$= S_{TOT} (U_A, U_B, N) \quad \leftarrow \textcircled{2}$$

there 3 variables can vary ☺

• TOTAL U fixed

• N_A, N_B, N_C can change under the constraint

$$V_A \cdot A \leftrightarrow V_B \cdot B + V_C \cdot C$$

OK. SO want to maximize $\textcircled{2}$

as a function of U_A, U_B, N

$$\left(\frac{\partial S_{TOT}}{\partial U_A}\right)_{U_B, N_A} = 0 = \underbrace{\left(\frac{\partial S_A}{\partial U_A^k}\right)_{N_A} - \left(\frac{\partial S_C}{\partial U_C^k}\right)_{N_C}}_{\text{just } \frac{1}{T_A} = \frac{1}{T_C}}$$

$$\left(\frac{\partial S_{TOT}}{\partial U_B}\right)_{U_A, N_B} = 0 = \underbrace{\left(\frac{\partial S_B}{\partial U_B^k}\right)_{N_B} - \left(\frac{\partial S_C}{\partial U_C^k}\right)_{N_C}}_{\frac{1}{T_B} = \frac{1}{T_C}}$$

So $T_A = T_B = T_C$

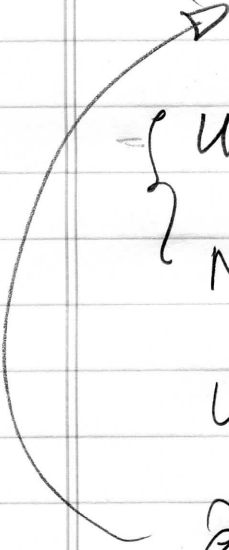
$\left(\frac{\partial S_{TOT}}{\partial N}\right)_{U_A, U_B}$ is slightly more complex.

Look at first term as an example

$$\left(\frac{\partial S_{TOT}}{\partial N}\right)_{U_A, U_B} = 0 = \left(\frac{\partial S_{TOT}}{\partial N}\right)_{U_A^k, N_B} + \dots$$

$$= \left(\frac{\partial S_A}{\partial N}\right)_{N_A} + \left(\frac{\partial S_A}{\partial N}\right)_{U_A^k}$$

$$= \left(\frac{\partial S_A}{\partial U_A^k}\right)_{N_A} \left(\frac{\partial U_A^k}{\partial N}\right) + \left(\frac{\partial S_A}{\partial N_A}\right)_{U_A^k} \left(\frac{\partial N_A}{\partial N}\right)$$



$$\begin{cases} U_A^k = U_A - N_A m_A c^2 \\ N_A = N_A^0 - V_A N \end{cases}$$

$$U_A^k = U_A - N_A^0 m_A c^2 + V_A N m_A c^2$$

$$\frac{\partial U_A^k}{\partial N} = V_A m_A c^2$$

$$N_A = N_A^0 - V_A N$$

$$\frac{\partial N_A}{\partial N} = (-V_A)$$

putting in all the other terms

$$\left(\frac{\partial S_{tot}}{\partial N}\right)_{U_A, U_B} = 0 = \underbrace{\left(\frac{\partial S_A}{\partial U_A^k}\right)_{N_A}}_{\frac{1}{T}} \cdot v_A m_A c^2 + \underbrace{\left(\frac{\partial S_A}{\partial N_A}\right)_{U_A^k}}_{-\frac{\mu_A}{T}} (-v_A)$$

$$+ \left(\frac{\partial S_B}{\partial U_B^k}\right)_{N_B} (-v_B m_B c^2) + \underbrace{\left(\frac{\partial S_B}{\partial N_B}\right)_{U_B^k}}_{-\frac{\mu_B}{T}} v_B$$

$$+ \left(\frac{\partial S_C}{\partial U_C^k}\right)_{N_C} (-v_C m_C c^2) + \underbrace{\left(\frac{\partial S_C}{\partial N_C}\right)_{U_C^k}}_{-\frac{\mu_C}{T}} v_C$$

$$0 = \frac{v_A m_A c^2}{T} + \frac{v_A \mu_A}{T} \quad (3)$$

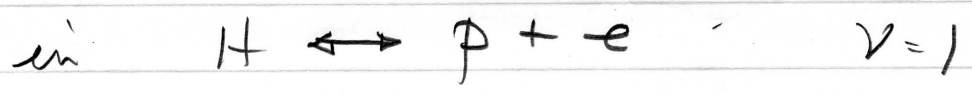
$$- \frac{v_B m_B c^2}{T} - \frac{v_B \mu_B}{T}$$

$$- \frac{v_C m_C c^2}{T} - \frac{v_C \mu_C}{T}$$

$$v_A \mu_A - v_B \mu_B - v_C \mu_C = - (v_A m_A c^2 - v_B m_B c^2 - v_C m_C c^2) -$$



LAW of MASS ACTION



$$m_H c^2 - m_p c^2 - m_e c^2 = -T$$



13.6 eV

Can also write (3) as

$$V_A \mu_A' = V_B \mu_B' + V_C \mu_C'$$

where $-\frac{\mu_A'}{T} = \frac{V_A m_A c^2}{T} + V_A \frac{\mu_A}{T}$

$$= V_A \left\{ \frac{m_A c^2}{T} + \frac{\mu_A}{T} \right\}$$

$$\frac{\partial S_A}{\partial U_A^k} \cdot \frac{\partial U_A^k}{\partial N_A}$$

$$\frac{\partial S_A}{\partial N_A} \cdot \frac{1}{T}$$

$$\frac{1}{T}$$

$$U_A^k = U_A - N_A m_A c^2$$

$$\partial U_A^k = -m_A c^2$$

$$-\mu_A$$

$$-\frac{\mu_A'}{T} = \left(\frac{1}{T}\right)(-m_A c^2) - \frac{\mu_A}{T}$$

$$\mu_A' = \mu_A + m_A c^2$$

Rest Energy modifies Chemical Potential

CHEMICAL POTENTIAL → Central to
Study of Equilibrium Chemical Reactions

$$\mu_{A'} = \mu_A + \underbrace{mAc^2}_{\substack{\uparrow \\ \text{Ionization changes} \\ \text{CHEMICAL POTENTIAL}}}$$

μ_A can shift due to other energy

Sources → prob 3.37
↓
potential energy

$$\textcircled{1} \mu_H - \mu_P - \mu_e = -(-I) = I$$

13.6 eV

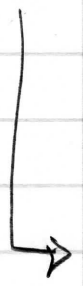
LAW OF MASS ACTION

Treat H, P, e as mono-atomic ideal gas.

$$\mu_H = T \left(\frac{-\partial S}{\partial N} \right)_{UV}$$

For an ideal gas:

$$\mu_{H, P} = -kT \ln \left[\left(\frac{4\pi m_H}{h^2} \right)^{3/2} \left(\frac{kT}{2} \right)^{3/2} + \frac{1}{P_{H,P}} \right]$$



Subst into ① above.

$$-kT \ln \left[\left(\frac{4\pi m_H}{h^2} \cdot \frac{kT}{2} \right)^{3/2} \cdot \frac{1}{P_H} \right]$$

$$+ kT \ln \left[\left(\frac{4\pi m_P}{h^2} \cdot \frac{kT}{2} \right)^{3/2} \cdot \frac{1}{P_P} \right]$$

$$+ kT \ln \left[\left(\frac{4\pi m_e}{h^2} \cdot \frac{kT}{2} \right)^{3/2} \cdot \frac{1}{P_e} \right] = I$$

$$\ln \left[\frac{P_H}{P_P P_e} \left(\frac{4\pi m_P m_e}{m_H} \cdot \frac{kT}{2} \right)^{3/2} \right] = \frac{I}{kT}$$

$\ln A = I, \quad \ln (A^{-1})^{-1} = I, \quad -\ln A^{-1} = I$
 $\ln A^{-1} = -I$

$$\ln \left[\frac{P_H}{P_P P_e} \left(\frac{4\pi m_p m_e}{m_H} \cdot \frac{kT}{2} \right)^{3/2} \right]^{-1} = \frac{-I}{kT}$$

$$\frac{P_P P_e}{P_H} \left(\frac{4\pi m_p m_e}{m_H} \cdot \frac{kT}{2} \right)^{-3/2} = e^{-\frac{I}{kT}}$$

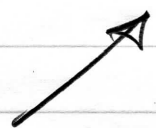


Saha Equation

Relates Equilibrium Concentration
of P, e, H as function of Temp
and ionization energy.

$$m_p \approx m_H$$

$$\frac{P_P P_e}{P_H} \left(\frac{4\pi m_e}{h^2} \cdot \frac{kT}{2} \right)^{-3/2} = e^{-I/kT}$$



BOLTZMAN
FACTOR.

for neutral gas

$$P_P = P_e$$

$$e^{-\frac{\text{ENERGY}}{kT}}$$

$$\frac{P_P^2}{P_H}$$

$$\frac{p_p}{p_H} \frac{1}{(\dots)} = e^{-I/kT}$$

$$I = 13.6 \text{ eV} \rightarrow \sim T \sim 10^5 \text{ K}$$

So $kT \ll 10^5 \text{ K} \rightarrow$ NO IONIZATION

e p at 10^5 K non relativistic

$$\left. \begin{aligned} m_e &= 0.5 \times 10^6 \text{ eV} \\ m_p &= 10^9 \text{ eV} \end{aligned} \right\} \text{ cf } 13.6 \text{ eV}$$

$$m^2 c^4 \sim E^2$$

SCHRÖDINGER derives CHEMICAL POTENTIAL
using GIBBS FREE ENERGY

above is false.