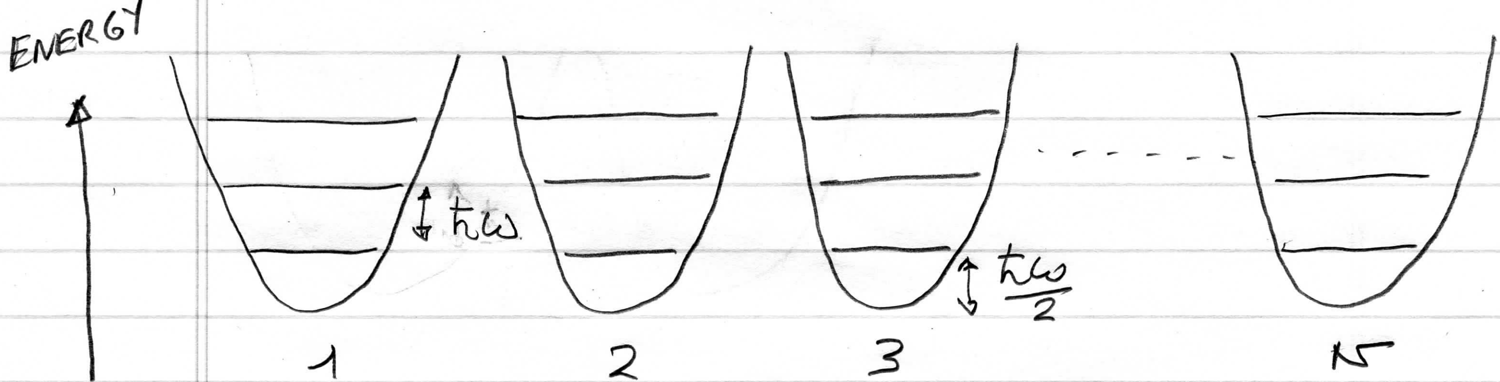
EINSTEIN MODEL OF SOLID

Unrealistic but useful



Quadratic potential

↳ Equally spaced energy levels $\hbar\omega$ ← frequency

↳ Simple Harmonic Oscillator
SHO

EINSTEIN MODEL — N 3-dimensional QUANTUM SHO

POTENTIAL $V(x,y,z) = \frac{m\omega^2}{2}x^2 + \frac{m\omega^2}{2}y^2 + \frac{m\omega^2}{2}z^2$

m = mass of oscillator.

– OSCILLATORS are quantum, but for brevity set $\hbar\omega = 1$ ← can put back later if we want

— each atom → 3 independent SHO.

– For n atoms, # OSC = $N = 3n$

↳ EINSTEIN SOLID — N INDEP SHOs

? What are the MICROSTATES

$$(n_1, n_2, n_3 \cdots\cdots n_N)$$

↳ number of energy units in that OSC

n_1 

FOR EACH ATOM $\quad n_i = 0, 1, 2, 3, \cdots\cdots \infty \quad (\times \hbar\omega)$ ↳ $=1$

↳ specifying all n_i is an Energy STATE

EACH MICROSTATE is a collection of N non-negative integers

MICROSTATE

$$(n_1, n_2, n_3 \cdots\cdots n_N)$$

A MACROSTATE is a state of a

Particular ENERGY $\quad q$ = TOTAL ENERY
$= q \times \underbrace{(\hbar\omega)}_{=1}$

$$q = n_1 + n_2 + n_3 + \cdots + n_N.$$

ACTUALLY EACH OSC ENERGY $= \hbar\omega(n + \frac{1}{2})$

ignore.

IN contrast to N spin LATTICE

→ Einstein solid has ∞ number of micro states — even for one simple oscillator.

→ energy of oscillator is not BOUNDED from ABOVE

this is important

WHAT IS MULTIPLICITY of MACROSTATE

We call this as before

$$\Omega(N, q)$$

↳ TOTAL energy
= q quanta
→ labels MACROSTATE

N OSCILLATOR

As usual this is just a case
of COMBINATORICS.

– image have N boxes

each corresponds to
one of the oscillators

– and have
q balls to distribute

each ball is a quantum
of energy.

How many way can we put q balls into N boxes.

→ this is WHAT WE CALL $\Omega(N, q)$

MULTIPLICITY

$$\begin{array}{ccccccccccccc} & 1 & & 2 & & 3 & & & & N-2 & & N-1 & \\ \text{box}_1 & | & \text{box 2} & | & \text{box 3} & | & \text{----} & & \text{box}_{N-2} & | & \text{box}_{N-1} & | & \text{box } N \end{array}$$

Have $N-1$ partitions & N boxes.

represent quantum of energy by •

$$\begin{array}{ccccccccc} & 1 & & 2 & & 3 & & N-1 & \\ \bullet\ \bullet & | & \bullet & | & \bullet\ \bullet\ \bullet & | & \text{---} & | & \bullet \end{array}$$

quantum of energy

This picture completely specifies MICROSTATE

→ $n_1 = 2$, $n_2 = 1$, n_3 ..... $n_N = 1$

there are always q •

and $N-1$ | ← PARTITION.

always q • and $N-1$ |

so there are $q+N-1$ symbols.

given q and N the number of possible arrangements =

Number of ways of choosing q of $q+N-1$ symbols to be dots •

(q out of $q+N-1$)

$$= \binom{q+N-1}{q}$$

$(N-1+q)!$ → ways to order $N-1$ partitions + q balls.

But relabelling the partitions makes NO DIFFERECE → SAME MICROSTATE

$(N-1)!$ WAYS to arrange partition

$$\rightarrow \frac{(N-1+q)!}{(N-1)!}$$

— all the balls are identical so if we relabelled them

I.E. $|\,\circ\ \circ\,|\,\circ\,|\,\circ\ \circ\ \circ\,|$

$\downarrow$

$|\,\circ\,|\,\circ\ \circ\ \circ\,|\,\circ\ \circ\,|$ etc

SAME MICROSTATE — $q!$ WAYS

SO $$\Omega(N,q) = \frac{(N-1+q)!}{(N-1)!\,q!}$$

Multiplicity of MACROSTATE with q quanta

Coin Choosing Example in Text

→ see page 22

H T ------- T T ------- T

↓

H T

same as T H $\frac{1}{2} \rightarrow \frac{1}{n!}$

in case of

| • • | • • • |

↑ we "choose" these in some order.

| | → | |

| • | | • |

| • • | | • • |

no I was wrong

All this stuff is just Combinatorics of choosing n out of N

OBJECTS

$$\Omega(N,n) = \frac{N!}{n!\,(N-n)!} = \binom{N}{n}$$

choose

In case of Einstein Solid

OBJECTS $\longrightarrow$ $N_A - 1 + q_A'$

CHOOSE q_A'

so $N \longrightarrow N_A - 1 + q_A'$

$n! \longrightarrow q_A!$

$(N-n)! \longrightarrow N_A - 1 + q_A' - q_A'$

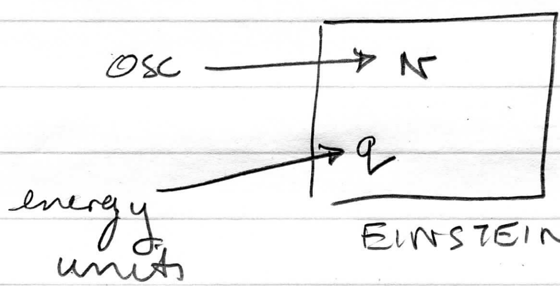
$$\text{So } \Omega(N_A, q_A') = \frac{(N_A - 1 + q_A)!}{q_A'\,(N_A - 1)!}$$

POSTULATE of STAT MECH

— TD Equilibrium in a SYSTEM OF N SHO

→ Reached through some unspecified interaction between the oscillators.

FOR A SINGLE SYSTEM OF OSCILLATORS



EINSTEIN SOLID

this ISOLATED system can be found with EQUAL PROBABILITY IN each of its $\Omega(N,q)$ accessible Microstates with

$$\text{PROBABILITY} \quad \frac{1}{\Omega(N,q)}$$

the system is IN SOME microstate

→ SO SUM over all PROBS = 1

SO each MICROSTATE PROB = $\frac{1}{\Omega}$

look at it this way.

$$\frac{1}{\#\text{ accessible } \mu\text{state}} = \frac{1}{\Omega(N,q)}$$

all equally probable