HEAT ENGINES

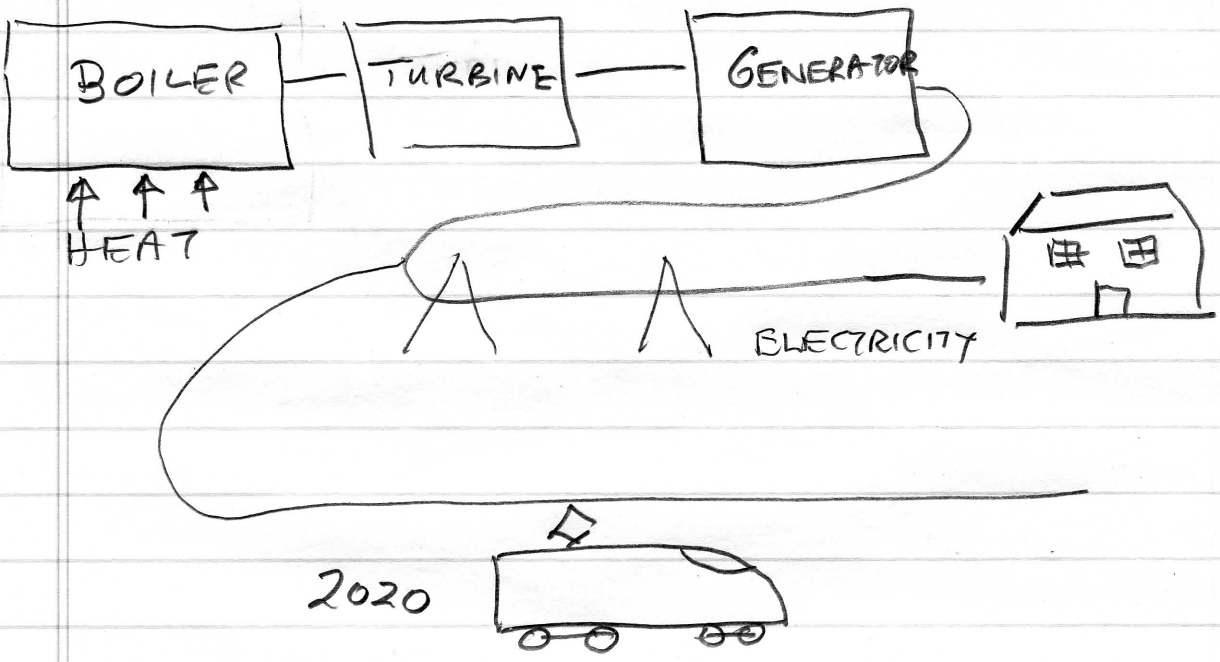
KARL MARX → Philosophers has discussed the world, the point, however, is to change it.

"Clausius" → Physicists have discussed entropy the point, however, is to change.

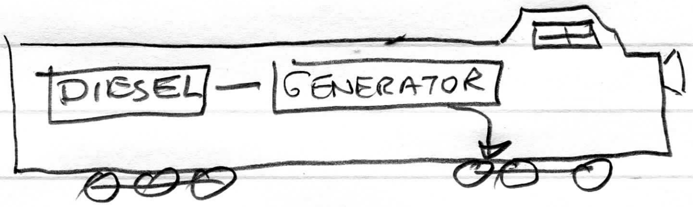
In a "traditional" course in thermodynamics Heat Engines would form a major part of the course. In our course we have concentrated on "theoretical Physics" aspects

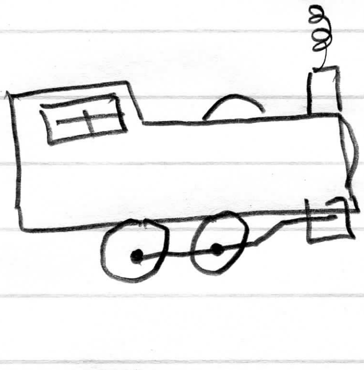
However . . .

The USE of HEAT → Doing USEFUL WORK



2018



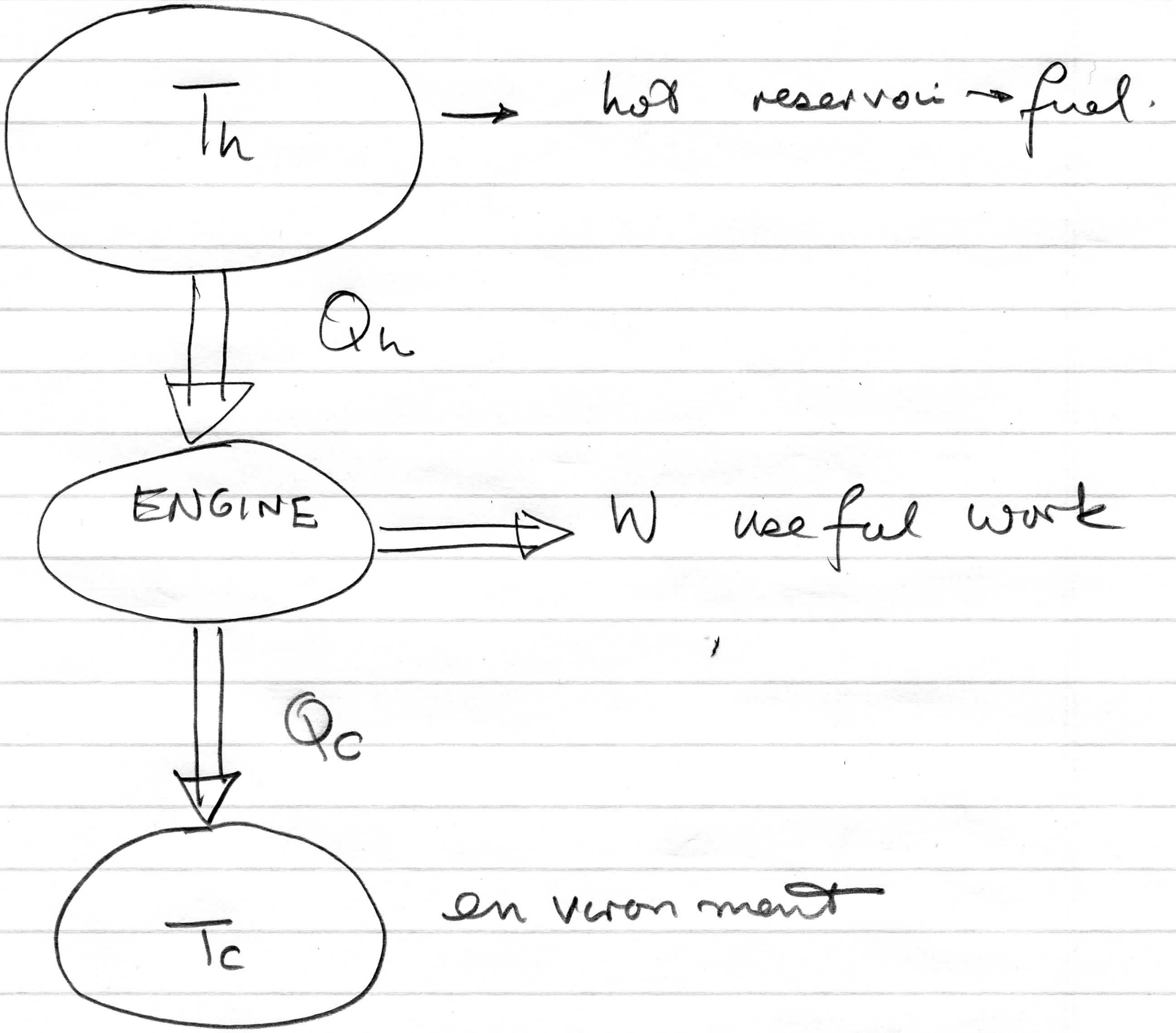


1920
1820

etc Automobiles
refrigerators
aircraft.

LIMITATION on Efficiency of heat Engines

Follows from Second Law



- Heat Engine absorbs some heat Q_h from HOT object (reservoir at T_h)

- uses it to do work W

- when heat engine absorbs Q_h at T_h then entropy of the HOT RESERVOIR DECREASES $\rightarrow -\frac{Q_h}{T_h}$

- By 2ND Law entropy of heat Engine INCREASES by at LEAST THAT AMOUNT

$$+\frac{Q_h}{T_h}$$

- Heat engine come back to initial State $\rightarrow$ Dump heat Q_c @ T_c into environment

  ↳ increase entropy of environm by $\frac{Q_c}{T_c}$

— at end of Cycle

↳ Engine in SAME STATE

↳ Entropy unchanged

But Total entropy ΔS

$\Delta S =$ Decrease of Hot Reservoir + Increase of Cold reservoir

$$-\frac{Q_h}{T_h} + \frac{Q_c}{T_c} \geq 0$$

↑ 2ND law.

So $\frac{Q_c}{T_c} \geq \frac{Q_h}{T_h} \Rightarrow \frac{Q_c}{Q_h} > \frac{T_c}{T_h}$

1ST LAW → ENERGY CONSERVATION

→ Heat Engine comes back to original state

$$Q_h = W + Q_c$$

Rotate wheels

$$\mathcal{E} = \frac{\text{BENEFIT}}{\text{COST.}} = \frac{W}{Q_h}$$

Burn oil = $ $

Since $Q_h = W + Q_c$

$$\mathcal{E} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

BUT

$$\frac{Q_c}{Q_h} \geqslant \frac{T_c}{T_h}$$

Room Temp

So $\mathcal{E} \leqslant 1 \; ; = 1 - \frac{T_c}{T_h}$

1000°K

$\approx 70\%$

Real Engine
much lower

$\sim 40\%$.

Many PRACTICAL heat engines

STUDY IMPRACTICAL one → get basic idea

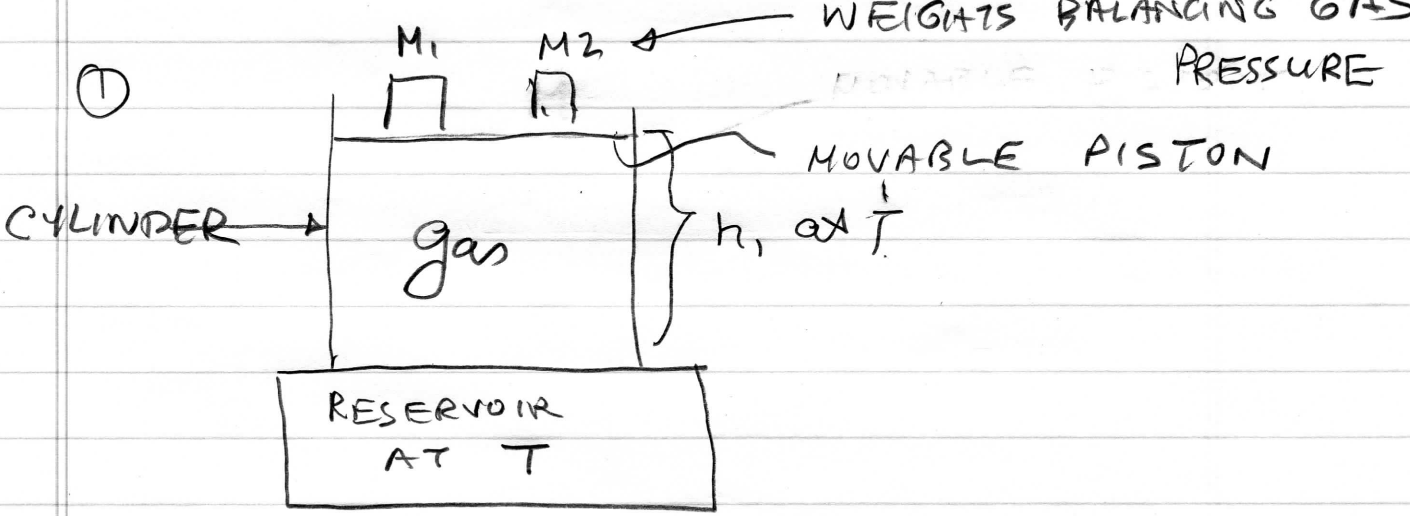
→ STIRLING ENGINE

Heat engines work in a cyclic manner

↓

STUDY STIRLING CYCLE

①



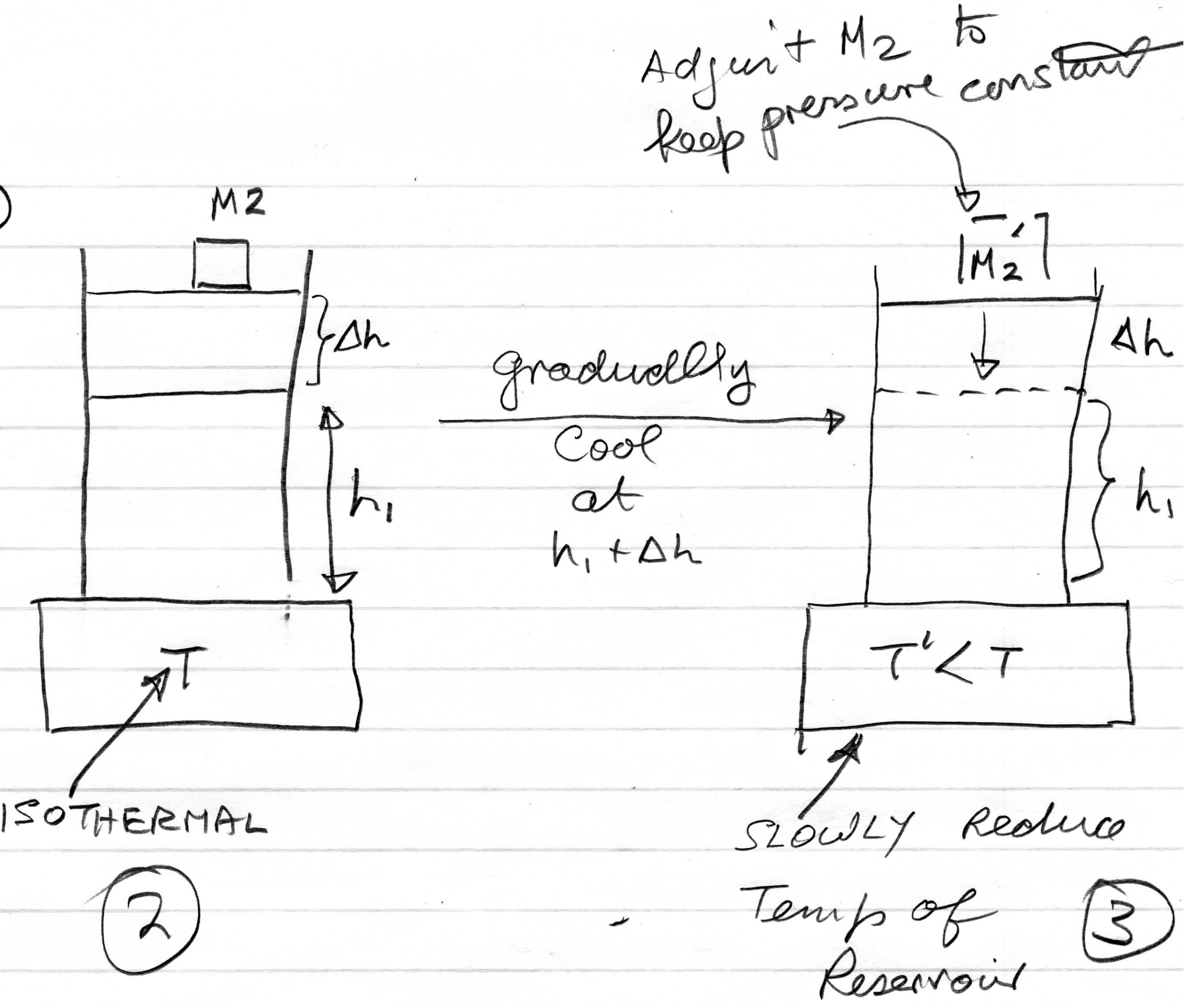
① → ② SLOWLY slide M1 off

→ gas EXPANDS ISOTHERMALLY at T

from $h_1 \rightarrow h_1 + \Delta h$

gas does useful work in raising M2 + ABSORBS heat from Reservoir

② → ③



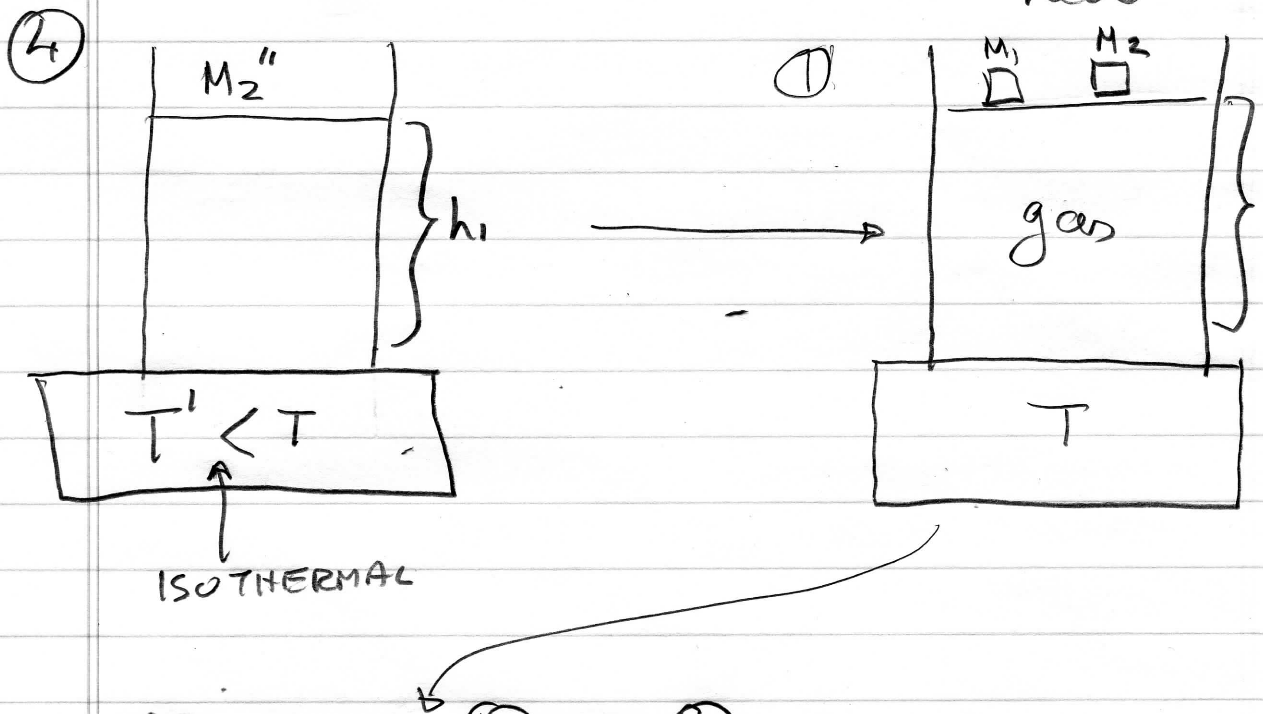
See REGENERATOR in Text fig 4·7

③ → ④ let gas ISOTHERMALLY CONTRACT → h_1

Reservoir at $T' < T$

↳ adjust M_2 → doing work on gas

→ gas gives away heat

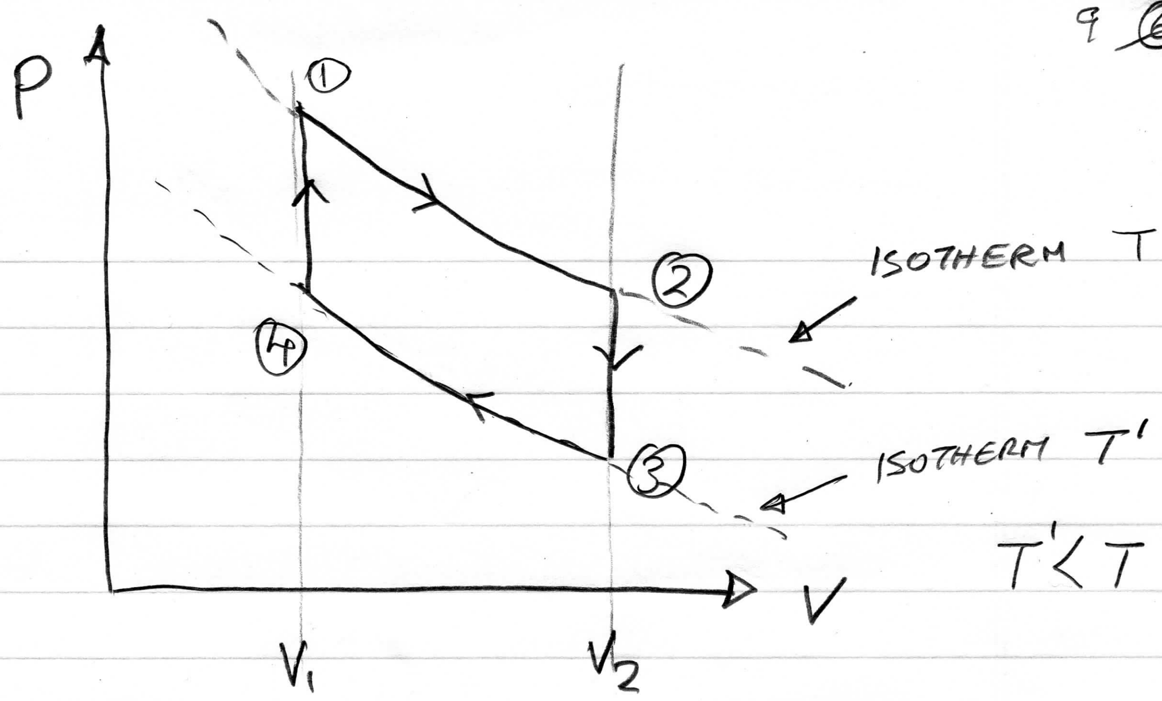


④ → ①

Heat Reservoir + gas at Constant V

Temp of Reservoir $T' \to T$ ← original

CYCLE.



ASSUME IDEAL GAS

① → ②
$$W_{12}^{gas} = kT\ln\frac{V_2}{V_1}$$ → work gas does on M_2

$$Q_{12}^{absorbed} = kT\ln\frac{V_2}{V_1}$$ heat

② → ③
$$Q_{23}^{\text{"Absorbed"}} = C_V(T'-T) < 0$$ gas realeases Q

$$W_{23} = 0$$

③ → ④
$$W_{gas} = -kT'\ln\frac{V_2}{V_1}$$ work done on gas by M_2'

$$Q_{34}^{Absorbed} = -kT'\ln\frac{V_2}{V_1}$$ gas gives away heat in ISOTHERMI PROCESS

④ → ①
$$Q_{41} = C_V(T-T') > 0$$ ISOCHORIC HEATING

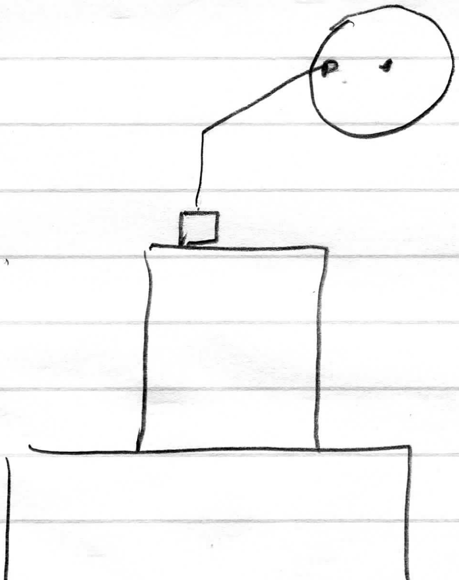
$$W_{41} = 0$$

STIRLING CYCLE → energy of gas always ends up same at ①

① → T.D EQUILIBRIUM P_1 V_1 SAME

in ① → ② gas does work. by raising weight

↳ Can imagine some set of levers & gears which change motion into Rotation of wheel



Gas has work done ON it in ③ → ④

TOTAL WORK $= W_{12}^{gas} + W_{34}^{gas}$

↓

Benefit $= k(T-T')\ln\frac{V_2}{V_1}$

↳ Total Area of PV diagram.

You have to put heat in ④ → ①

↳ costs \$\$ (or FrF)

not a cost

$$Q^{ABSORBED}_{1-2-3-4} = Q_{12}^{ABSORBED} + \cancel{Q_{23}^{GIVEN\ OFF}}$$

$$+ \cancel{Q_{34}^{GIVEN\ OFF}} + Q_{41}^{ABSORBED}$$

not a cost.

What reservoir has to provide is

$$Q_{12}^{Absorbed} + Q_{41}^{absorbed}$$

$$\text{Cost} = kT\ln\frac{V_2}{V_1} + C_V(T-T')$$

$$\text{efficiency} = \frac{\text{benefit}}{\text{cost}} = \frac{K(T-T')\ln V_2/V_1}{kT\ln\frac{V_2}{V_1} + C_V(T-T')}$$

$$e = \frac{kT\ln V_2/V_1 + C_v(T-T') - \left(kT'\ln\frac{V_2}{V_1} + C_v(T-T')\right)}{kT\ln\frac{V_2}{V_1} + C_v(T-T')}$$

$$= 1 - \frac{C_v(T-T') + kT'\ln V_2/V_1}{\underbrace{C_v(T-T') + kT\ln V_2/V_1}_{<0 \text{ since } T'<T}}$$

If $Q^{\text{absorb}}_{4\to 1}$ is taken from Regenerator (see text), and given away to it in $2\to 3$

↳ $Q_{41}^{\text{"Absorbed"}}$ not a "cost"

↳ then $\underbrace{e = 1 - \frac{T'}{T}}_{\text{CARNOT CYCLE}}$

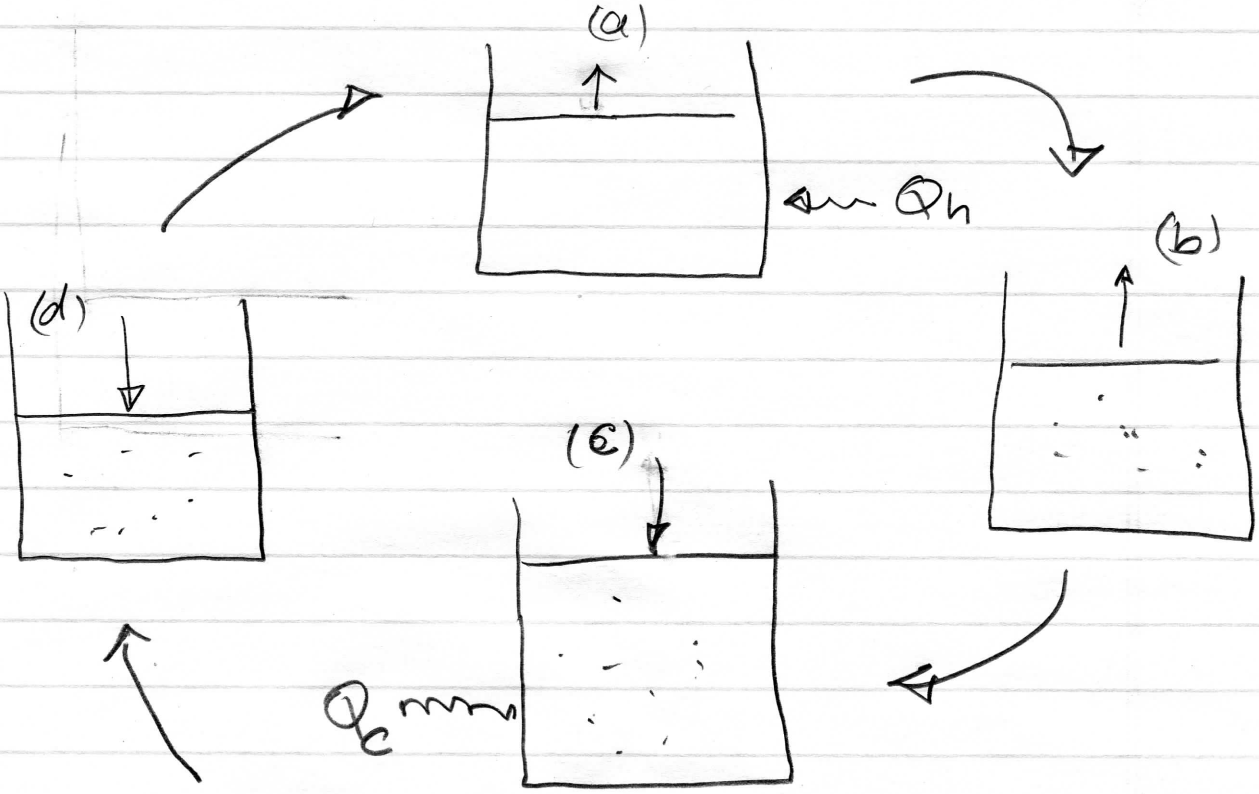
To MAKE EFFICIENCY → 1

Need to minimize Second term in

$$\varepsilon = 1 - \frac{C_V(T-T') + kT'\ln\frac{V_2}{V_1}}{C_V(T-T') + kT\ln V_2/V_1}$$

easiest to see from CARNOT CYCLE

$$e = 1 - \frac{T'}{T} \begin{matrix}\leftarrow \text{small}\\ \leftarrow \text{large.}\end{matrix}$$



a) Isothermal Expansion absorbing Q_h

b) Adiabatic expansion to T_c

c) Isothermal Compression at T_c while

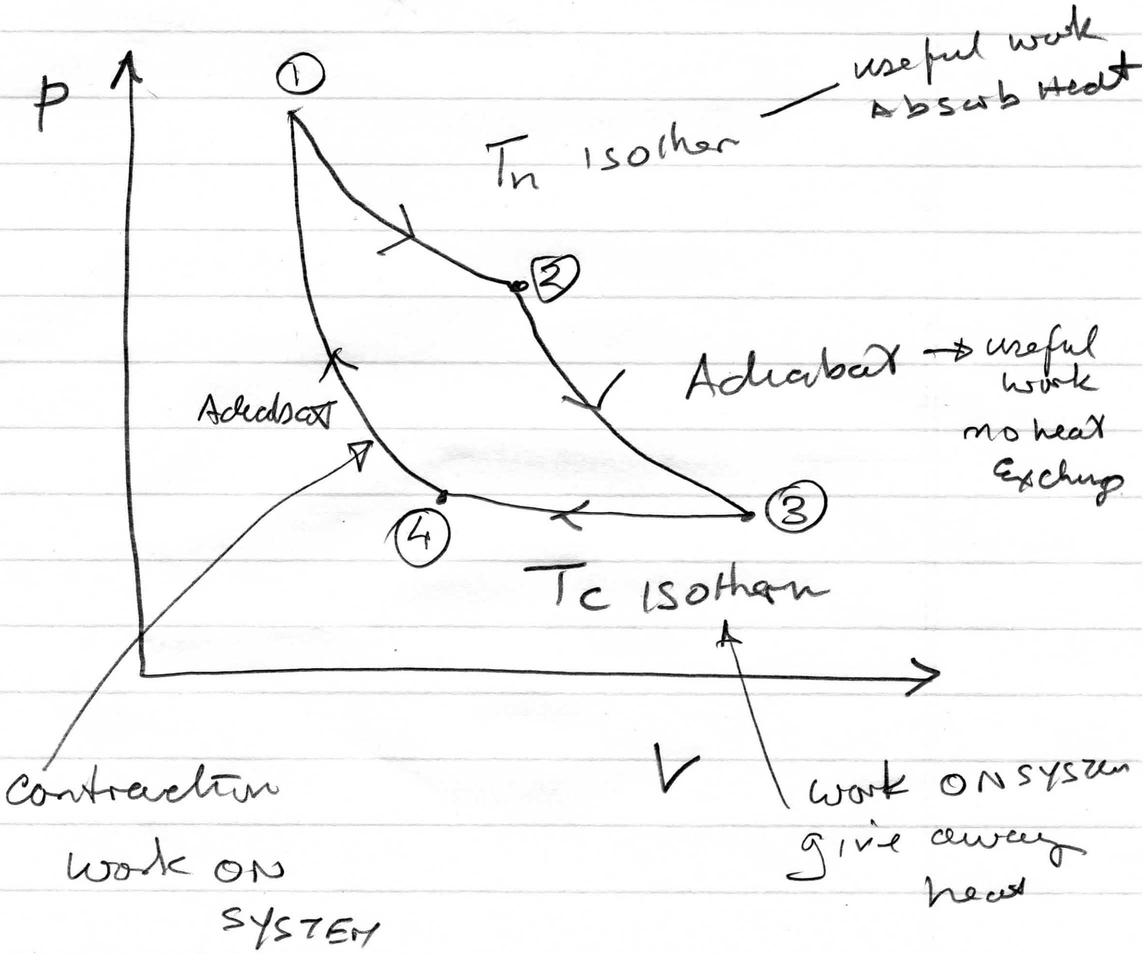
expelling heat

d) Adiabatic compresssin back to T_h

System in contact with hot reservoir ⓐ

System in contact with cold reservoir ⓑ

$$\varepsilon = 1 - \frac{T_c}{T_h}$$



b) Absorb heat from hot reservoir

S of reservoir $-\frac{Q_h}{T_c}$

S of gas $+\frac{Q_h}{T_{gas}}$

To avoid making New ENTROPY

$$T_{gas} = T_h$$

But heat does not flow between bodies at same temp

↳ $T_{gas} \lesssim T_h$ → Expand isothermally

c) gas dumps heat into cold reservoir

$$T_{gas} \gtrsim T_c$$

Compress isothermally

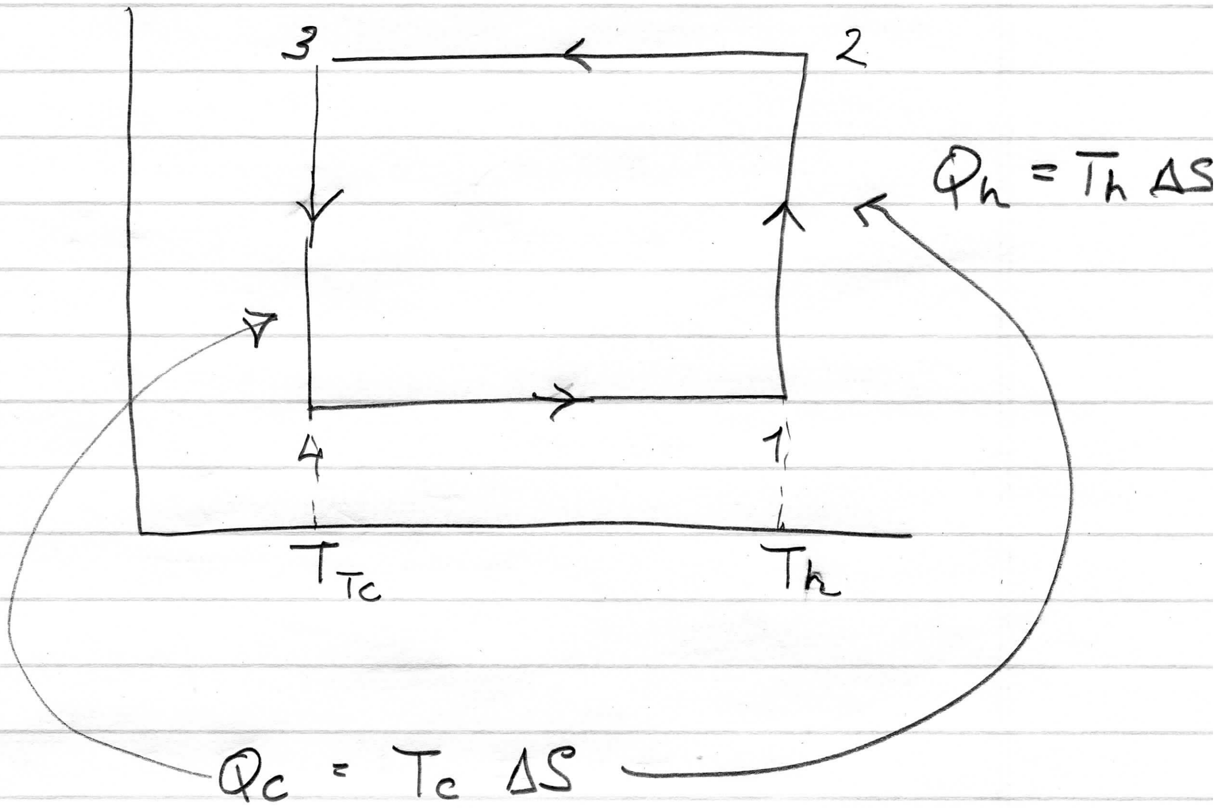
→ How to get gas from one temp to the other?

↳ do not want heat flow during "connecting" processes

↳ ⓑ + ⓓ ADiabatic

→ impractical because heat flows during ISOTHERMAL → VERY SLOW

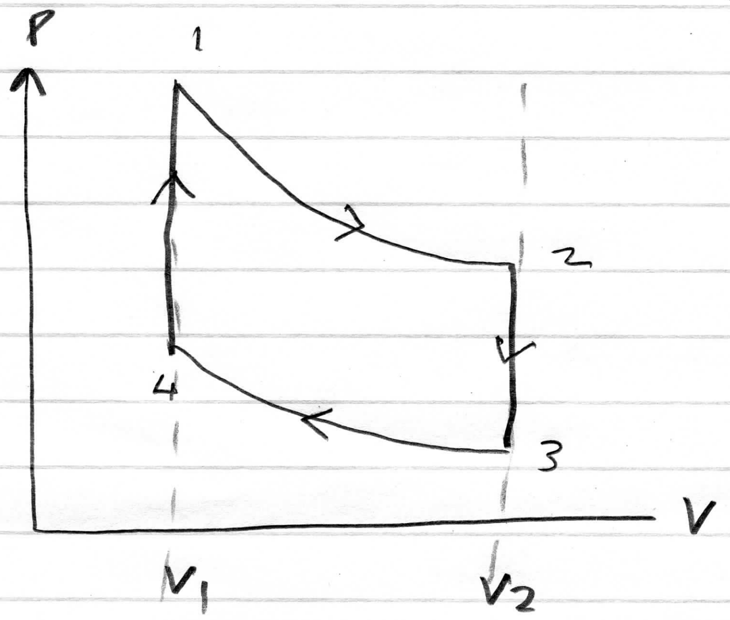
Entropy – Temp diagram.



hence $\varepsilon = 1 - \frac{Q_c}{Q_h} = 1 - \frac{T_c}{T_h}$

"Cartoon" of OTTO CYCLE → Real Internal Combustion Engine

4 STROKE ENGINE



4 → 1 "IGNITION" air + gasoline ignited quickly

↳ increase P at fixed V_1

1 → 2 "POWER STROKE" — fast expansion $V_1 \rightarrow V_2$

↳ useful work

2→3 "Exhaust" Expel burned gas and replace with new mixture at lower P & T

↳ same V

3→4 "Compressi" Adiabatic Compression to V_1 then IGNITION.

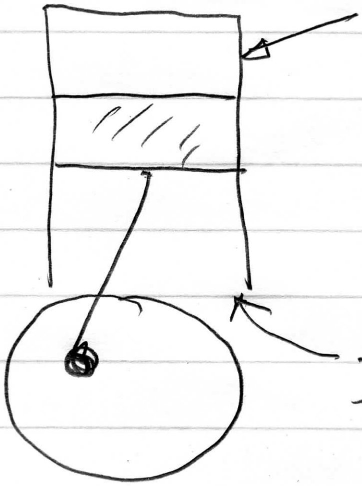
No HOT Reservoir T, nor Cold T'

↳ highest and lowest Ts in cycle correspond to T & T

for an IDEAL gas $e = 1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

TWO - STROKE

1)



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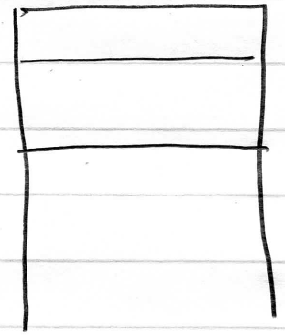

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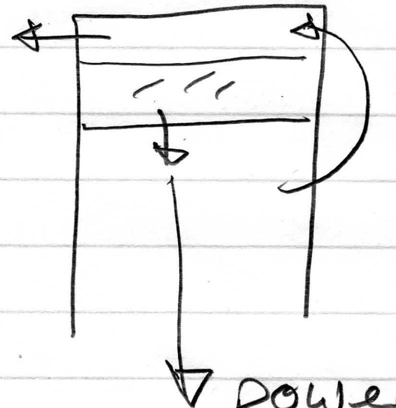

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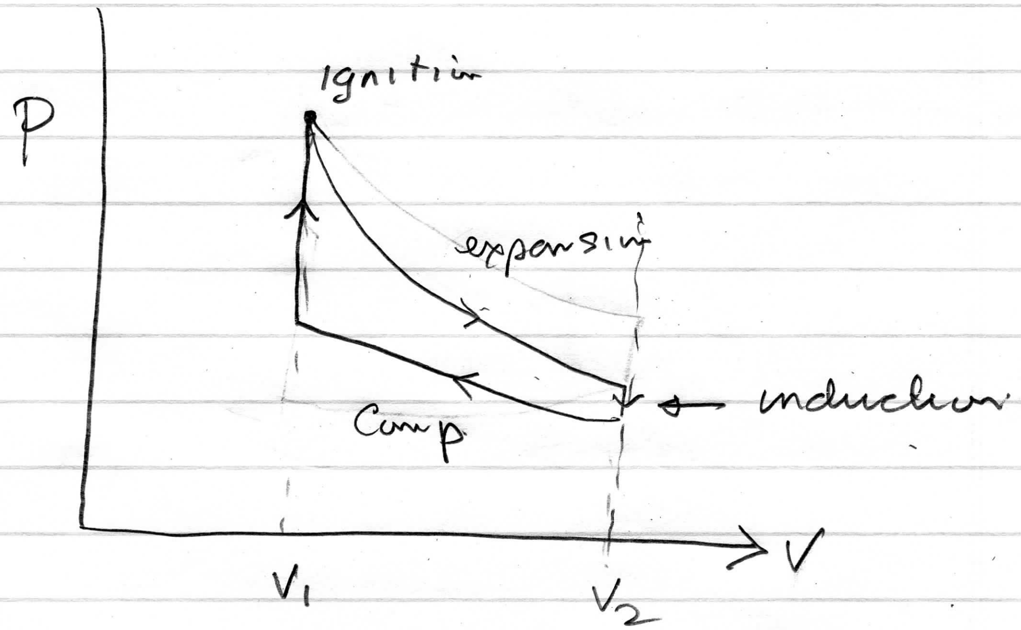

Lawn mower, some motor cycles.