PHY140Y

Solutions to Problem Set 7

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1. Problem 34, Chapter 23

   (a) For points on the x axis with $x > a$, the electric field from the charge $+q$ is in the x direction, and that of the charge $-q$ is in the negative x direction. Thus,

   $$\vec{E} = kq\,\hat{x}\left[\frac{1}{(x+a)^2} - \frac{1}{(x-a)^2}\right] \tag{1}$$

   for $x > a$.

   (b) For $q = 1\ \mu$C and $a = 1$ m,

   $$\vec{E} = \left(9\times 10^4\right)\hat{x}\left[\frac{1}{(x+1)^2} - \frac{1}{(x-1)^2}\right]\ \text{N/C}. \tag{2}$$

2. Problem 42, Chapter 23

   (a) The electric field on the y axis outside the triangle (ie. $y > \sqrt{3}a/2$) due to the two charges on the x axis is calculated noting that only the components of the Coulomb force along the y axis matters as the x axis components cancel. This results in a field due to these charges of

   $$\vec{E} = 2kqy\,\hat{y}\left(y^2 + \frac{a^2}{4}\right)^{-3/2}. \tag{3}$$

   The electric field resulting from the charge on the y axis is

   $$\vec{E} = kq\,\hat{y}\left(y - \sqrt{3}a/2\right)^{-2}. \tag{4}$$

   The total electric field on a charge on the y axis is

   $$\vec{E} = kq\,\hat{y}\left[2y\left(y^2 + \frac{a^2}{4}\right)^{-3/2} + \left(y - \sqrt{3}a/2\right)^{-2}\right]. \tag{5}$$

   (b) We can rewrite the expression for the electric field as

   $$\begin{aligned}\vec{E} &= kq\,\hat{y}\left[2y\left(y^2\left\{1 + \frac{a^2}{4y^2}\right\}\right)^{-3/2} + \left(y\left\{1 - \sqrt{3}a/(2y)\right\}\right)^{-2}\right] && (6)\\ &= \frac{kq}{y^2}\,\hat{y}\left[2\left(1 + \frac{a^2}{4y^2}\right)^{-3/2} + \left(1 - \sqrt{3}a/(2y)\right)^{-2}\right] && (7)\\ &\simeq \frac{3kq}{y^2}\,\hat{y}, && (8)\end{aligned}$$

since the terms in the brackets both approach 1 as $y \gg a$. Thus, the charge distribution looks like 3 point charges at the origin once one gets far from the distribution compared to the size of the distribution.

3. Problem 22, Chapter 24

   (a) At a radius of $r = R/2$, the electric field can be calculated from Gauss's Law. First note that for a uniformly charged sphere with total charge Q, the charge contained within a sphere of radius $r \leq R$ is

$$q(r) = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) \tag{9}$$

$$= Q\left(\frac{r^3}{R^3}\right), \tag{10}$$

   where the first term in the first line is the charge density of the sphere and the second term is the volume of a sphere of radius r. The electric field strength $E(r)$ at a radius r is given by Gauss's Law:

$$\int_{\substack{\text{sphere}\\\text{surface}}} \vec{E(r)} \cdot \hat{r}\, dS = 4\pi k \int_{\substack{\text{sphere}\\\text{volume}}} \rho\, dV \tag{11}$$

$$\Rightarrow E(r) = \left(\frac{kQ}{r^2}\right)\left(\frac{r^3}{R^3}\right) \tag{12}$$

$$= \left(\frac{kQr}{R^3}\right). \tag{13}$$

   The field strength at a distance $r > R$ is

$$E(r) = \frac{kQ}{r^2}. \tag{14}$$

   So the field strength at a given point $r_1 < R$ is equal to the field strength of a point $r_2 > R$ when

$$\frac{kQr_1}{R^3} = \frac{kQ}{r_2^2} \tag{15}$$

$$\Rightarrow r_2 = \sqrt{\frac{R^3}{r_1}} \tag{16}$$

$$\Rightarrow r_2 = \sqrt{\frac{R^3}{R/2}} = \sqrt{2}R = 2.83 \text{ cm}, \tag{17}$$

   when we substitute $r_1 = R/2$.

   (b) The net charge on the sphere, Q, can be obtained from equation 13:

$$Q = \frac{E(r)R^3}{kr} \tag{18}$$

$$= \frac{2E(r)R^2}{k} = \frac{2(39)(0.02)^2}{9 \times 10^9} = 3.47 \text{ nC}. \tag{19}$$

4. Problem 36, Chapter 24

   (a) Since the slab is infinite in size, the electric field has to be perpendicular to the slab at all points, and should be symmetric about the plane going through the centre of the slab. We therefore define $z = 0$ as this plane. We can now apply Gauss's Law to the slab, imaging a small "Gaussian cylinder" that straddles the midplane of the slab with its axis perpendicular to the midplane, and with length l. There is no electric field flux through the sides of the cylinder, but there is flux through the top and bottom. Thus, the electric field at a distance $z = \pm l/2$ satisfies

$$\begin{aligned} 2E(z)A &= Al\rho = 4\pi kA\,(2z)\,\rho && (20)\\ E(z) &= 4\pi kz, && (21) \end{aligned}$$

   where ρ is the charge density in the slab and A is the surface area of the top and bottom of the cylinder.

   (b) Outside the slab, the same argument leads to

$$E(z) = 4\pi k(d/2) = 2\pi kd. \qquad (22)$$

5. Problem 20, Chapter 25
   We can use the results of Problem 3, where we solve for the electric field of a spherical charge distribution, to determine the voltage associated with this field as a function of $r > R_e$, where R_e is the radius of the earth. This gives us

$$\begin{aligned} V(r) &= \frac{kQ}{r} && (23)\\ \Rightarrow V(r_{ion}) - V(R_e) &= kQ\left(r_{ion}^{-1} - R_e^{-1}\right) && (24)\\ &= (9\times10^9)(-4.3\times10^5)\left(\frac{1}{6.45\times10^6} - \frac{1}{6.37\times10^6}\right) && (25)\\ &= 7.54 \text{ MV}. && (26) \end{aligned}$$

6. Problem 2, Chapter 39

   (a) To a good approximation, we can consider Rigel to be a black body, in which case the power radiated per square meter is

$$P/A = \sigma T^4 = (5.67\times10^{-8})(10^4)^4 = 5.67\times10^8 \text{ W/m}^2. \qquad (27)$$

   (b) The maximum power is radiated at the wavelength λ_{max}, given by

$$\lambda_{max} = \frac{2.898\times10^{-3}}{10^4} = 290 \text{ nm}. \qquad (28)$$

7. Problem 8, Chapter 39
The power emitted over a range of wavelength $d\lambda$ is given by the spectral density

$$\begin{aligned} dP &= R(\lambda, T)\, d\lambda \qquad (29) \\ &= \frac{2\pi hc^2}{\lambda^5\left(e^{\hbar c/[\lambda kT]} - 1\right)}\, d\lambda. \qquad (30) \end{aligned}$$

Since the interval is so small, $R(\lambda, T)$ can be taken to be a constant and we can just evaluate dP and multiply it by $d\lambda = 2$ nm. Thus,

$$dP = \frac{3.74 \times 10^{-16}}{(500)^5\left(e^{(1.44\times10^{-2}/[(500)(3000)])} - 1\right)} \times (2 \times 10^{-9}) = 1.64\ \mathrm{kW/m^2}. \qquad (31)$$

8. Problem 18, Chapter 39

   (a) The proton's Compton wavelength is given by

   $$\lambda_c = \frac{h}{m_p c} = \frac{hc}{m_p c^2} = \frac{1.240 \times 10^{-12}}{938} = 1.32\ \mathrm{fm}. \qquad (32)$$

   (b) The energy of a photon, E_γ, with this wavelength is

   $$E_\gamma = \frac{hc}{\lambda} = 938\ \mathrm{MeV}. \qquad (33)$$

   The fact that the wavelength of a photon with the same energy as the mass of the particle is equal to the particle's Compton wavelength should not be a surprise, given that this wavelength represents the shift in energy of a photon scattering at 90° off a proton.

9. Problem 24, Chapter 39

   (a) The stopping potential, V_s, is equivalent to the kinetic energy that the electron must have when leaving the surface since all the kinetic energy is expended crossing the potential gap. Since the total energy of the photon goes into the work function, ϕ, and this kinetic energy, we have

   $$\begin{aligned} E_\gamma &= \phi + eV_s \qquad (34) \\ \Rightarrow \phi &= E_\gamma - eV_s \qquad (35) \\ &= \frac{hc}{\lambda} - eV_s = \frac{1240}{365} - 1.8 = 1.60\ \mathrm{eV}. \qquad (36) \end{aligned}$$

   (b) At the new wavelength,

   $$\begin{aligned} eV_s &= \left(\frac{hc}{\lambda} - \phi\right) \qquad (37) \\ &= \left(\frac{1240}{280} - 1.8\right) = 2.79\ \mathrm{eV}, \qquad (38) \end{aligned}$$

   so the stopping potential would be $V_s = 2.79$ V.

10. Problem 62, Chapter 39

The uncertainty in the electron's vertical position, Δy, leads to an uncertainty in the vertical component of their momentum, Δp_y, that must satisfy

$$\Delta p_y \geq \frac{\hbar}{\Delta y}. \tag{39}$$

Since the electrons have a horizontal momentum of p_x, we can determine the minimum spread in the electron beam, $\Delta\theta$, by noting that

$$\begin{aligned}
\tan \Delta\theta &= \frac{\Delta p_y}{p_x} && (40)\\
&\geq \frac{\hbar}{p_x \Delta y} && (41)\\
&= \frac{\hbar}{m_e v_x \Delta y} = \frac{1.055 \times 10^{-34}}{(9.11 \times 10^{-31})(2.2 \times 10^{7})(9 \times 10^{-8})} && (42)\\
&= 5.85 \times 10^{-5} \text{ radians.} && (43)
\end{aligned}$$

Note that the Heisenberg Uncertainty Principle applies to each coordinate separately!