PHY140Y

Spring Term – Week 12 Discussion

5 April, 1999

1. A proton in a hydrogen atom is a spin-1/2 particle, and so can be found in one of two quantum states, one with spin aligned in the $+\hat{z}$ direction ("spin up") and the other with spin aligned in the $-\hat{z}$ direction ("spin down"). The magnetic moment of the proton, which is in the same direction as the spin, has a magnitude

$$\mu_p = 2.793 \left(\frac{e\hbar}{2m_p} \right) = 1.41 \times 10^{-26} \text{ J/T}, \tag{1}$$

where m_p is the mass of the proton and the units are Joules per Tesla. Because of this dipole moment, the proton has different energies in its two spin states when sitting in an external magnetic field aligned along the $\hat{z}$ direction, $\vec{B}$:

$$U = -\vec{\mu_p} \cdot \vec{B}, \tag{2}$$

where the magnetic dipole vector $\vec{\mu_p}$ is either $+\mu_p\hat{z}$ or $-\mu_p\hat{z}$.

   (a) The energy difference of these two spin states in hydrogen due to the effective magnetic field created by the electron orbiting the hydrogen atom is $\Delta E = 5.9 \times 10^{-6}$ eV. What is the effective magnetic field due to the electrons?

   (b) What frequency does this correspond to?

   (c) This "hyperfine splitting" is the basis for MRI, Magnetic Resonance Imaging. A large magnetic field $B = 2.0$ T is used to enhance the energy splitting, and then a EM wave is used to excite the protons from their ground state to their excited state. What is the frequency of this EM wave?

   (d) The protons are detected by the emission of the photons when they fall from the excited state to the ground state. This is controlled by "shaping" the magnetic field in space and detecting the radiation from the protons as the EM wave frequency is modified. If we can detect photons that differ in frequency by 1×10^7 Hz, how rapidly must the B field change in order for us to detector protons that differ in position by 1 mm?

2. Fission is usually considered to be the process that starts with ^{235}U "fissioning" into two smaller nuclei via a neutron-induced reaction. Let's work out some of the details for those eager to build a CANDU-II.

   (a) When a ^{235}U nucleus decays into two smaller nuclei, assume that the two nuclei have equal numbers of protons and appear 10×10^{-14} m apart initially. They then are mutually repelled due to the electromagnetic repulsion of the two nuclei. What is the electromagnetic energy released when these nuclei are able to separate a long distance?

   (b) When a neutron at thermal energies ($\sim 10^{-2}$ eV) is released in a block of ^{235}U, each ^{235}U nucleus has a cross section of about $\sigma = 10^{-25}$ m^2. A cross section is a measure of the effective size of the nucleus. How many centimeters of ^{235}U should this neutron pass through before the mean number of times it interacts with a ^{235}U nucleus is greater than 1.

(c) From this, if the neutrons are produced by spontaneous fission, estimate (order of magnitude is OK) the mass of ^{235}U needed to sustain a chain reaction (this is the "critical mass").