PHY140Y

Spring Term – Week 3 Solutions

January 20, 1999

1. The centre of mass of the system is the point exactly between the two suns. We could have been pedantic about it and use the formal definition of the centre of mass, $\vec{R_{cm}}$,

$$M_{tot}\vec{R_{cm}} = \sum_i m_i \vec{r_i}, \tag{1}$$

where $M_{tot} = \sum_i m_i$ is the total mass of the system.

To solve this problem, note that each sun is in a circular orbit so the normal relationships for uniform circular motion must be satisfied. In particular, the centripetal acceleration should satisfy

$$a_c = \frac{v^2}{R/2}. \tag{2}$$

This has to be equal to the gravitational attraction of the other sun, which is a distance R away:

$$a_g = \frac{GM}{R^2}. \tag{3}$$

Using the relationship $vT = 2\pi(R/2)$ for the period T, we get the result

$$T^2 = \frac{2\pi^2 R^3}{GM}. \tag{4}$$

(2) (a) Since the field is oriented at right angles to the surface, $\vec{F} \cdot \hat{n} = F$. Thus the flux is

$$\phi = \int_S (F(\vec{x}) \cdot \hat{n}) dS = FA. \tag{5}$$

When the surface is oriented with its normal perpendicular to the field, $\vec{F} \cdot \hat{n} = 0$. Thus, the flux is zero.

(b) Orient the cube so that one set of faces have their unit normals parallel to the field vector. In this orientation, four of the six surfaces have $\vec{F} \cdot \hat{n} = 0$. For the other two,

$$\begin{aligned} \vec{F} \cdot \hat{n} &= FA \text{ and} \qquad (6) \\ \vec{F} \cdot \hat{n} &= -FA. \qquad (7) \end{aligned}$$

Thus, the total flux through this object is zero in this constant field.

For the sphere, we can use symmetry arguments similar to what we used in the case of the cube to show that the flux has to be zero.

(c) The electric field of a point charge at a distance r is given by

$$\vec{E}(\vec{r}) = \frac{kq}{r^2}\hat{r}, \tag{8}$$

where $k = 9.0 \times 10^9\ \mathrm{N \cdot m^2/C}$. The normal to the sphere of radius r is also $\hat{r}$. Thus the flux of the electric field through the sphere is

$$\begin{aligned} \phi &= \int_S (\vec{E} \cdot \hat{n}) dS && (9) \\ &= \frac{kq}{r^2} 4\pi r^2 = 4\pi kq. && (10) \end{aligned}$$

Note that this is independent of the size of the sphere.