

PHY140Y

Spring Term – Week 6 Discussion Solutions

8 February, 1999

1. (a) The forces acting on the ladder are:
- the force of gravity, \vec{F}_g , acting through the centre of mass of the ladder,
 - the normal force, \vec{F}_N , responding to the weight of the ladder on the ground,
 - the force of friction, \vec{F}_f , preventing the ladder from sliding on the ground, and
 - the normal force, \vec{F}_w , exerted by the wall on the ladder to keep it stationary.

These forces are shown in Fig. 1.

- (b) Choose a coordinate system with \hat{x} pointing toward the wall and \hat{y} pointing upwards along the wall. The net force in the \hat{y} direction is

$$0 = F_N + F_g \quad (1)$$

$$= F_N - mg. \quad (2)$$

The net force in the horizontal direction is

$$0 = F_f - F_w \quad (3)$$

$$= \mu F_N - F_w. \quad (4)$$

- (c) The best pivot axis to choose for this problem is the one located at the base of the ladder. This is because two of the four forces act through this point, so that the torque they generate on the ladder is zero. The net torque is then given by the force of gravity acting on the centre of mass and the force exerted by the wall at the end of the ladder:

$$\tau_{net} = \vec{r}_{cm} \times \vec{F}_g + \vec{r}_{end} \times \vec{F}_w \quad (5)$$

$$= \left(-\frac{LF_g \cos \phi}{2} + LF_w \sin \phi \right) \hat{z}, \quad (6)$$

where \vec{r}_{cm} and \vec{r}_{end} are vectors from the pivot point to the centre of mass and the end of the ladder, respectively. The axis \hat{z} points out of the page.

- (d) We can use the force relations to find

$$F_N = mg \quad (7)$$

$$\Rightarrow F_f = \mu F_N = \mu mg, \quad (8)$$

and

$$F_w = \mu F_N = \mu mg. \quad (9)$$

Substituting that into the expression for net torque, we find

$$0 = -\frac{LF_g \cos \phi}{2} + LF_w \sin \phi \quad (10)$$

$$= -\frac{Lmg \cos \phi}{2} + L\mu mg \sin \phi \quad (11)$$

$$\Rightarrow \mu = \frac{\cos \phi}{2 \sin \phi} = \frac{1}{2} \cot \phi. \quad (12)$$

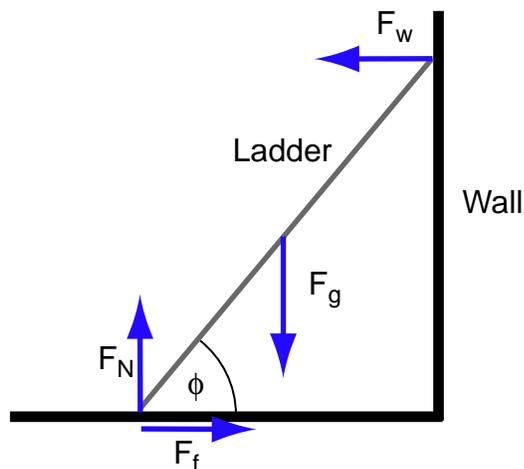


Figure 1: The forces acting on the ladder.

This is pretty remarkable, in that it doesn't depend on the length of the ladder, it's mass or the force of gravity. So that is why there is a "universal" best angle at which a ladder should be placed leaning up against the wall. We also see that the force of friction has to rise very rapidly as ϕ decreases, especially once you have angles less than $\pi/4$.

Also note that we didn't even have to compute a moment of inertia in this problem!

2. (a) The moment of inertia of the bicycle wheel is

$$I_b = m_b r_b^2 = 0.32 \text{ kg m}^2. \quad (13)$$

- (b) The moment of inertia of the motorcycle wheel is

$$I_m = m_m r_m^2 = 2.26 \text{ kg m}^2. \quad (14)$$

- (c) The angular velocity of the bicycle wheel is

$$\omega_b = \frac{v_b}{r_b} = \frac{3 \times 10^4}{3600 \times 0.40} = 20.8 \text{ rad/s}. \quad (15)$$

The angular momentum of the bicycle wheel is

$$L_b = I_b \omega_b \quad (16)$$

$$= 0.32 \times 20.8 = 6.67 \text{ kg m}^2/\text{s}. \quad (17)$$

The angular velocity and momentum of the motorcycle wheel are

$$\omega_m = \frac{v_m}{r_m} = \frac{1.3 \times 10^5}{3600 \times 0.35} = 103 \text{ rad/s}, \quad (18)$$

$$\Rightarrow L_m = I_m \omega_m \quad (19)$$

$$= 2.26 \times 103 = 233 \text{ kg m}^2/\text{s}. \quad (20)$$

The motorcycle wheel has an angular momentum that is forty times that of the bicycle. In both cases, the angular momentum points along the axis of the wheel (use the right-hand rule to establish the direction of the vector).

- (d) Suppose that the angular momentum of the bicycle played a key role in the stability of the rider and bicycle. This implies that any change in the angular momentum of the wheel (such as a sudden turn) would be associated with a torque on the wheel, and there would be corresponding torque exerted on the rider in response. Does this occur? Not from our experience. In fact, the way we turn on a bicycle is to shift our centre of mass from one side of the bike to the other. This causes the bike to fall to one side and we then steer the bike in that direction to match the acceleration of gravity on our body with the centripetal acceleration needed to maintain a curved trajectory. Otherwise, how could riding “with no-hands” be possible?
- (e) With a motorcycle, the situation is a little different because now the angular momentum of the wheel is significantly larger. The proper way to turn at high speed on a motorcycle is to push on the handle bars in the opposite direction to the direction you want to turn. This exerts a torque on the front wheel, which in turn causes the bike to “heel” over in the direction of the intended turn. At that point you then steer through the turn.