PHY140Y

Solutions to Problem Set 6

22 February 2000

1. Chapter 23, Problem 34, Page 596

   (a) For points on the x-axis with $x > a$, the electric field from the charge $+q$ is in the x direction and that of of the charge $-q$ is in the negative x direction. Thus

$$\vec{E}(x) = kq\left[\frac{2}{(x+a)^2} - \frac{1}{(x-a)^2}\right]\hat{x} \text{ for } x > a. \tag{1}$$

   (b) For $q = 1.0\ \mu$C, and $a = 1$ m, the electric field has the explicit form

$$\vec{E}(x) = (9 \times 10^3)\left[\frac{2}{(x+1)^2} - \frac{1}{(x-1)^2}\right]\hat{x} \tag{2}$$

   shown plotted in Fig. 1.

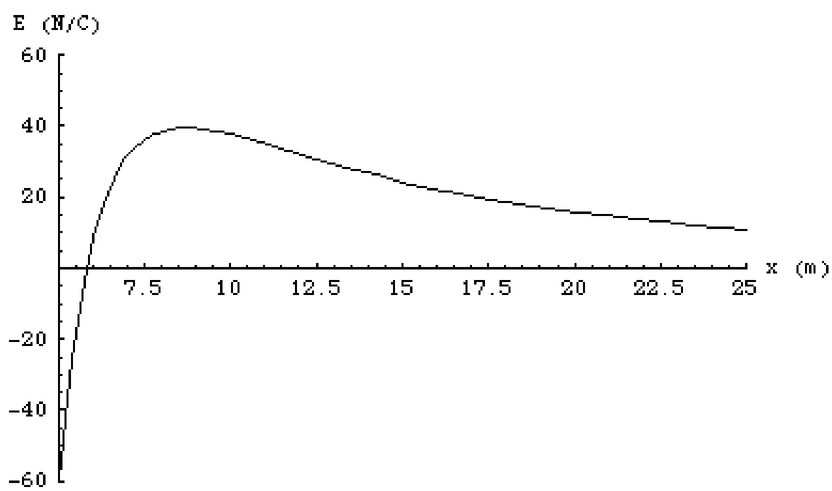

Figure 1: The electric field strength for Problem 1b)

2. Chapter 23, Problem 50, Page 597

   Using the variables shown in Fig. 23-45 in the text, one can see that the electric field at the centre of the semi-loop has to be in the x direction. We divide the loop up into small elements defined by the angle $d\theta$, and the total charge in that element is then

$$dq = Q\frac{a\,d\theta}{\pi a} = \frac{Q\,d\theta}{\pi}. \tag{3}$$

The magnitude of the electric field at the centre of the loop is then

$$dE = \frac{kQd\theta}{\pi a^2} \tag{4}$$

and the x component of this is then

$$dE_x = \frac{kQd\theta}{\pi a^2}\sin\theta. \tag{5}$$

We now just integrate this from $\theta = 0$ to $\theta = \pi$ to get the total magnitude of the electric field (which is in the x direction):

$$\begin{aligned} E &= \int_0^\pi \frac{kQ}{\pi a^2}\sin\theta\, d\theta && (6)\\ &= \frac{kQ}{\pi a^2}\int_0^\pi \sin\theta\, d\theta && (7)\\ &= \frac{kQ}{\pi a^2} \left. -\cos\theta \right]_0^\pi && (8)\\ &= \frac{2kQ}{\pi a^2}. && (9) \end{aligned}$$

3. Chapter 24, Problem 18, Page 625

The example in the text (Example 24-1) shows how to work out this problem. From that example, we can show that

(a) at 15 cm $= r < R = 25$ cm, the electric field is radial and has strength

$$E = \frac{kQr}{R^3} = (1.21 \times 10^6)\text{ N/C}, \tag{10}$$

(b) at $r = 25$ cm (at the surface) the electric field strength is

$$E = \frac{kQ}{R^2} = (2.02 \times 10^6)\text{ N/C}, \quad \text{and} \tag{11}$$

(c) at $r = 50$ cm, the electric field strength is

$$E = \frac{kQ}{r^2} = (5.04 \times 10^5)\text{ N/C}. \tag{12}$$

4. Chapter 24, Problem 34, Page 626

We can assume that the electric field has place symmetry and we can ignore the edge effects. Then in that case, the electric field strength is

$$E = \frac{\sigma}{2\epsilon_0}. \tag{13}$$

Then,

(a) the surface charge density is

$$\sigma = 2\epsilon_0 E \quad (14)$$
$$= 2(8.85 \times 10^{-12})(-430) = -7.61 \times 10^{-9} \text{ C/m}^2, \quad (15)$$

(b) the charge on the plate q is found by noting that

$$\sigma = q/A \quad (16)$$
$$\Rightarrow q = \sigma A \quad (17)$$
$$= (-7.61 \times 10^{-9})(4.5)^2 = -1.54 \times 10^{-7} \text{ C}, \quad (18)$$

where A is the area of the plate, and

(c) the electric field strength remains -430 N/C everywhere where we can assume edge effects are negligible. This is certainly true 20 cm from this large plate.

5. Chapter 12, Problem 6, Page 307

At constant angular acceleration, the relationship between the angular displacement and the change in angular velocity is

$$\Delta\theta = \frac{1}{2}(\omega_i + \omega_f)\,\Delta t \quad (19)$$
$$= \frac{1}{2}(3600 + 1800)(1.4) = 63.0 \text{ revolutions}. \quad (20)$$

6. Chapter 12, Problem 24, Page 308

(a) In this case, the masses located on the axis of rotation contribute nothing to the moment of inertia, so the only contributions come from the two masses, exactly l away from the axis. Thus,

$$I = 2ml^2. \quad (21)$$

(b) In the case where the axis passes between two sides, each mass is now a distance $l/2$ from the axis and so the moment of inertia is

$$I = 4 \times ml^2/4 = ml^2. \quad (22)$$

7. Chapter 12, Problem 44, Page 309

The frictional torque decelerates the flywheel, so we have that

$$\omega_f = \omega_i + \alpha t, \quad (23)$$

and

$$\tau = -f_k R_s = I\alpha \quad (24)$$
$$\Rightarrow t = \frac{-\omega_i}{\alpha} \quad (25)$$
$$= \frac{I\omega_i}{f_k R_s}, \quad (26)$$

where R_s is the radius of the shaft. We can approximate the moment of inertia of the entire flywheel by ignoring the much small shaft and treating it as a uniform, solid disk. Thus,

$$I = \frac{MR_w^2}{2} \quad (27)$$

$$\Rightarrow t = \frac{MR_w^2\omega_i}{2f_kR_s} \quad (28)$$

$$= \frac{(7.7 \times 10^4)(2.4)^2(37.7)}{2(34)(0.205)} = 1.20 \times 10^3 \text{ s}, \quad (29)$$

or about 20 minutes.

8. Chapter 13, Problem 6, Page 330

Our definition of the torque gives us

$$\tau = rF \sin\theta \quad (30)$$

$$= (0.80)(5.0)\sin(160^\circ) = 1.37 \text{ N} \quad (31)$$

pointing into the page.

9. Chapter 13, Problem 20, Page 331

(a) If we neglect the mass of the rod and treat the cups like point masses, then the moment of inertia of the anemometer about its axle is

$$I = 4mr^2 \quad (32)$$

$$= 4(0.12)(0.16)^2 = 1.23 \times 10^{-2} \text{ kg m}^2. \quad (33)$$

(b) The angular momentum about the axle has magnitude

$$L = I\omega = (1.23 \times 10^{-2})(75.4) = 0.926 \text{ Js} \quad (34)$$

10. Chapter 13, Problem 30, Page 331

If the clay is dropped vertically into a horizontal spinning wheel, the angular momentum of the rotation system is conserved. Then

$$I_i\omega_i = I_{wheel}\omega_i \quad (35)$$

$$= I_f\omega_f \quad (36)$$

$$= \left(I_{wheel} + m_{clay}r^2\right)\omega_f \quad (37)$$

$$\Rightarrow \omega_f = \frac{(6.40)(19)}{6.40 + (2.7)(0.46)^2} = 17.4 \text{ rpm}. \quad (38)$$