PHY140Y

Solutions to Problem Set 7

13 March 2000

1. Chapter 39, Problem 4, Page 1070

   The temperature of a blackbody corresponding to a peak wavelength of $\lambda_{max} = 40$ μm is given by the Wein Displacement Law:

$$T\lambda_{max} = 2.898 \times 10^{-3} \text{ m K} \quad (1)$$

$$\Rightarrow T = \frac{2.898 \times 10^{-3}}{\lambda_{max}} = 72.5 \text{ K.} \quad (2)$$

2. Chapter 39, Problem 16, Page 1071

   (a) The energy of each photon is

$$E_\gamma = hf \quad (3)$$

$$= (4.136 \times 10^{-15})(2.4) = 9.93 \times 10^{-6} \text{ eV,} \quad (4)$$

   or 1.59×10^{-24} J.

   (b) The number of photons given off per second will be the total power output, P, divided by the energy of each photon:

$$N_\gamma = \frac{P}{E_\gamma} \quad (5)$$

$$= \frac{625}{1.59 \times 10^{-24}} = 3.94 \times 10^{26} \text{ s}^{-1}. \quad (6)$$

   Prodigious...

3. Chapter 39, Problem 28, Page 1071

   The maximum energy of the electrons leaving the surface is

$$K_{max} = \frac{1}{2} m_e v_e^2 \quad (7)$$

$$= \frac{1}{2}(9.11 \times 10^{-31})(4.2 \times 10^5)^2 = 8.03 \times 10^{-20} \text{ J,} \quad (8)$$

   or $K_{max} = 0.502$ eV. From the formula for the work function of a metal and the energy of a photon, we have

$$\lambda = \frac{hc}{\phi + K_{max}} \quad (9)$$

$$= \frac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{(2.30 + 0.502)(1.602 \times 10^{-19})} = 4.43 \times 10^{-7} \text{ m,} \quad (10)$$

   where in the last step, we have converted eV to J in the denominator.

4. Chapter 39, Problem 44, Page 1071

The energy of the excited state and its principal quantum number, n_e, is given by

$$\begin{aligned} E_e &= -13.6 + 12.75 && (11) \\ &= -0.85 = \frac{-13.6}{n_e^2} && (12) \\ \Rightarrow n_e &= \sqrt{\frac{13.6}{0.85}} = 4. && (13) \end{aligned}$$

The transition to the next lower energy level given by $n = 3$ would require the emission of a photon with energy

$$E_\gamma = -0.85 - \frac{-13.6}{3^2} = 0.66 \text{ eV}. \qquad (14)$$

5. Chapter 39, Problem 52, Page 1071

The resolution depends on the wavelength, so if $\lambda = h/p < \lambda_l = 450$ nm, the electron microscope is better. Thus

$$\begin{aligned} p = m_e v &> \frac{h}{\lambda_l} && (15) \\ v &> \frac{h}{m_e \lambda_l} && (16) \\ &= \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(4.5 \times 10^{-7})} = 1.62 \times 10^3 \text{ m/s}. && (17) \end{aligned}$$

6. Chapter 39, Problem 56, Page 1072

If we let $\Delta x = 2$ μm and $\Delta v = 2$ m/s, then the uncertainty in the momentum will be

$$\Delta p = m \Delta v, \qquad (18)$$

where we assume that the electron is moving non-relativistically. Heisenberg's uncertainty principle requires

$$\begin{aligned} \Delta x \Delta v &\geq \frac{\hbar}{m} && (19) \\ m &\geq \frac{\hbar}{\Delta x \Delta v} && (20) \\ &= \frac{1.06 \times 10^{-34}}{(2 \times 10^{-6})(2)} = 2.64 \times 10^{-29} \text{ kg}. && (21) \end{aligned}$$

Since this minimum mass is heavier than an electron, it is not possible to localize and measure the velocity of an electron to this precision. However, a proton is heavier than this limit and therefore its properties can be measured to this accuracy.

7. Chapter 40, Problem 10, Page 1096

In a one-dimensional square well, the lowest energy of the particle is

$$E_1 = \frac{h^2}{8mL^2}, \tag{22}$$

where L is the length of the well. If we set this energy to be the kinetic energy of a person with mass $m = 60$ kg travelling at a speed $v = 1.0$ m/s, we find that

$$\begin{aligned} E_1 &= \frac{1}{2}mv^2 && (23) \\ &= \frac{h^2}{8mL^2} && (24) \\ \Rightarrow h &= \sqrt{4m^2L^2v^2} && (25) \\ &= 2mLv = (2)(60)(2.6)(1.0) = 312 \text{ J s}, && (26) \end{aligned}$$

which is about 10^{36} times larger than the true value.

8. Chapter 40, Problem 20, Page 1096

Since the energy of each state is given by

$$E_n = \frac{h^2}{8mL^2}\, n^2, \tag{27}$$

the energy E_γ and wavelength λ of the photons emitted when transitions are made from the state n_1 to the state n_2 are

$$\begin{aligned} E_\gamma &= \left(n_1^2 - n_2^2\right)\frac{h^2}{8mL^2} && (28) \\ \Rightarrow &= \frac{hc}{\lambda_{n_1 \to n_2}} && (29) \\ \Rightarrow \lambda_{n_1 \to n_2} &= \frac{8mL^2c}{h\left(n_1^2 - n_2^2\right)} && (30) \\ &= \frac{8(9.11 \times 10^{-31})(1 \times 10^{-10})^2(3.0 \times 10^8)}{(6.63 \times 10^{-34})}\frac{1}{(n_1^2 - n_2^2)} && (31) \\ &= (3.30 \times 10^{-8})\frac{1}{(n_1^2 - n_2^2)}. && (32) \end{aligned}$$

The possible values for n_1 and n_2 if the square wells all start in the $n = 4$ state are:

$$\begin{aligned} n_1 = 4 \text{ and } n_2 = 1 &\Rightarrow \lambda = 2.20 \text{ nm} && (33) \\ n_1 = 4 \text{ and } n_2 = 2 &\Rightarrow \lambda = 2.75 \text{ nm} && (34) \\ n_1 = 4 \text{ and } n_2 = 3 &\Rightarrow \lambda = 4.71 \text{ nm} && (35) \\ n_1 = 3 \text{ and } n_2 = 1 &\Rightarrow \lambda = 4.12 \text{ nm} && (36) \\ n_1 = 3 \text{ and } n_2 = 2 &\Rightarrow \lambda = 6.60 \text{ nm} && (37) \\ n_1 = 2 \text{ and } n_2 = 1 &\Rightarrow \lambda = 11.0 \text{ nm}. && (38) \end{aligned}$$

9. Chapter 40, Problem 31, Page 1097

(a) Since the wave functions are symmetric, we only need to consider the situation at $x = L/2$. To show that ψ_1 is continuous, we evaluate the wave function using the form for the inner and outer edges:

$$\psi_1(L/2) = \frac{1.26}{\sqrt{L}}\cos(2.50/2) = \frac{0.397}{\sqrt{L}} \tag{39}$$

$$\psi_1(L/2) = \frac{17.9}{\sqrt{L}}\exp(-7.60/2) = \frac{0.400}{\sqrt{L}}. \tag{40}$$

Thus, since these agree to the number of significant figures we are given, the solution is continuous. Similarly, we can evaluate the derivative of ψ_1 at $x = L/2$:

$$\frac{d\psi_1}{dx}(L/2) = \frac{(-1.26)(2.50)}{L^{3/2}}\sin(2.50/2) = \frac{-2.99}{L^{3/2}} \tag{41}$$

$$\frac{d\psi_1}{dx}(L/2) = \frac{(-17.9)(7.60)}{L^{3/2}}\exp(-7.60/2) = \frac{-3.04}{L^{3/2}}, \tag{42}$$

which are equal to the precision provided to us.

(b) This plot is shown in Fig. 40-26 of the text.

10. Chapter 40, Problem 32, Page 1098

The probability that the particle will be found outside the well is

$$P = \int_{-\infty}^{-L/2}|\psi_1|^2 dx + \int_{L/2}^{\infty}|\psi_1|^2 dx \tag{43}$$

$$= 2\int_{L/2}^{\infty}|\psi_1|^2 dx \tag{44}$$

$$= 2\int_{L/2}^{\infty}\left(\frac{17.9}{\sqrt{L}}\exp(-7.60x/L)\right)^2 dx \tag{45}$$

$$= \frac{2(17.9)^2}{L}\left|\frac{-L}{2(7.60)}\exp(-15.2x/L)\right|_{l/2}^{\infty} \tag{46}$$

$$= 0.021, \tag{47}$$

or about 2.1% of the time.