PHY140Y

Solutions to Problem Set 8

28 March 2000

1. Chapter 41, Problem 14, Page 1130

   (a) The maximum orbital angular momentum quantum number is $l = n - 1$, so that the in terms of the principal quantum number,

$$\begin{aligned} L &= 30\sqrt{11}\hbar && (1)\\ &= \sqrt{(n-1)n}\hbar && (2)\\ \Rightarrow (n-1)n &= 30^2 \times 11 = 99 \times 100 && (3)\\ \Rightarrow n &= 100. && (4) \end{aligned}$$

   (b) The energy of this state is

$$E_{100} = \frac{-13.6}{100^2} = -1.36 \times 10^{-3} \text{ eV}. \quad (5)$$

2. Chapter 41, Problem 22, Page 1130

   For a $S = 2$ particle, the magnitude of its spin is

$$\begin{aligned} S &= \sqrt{2 \times 3}\hbar && (6)\\ &= \sqrt{6}\hbar = 2.58 \times 10^{-34} \text{ Js}. && (7) \end{aligned}$$

3. Chapter 41, Problem 28, Page 1130

   (a) The principal quantum number $n = 4$. Thus the energy of the state is $E_4 = -13.6/16 = 0.85$ eV.

   (b) The orbital quantum number $l = 3$, so the magnitude of the orbital angular momentum is

$$L = \sqrt{l(l+1)}\hbar = \sqrt{12}\hbar = 3.65 \times 10^{-34} \text{ Js}. \quad (8)$$

   (c) Since $j = 5/2$, the magnitude of the total angular momentum is

$$J = \sqrt{j(j+1)}\hbar = \frac{\sqrt{35}}{2}\hbar = 3.12 \times 10^{-34} \text{ Js}. \quad (9)$$

   (d) The orbital angular momentum is greater. This is because $j = l - 1/2$, which implies that the orbital angular momentum and the electron spin vector are anti-parallel.

4. Chapter 41, Problem 48, Page 1131

(a) The energy levels for an electron in a one-dimensional square well are

$$E_n = \frac{h^2}{8mL^2} n^2 \tag{10}$$

$$= \frac{(6.63 \times 10^{-34})^2}{8(9.11 \times 10^{-31})(2.0 \times 10^{-10})^2} \frac{1}{1.60 \times 10^{-19}} n^2 = 9.42 n^2 \text{ eV}. \tag{11}$$

The energy level diagram is then a set of levels with spacing given by this formula. The allowed transitions are shown in Fig. 1.

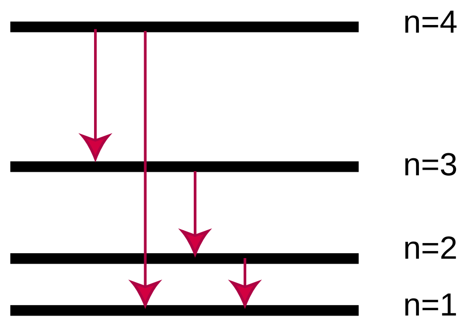

Figure 1: The energy level diagram showing all allowed transitions of the electron in a square well with $n = 4$ (Problem 4a).

(b) The possible photon energies are given by the difference in the energy levels in part a.

$$\Delta E_{4\to 3} = 9.42(16 - 9) = 66.0 \text{ eV} \tag{12}$$

$$\Delta E_{4\to 1} = 9.42(16 - 1) = 141 \text{ eV} \tag{13}$$

$$\Delta E_{3\to 2} = 9.42(9 - 4) = 47.1 \text{ eV} \tag{14}$$

$$\Delta E_{2\to 1} = 9.42(4 - 1) = 28.3 \text{ eV}. \tag{15}$$

5. Chapter 41, Problem 52, Page 1132

The probability for an electron to be found in the radial interval $0 \le r \le 3a_0$ is

$$P = \int_0^{3a_0} 4\pi r^2 |\psi|^2 dr \tag{16}$$

$$= 4\pi \left(\frac{1}{\pi a_0^3}\right) \left\{ \frac{(3a_0)^2 e^{-6}}{-2/a_0} - \frac{2}{-2/a_0} \left[\frac{e^{-6}}{(-2/a_0)^2} (-6 - 1) \right] - \frac{2}{(-2/a_0)^3)} \right\} \tag{17}$$

$$= 4 \left\{ -\frac{9}{2} e^{-6} - \frac{7}{4} e^{-6} + \frac{1}{4} \right\} \tag{18}$$

$$= 1 - 25e^{-6} = 0.938, \tag{19}$$

where we have performed the integration by parts.

6. Chapter 43, Problem 2, Page 1188

The distance of closest approach, r_{min}, is defined by where the Coulomb potential energy would equal the initial kinetic energy K_i. Since the potential energy of an α particle in the electric field of the ^{56}Fe nucleus is

$$\begin{aligned} U(r) &= \frac{k(2)(26)e^2}{r}, & (20) \\ \Rightarrow K_i &= \frac{52ke^2}{r_{min}} & (21) \\ \Rightarrow r_{min} &= \frac{52ke^2}{K_i} & (22) \\ &= \frac{52(9\times10^9)(1.602\times10^{-19})^2}{(6\times10^6)(1.602\times10^{-19})} = 1.25\times10^{-14}\text{ m} = 12.5\text{ fm}, & (23) \end{aligned}$$

or about 2.7 times the radius of the ^{56}Fe nucleus.

7. Chapter 43, Problem 20, Page 1189

The total binding energy is

$$\begin{aligned} E_b &= [26m_p + 30m_n - m_{Fe}] & (24) \\ &= [26(1.00728) + 30(1.00867) - 55.9206]\,(931.5) = 493\text{ MeV} & (25) \\ \Rightarrow E_b/A &= \frac{493}{56} = 8.80\text{ MeV/nucleon}. & (26) \end{aligned}$$

8. Chapter 43, Problem 30, Page 1189

(a) The time to decay a fraction N/N_0 of the original nuclei, t, is given by

$$\frac{t}{t_{1/2}} = \frac{\ln(N_0/N)}{\ln 2}. \quad (27)$$

Since the lifetime of ^{90}Sr is $t_{1/2} = 29$ years and the fraction that remains is 1 minus the fraction that decays, the time to decay 99% of the ^{90}Sr is

$$\begin{aligned} t &= t_{1/2}\frac{\ln(1/0.01)}{\ln 2} & (28) \\ &= (29)\frac{\ln(100)}{\ln 2} = 193\text{ years}. & (29) \end{aligned}$$

(b) The same calculation yields a time of

$$\begin{aligned} t &= t_{1/2}\frac{\ln(1/0.001)}{\ln 2} & (30) \\ &= (29)\frac{\ln(1000)}{\ln 2} = 289\text{ years}. & (31) \end{aligned}$$

9. Chapter 43, Problem 38, Page 1189

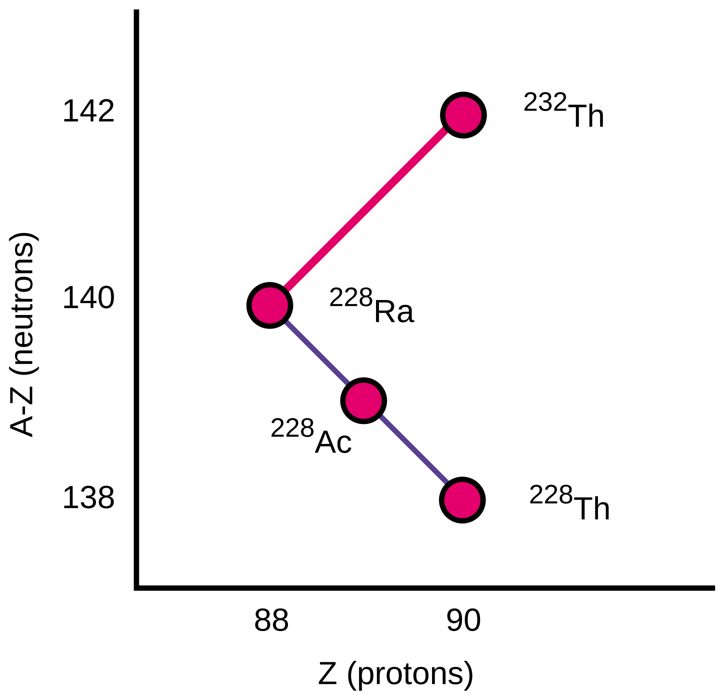

Figure 2: The energy level diagram showing all allowed transitions of the electron in a square well with $n = 4$ (Problem 9b).

(a) From counting the change in the number of neutrons and protons after each α and β decay, we find that the third daughter has four less neutrons but the same number of protons, so that it is ${}^{232}_{90}\text{Th}$.

(b) The decay chain is shown in Fig. 2.

10. Chapter 43, Problem 50, Page 1190

(a) We assume that the two radioactive isotopes, a and b, are independent of each other, so the total activity is given by

$$(\lambda N)_{tot} = (\lambda N)_a + (\lambda N)_b \tag{32}$$

$$= (\lambda N)_{a,0}\, 2^{-t/t_{a,1/2}} + (\lambda N)_{b,0}\, 2^{-t/t_{b,1/2}}, \tag{33}$$

where the initial activities, $(\lambda N)_{a,0}$ and $(\lambda N)_{b,0}$, are determined by the initial sample compositions. One of these two isotopes has a shorter lifetime, and therefore dominates the total activity at times near 0, whereas the other dominates the activity after the one with the shorter lifetime has decayed away. By approximating the initial and final activities with straight lines and solving for the slopes, we get values of

$$t_{a,1/2} \simeq 1.0 \text{ h} \tag{34}$$

$$t_{b,1/2} \simeq 36 \text{ h}. \tag{35}$$

(b) We can solve for the activity of the longer-lived species at $t = 0$ to get an initial activity of

$$(\lambda N)_{b,0} \simeq 0.3 \text{ MBq.} \tag{36}$$

Since the total activity at $t = 0$ is about 4.3 MBq, the initial activity of the shorter-lived species is

$$(\lambda N)_{a,0} \simeq 4.3 - 0.3 = 4.0 \text{ MBq.} \tag{37}$$