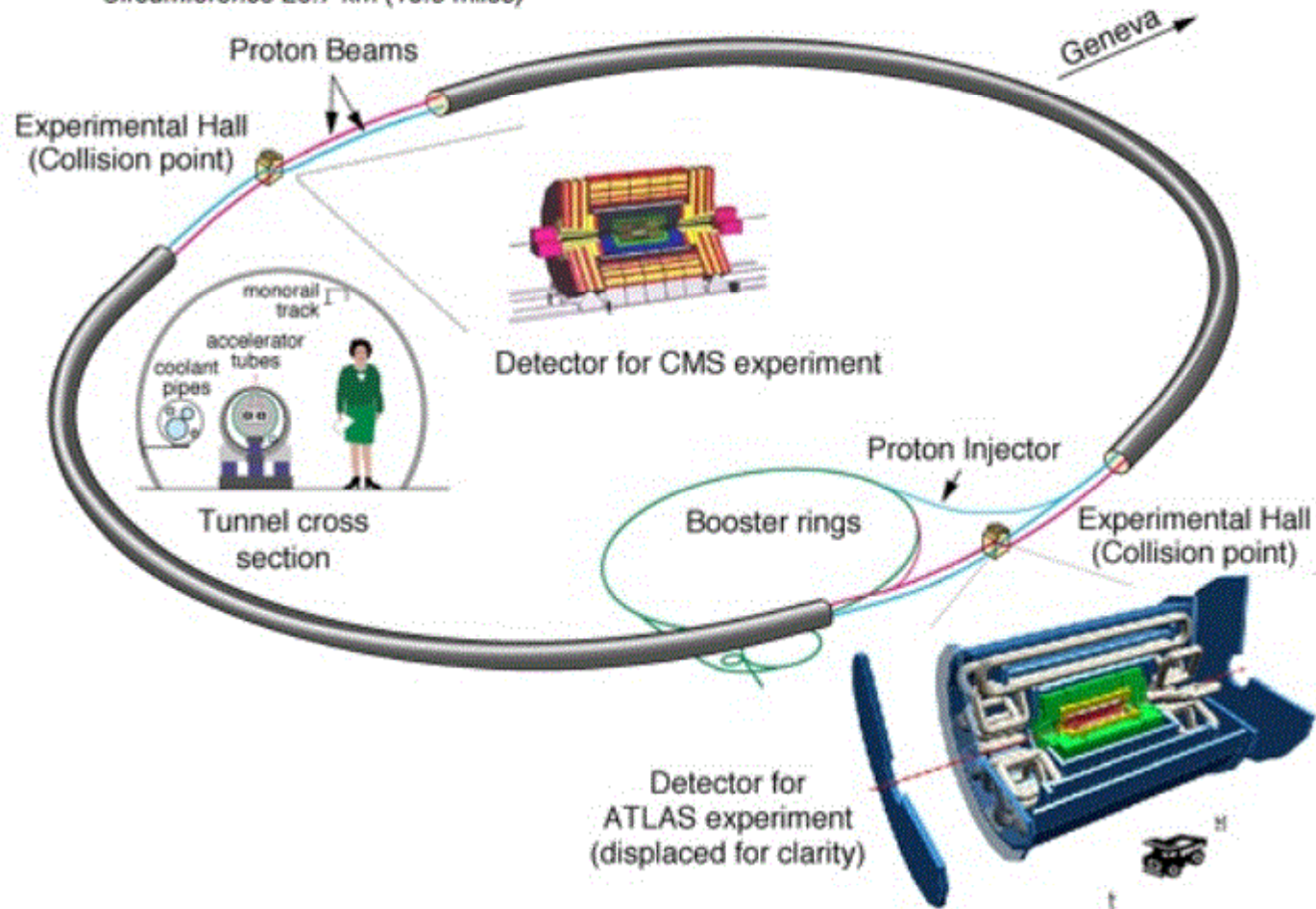


CARTOON OF MODERN ACCELERATOR COMPLEX

Large Hadron Collider at CERN

Circumference 26.7 km (16.6 miles)



Applications of Particle Accelerators

Nuclear Physics

Electron Proton Accelerators

High Energy Physics

Fixed Target Accelerators

Colliding Beam Storage Rings

Linear Colliders

Power Generation

Inertial Fusion

Reactor Fuel Breeding

Industry

X-Ray Radiography

Ion Implantation

Isotope Production

Materials Testing

Food Sterilization

X-Ray Lithography

Synchrotron Radiation

(Electron Storage Rings)

Atomic & Molecular Physics

Condensed Matter Physics

Earth Science

Chemistry

Molecular & Cell Biology

Surface Physics

Coherent Radiation

Free Electron Lasers

Medicine

Radiotherapy

Health Physics

Imaging

Microsurgery with tunable FEL

ACCELERATORS

LINEAR

ELECTROSTATIC
RF LINAC

CIRCULAR

CYCLOTRON
BETATRON
SYNCHROCYCLOTRON
SYNCHROTRON
STRONG FOCUSING
WEAK FOCUSING
FFAG
STORAGE RINGS

ELECTRONS, PROTONS, HEAVY IONS
POSITRONS, ANTI PROTON

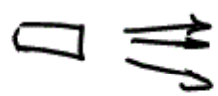
- DYNAMICS OF CHARGED PARTICLES IN EM FIELD
- SUPERCONDUCTING MAGNETS & RF CAVITIES
- VACUUM, RF, ETC ETC

⇒ COMPLEX SUBJECT.

ACCELERATOR DESIGN

• FOR US $\rightarrow$ PRODUCE ENERGY/INTENSITY TO PROBE INTERESTING PHYSICS.

• FIXED TARGET



$K \rightarrow \mu \nu$

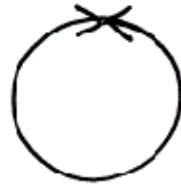
NEUTRINOS

SPS / KERN / TEV

JPARC / XFEL

$$E_{USE} \sim E_{CM} \sim \sqrt{E_b}$$

• COLLIDER



$$E_{USE} \sim E_{CM} = 2E_b$$

$$E \sim 56 \text{ eV} \quad \tau^+ \rightarrow B \bar{B}$$

CESR

DORIS

B-FACTORIES

SUPER B

$$E \sim 456 \text{ eV} \quad Z^0 \rightarrow \mu^+ \mu^-$$

LEP

$$E \sim 1 \text{ TeV} \quad p \bar{p} \rightarrow t \bar{t}$$

TEVATRON

$$E \sim 7 \text{ TeV}$$

$$pp \rightarrow H, \text{ SUSY}$$

LHC

INTENSITY / LUMINOSITY

- SUFFICIENT INTERACTION RATE TO OBSERVE PHYSICS IN PRACTICAL TIME SCALE

RATE $\longrightarrow R = \sigma \cdot \mathcal{L} \longleftarrow$ LUMINOSITY

$s^{-1} \quad cm^2 \quad s^{-1} cm^{-2}$

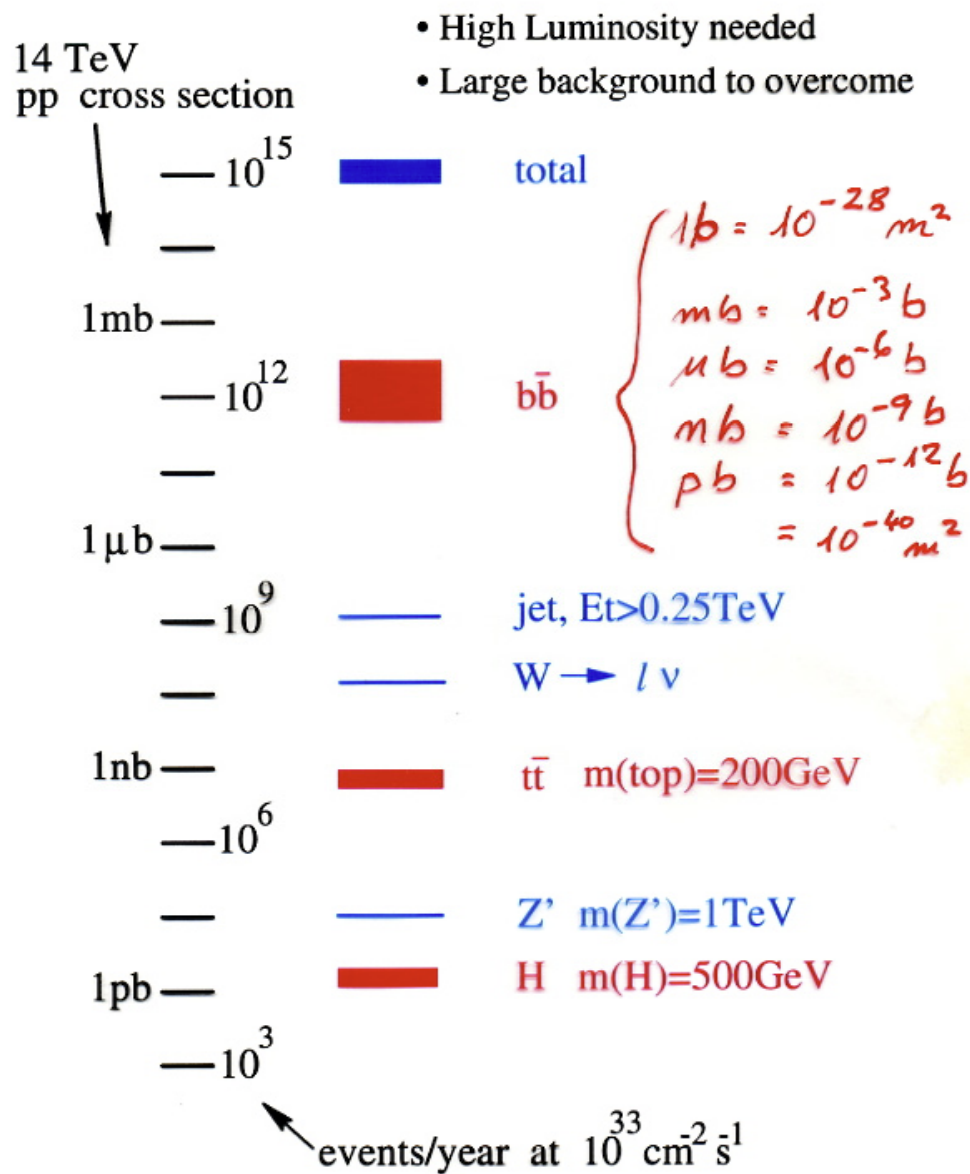
$$\sigma = 5 nb \quad e^+e^- \rightarrow T \rightarrow B\bar{B}$$

$$\sigma = 30 nb \quad e^+e^- \rightarrow Z^0$$

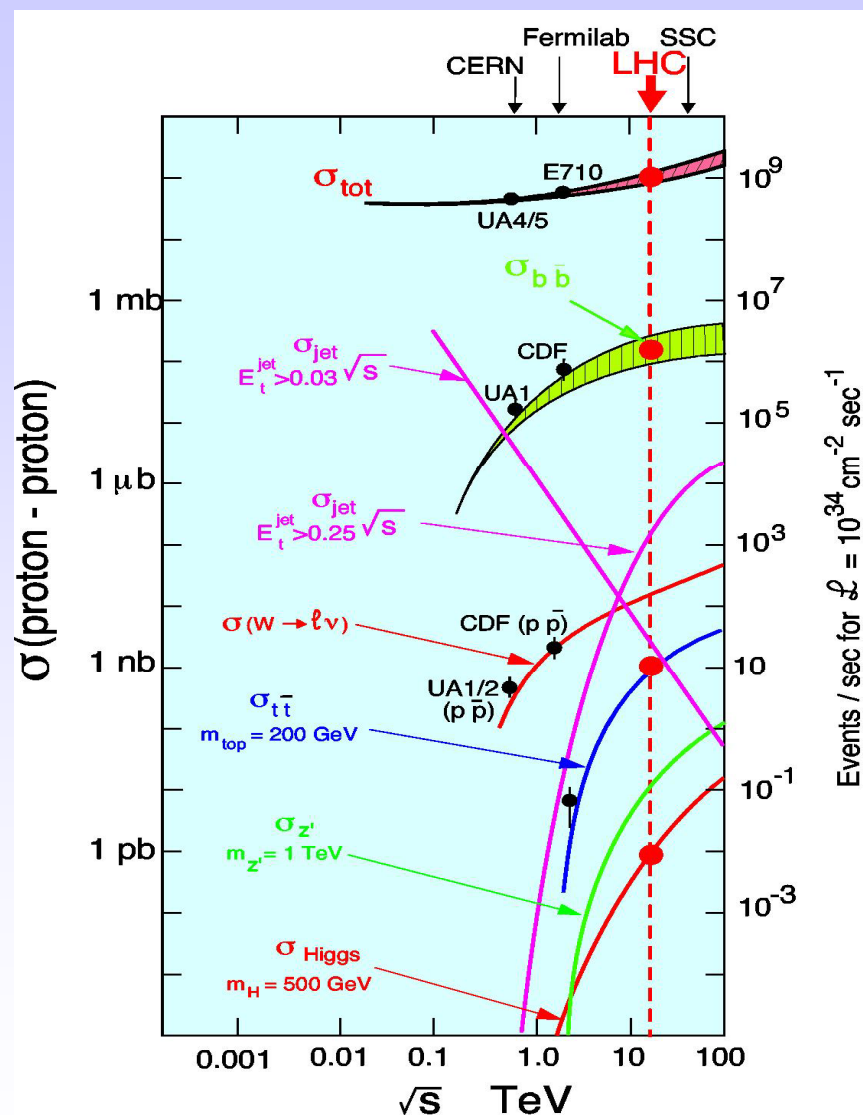
$$\sigma = 50 pb \quad e^+e^- \rightarrow W^+W^-$$

$$\sigma = 1 pb \quad pp \rightarrow \text{Higgs}$$

PP Cross Section



Cross Sections and Production Rates



Rates for $L = 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$: (LHC)

• Inelastic proton-proton reactions:	$10^9 / \text{s}$
• bb pairs	$5 \cdot 10^6 / \text{s}$
• tt pairs	$8 / \text{s}$
• $W \rightarrow e \nu$	$150 / \text{s}$
• $Z \rightarrow e e$	$15 / \text{s}$
• Higgs (150 GeV)	$0.2 / \text{s}$
• Gluino, Squarks (1 TeV)	$0.03 / \text{s}$

LHC is a factory for:
top-quarks, b-quarks, W, Z, Higgs,

The only problem: you have to detect them !

SYNCHROTRON RADIATION WILHE P. 33

FOR A CHARGED PARTICLE IN A CIRCULAR ORBIT

$$\text{POWER RADIATED} = P_s = \frac{e^2 c \gamma^2}{6\pi \epsilon_0} \cdot \frac{1}{(mc^2)^2} \cdot \left(\frac{dp}{dt} \right) \quad \text{MOMENTUM}$$

$$\text{MOTION THRU ANGLE } d\alpha \rightarrow dp = p d\alpha$$

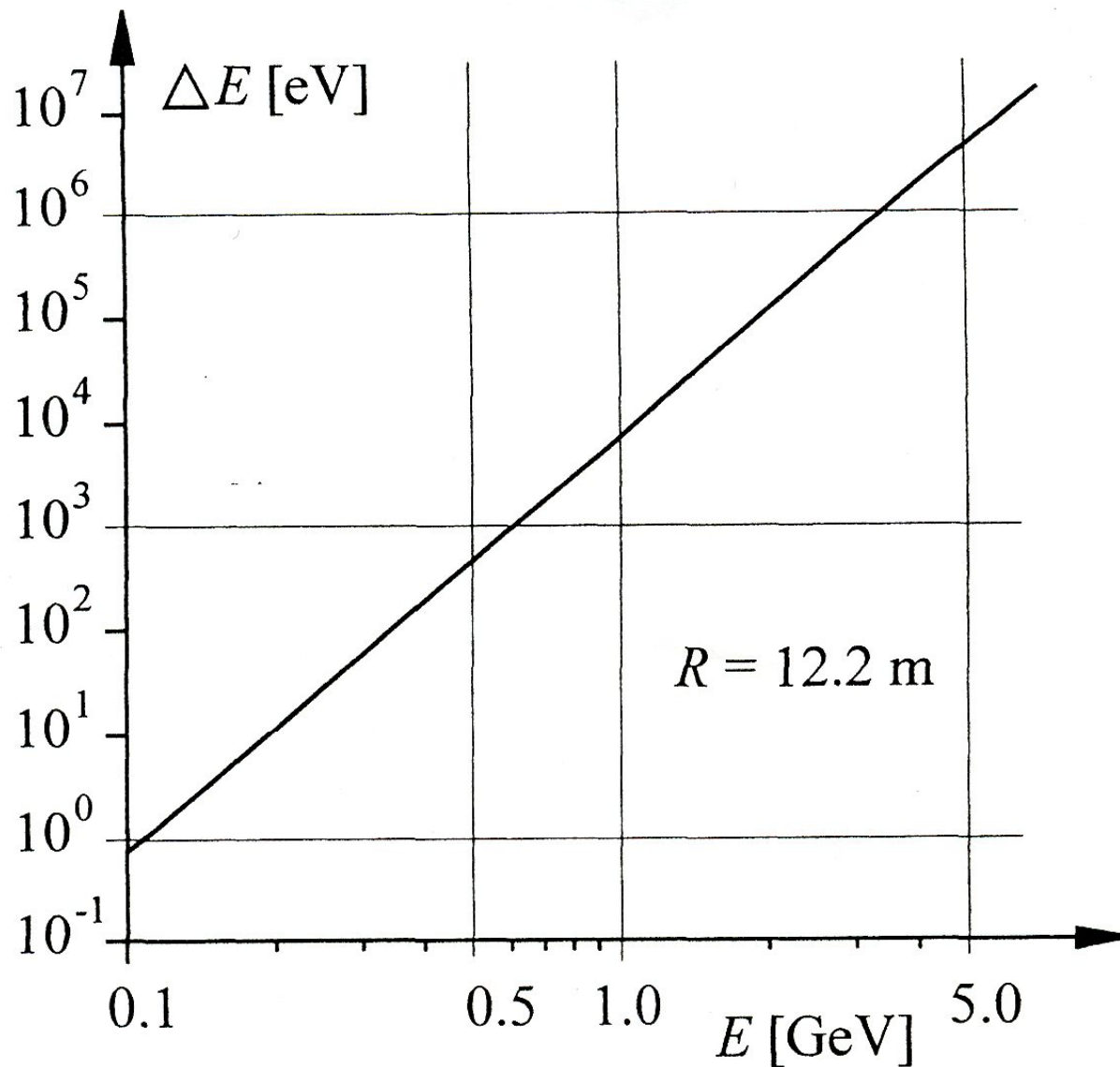
$$\frac{dp}{dt} = p \omega = p \frac{v}{R} \rightarrow P_s = \frac{e^2}{6\pi \epsilon_0} \frac{c}{(mc^2)^4} \cdot \frac{E^4}{R^2} \quad \left\{ \begin{array}{l} v = c \\ E = pc \\ \gamma = \frac{E}{mc^2} \end{array} \right.$$

$$\Delta E = \oint P_s dt = P_s t_b = P_s \frac{2\pi R}{c}$$

$$\Delta E = \frac{e^2}{3\epsilon_0 (mc^2)^4} \cdot \frac{E^4}{R} \rightarrow \frac{\Delta E_p}{\Delta E_e} = \left(\frac{m_e}{m_p} \right)^4 \sim 10^{-13}$$

$$\Delta E (\text{keV}) = 88.5 \frac{E^4 [\text{GeV}]^4}{R [\text{m}]}$$

SYNCHROTRON RADIATION IN A CIRCULAR MACHINE



SYNCHROTRON RADIATION IN VARIOUS ELECTRON MACHINES — LEP IS OBVIOUSLY LAST !

accelerator	L [m]	E [GeV]	R [m]	B [T]	ΔE [keV]
BESSY I (Berlin)	62.4	0.80	1.78	1.50	20.3
DELTA (Dortmund)	115	1.50	3.34	1.50	134.1
DORIS II (Hamburg)	288	5.00	12.2	1.37	4.53×10^3
ESRF (Grenoble)	844	6.00	23.4	0.855	4.90×10^3
PETRA (Hamburg)	2304	23.50	195	0.40	1.38×10^5
LEP (Geneva)	27×10^3	70.00	3000	0.078	7.08×10^5

ACCELERATOR BUZZ WORDS

LUMINOSITY

TRANSVERSE EMITTANCE

AMPLITUDE FUNCTION

BETATRON OSCILLATIONS

TUNE

SYNCHROTRON OSCILLATIONS

PHASE STABILITY

SYNCHRONOUS PHASE

LONGITUDINAL EMITTANCE

BEAM-BEAM TUNE SHIFT

STRONG/WEAK FOCUSING

FOODO STRUCTURES

DYNAMICS OF CHARGED PARTICLES IN ELECTRIC & MAGNETIC FIELDS

LORENTZ $\frac{d\mathbf{p}}{dt} = e(\mathbf{\bar{E}} + \mathbf{\bar{v}} \times \mathbf{\bar{B}})$

MAXWELL (MKS) $\nabla \cdot \mathbf{\bar{E}} = \frac{1}{\epsilon_0} \rho(\mathbf{\bar{r}}, t)$

$$\int \mathbf{\bar{E}} \cdot d\mathbf{\bar{s}} = \frac{1}{\epsilon_0} \int \rho dV$$

$$\nabla \cdot \mathbf{\bar{B}} = 0$$

$$\int \mathbf{\bar{B}} \cdot d\mathbf{\bar{s}} = 0$$

$$\nabla \times \mathbf{\bar{E}} = -\partial \mathbf{\bar{B}} / \partial t$$

$$\oint \mathbf{\bar{E}} \cdot d\mathbf{\bar{e}} = -\int \mathbf{\bar{B}} \cdot d\mathbf{\bar{s}}$$

$$\nabla \times \mathbf{\bar{B}} = \mu_0 \mathbf{\bar{j}}(\mathbf{\bar{r}}, t) + \frac{1}{c^2} \frac{\partial \mathbf{\bar{E}}}{\partial t}$$

$$\oint \mathbf{\bar{B}} \cdot d\mathbf{\bar{e}} = \mu_0 \int \mathbf{\bar{j}} d\mathbf{\bar{s}} + \frac{1}{c^2} \int \mathbf{\bar{E}} \cdot d\mathbf{\bar{s}}$$

**MAGNETIC
MATERIAL**

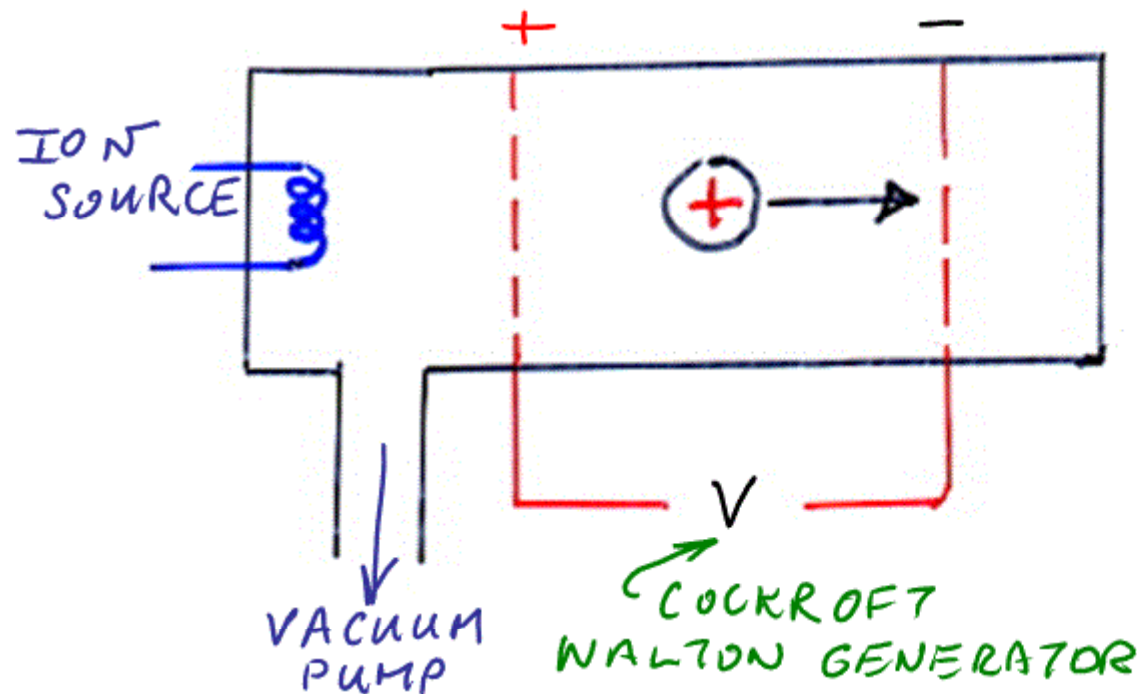
$$\nabla \times \mathbf{\bar{H}} = \mathbf{\bar{j}}(\mathbf{\bar{r}}, t) + \epsilon_0 \partial \mathbf{\bar{E}} / \partial t$$

$$\mathbf{\bar{B}} = \mu_0 (\mathbf{\bar{H}} + \mathbf{M})$$

$\uparrow$ **MAGNETIZATION**

SIMPLE ELECTROSTATIC ACCELERATOR

USED BY COCKROFT & WALTON — ARTIFICIAL RADIOACTIVITY



ELECTRIC FIELD

$$\vec{F} = q \vec{E}$$

CHARGE ON PARTICLE

$$|\vec{E}| = \frac{V}{d}$$

ENERGY GAINED BY CHARGED PARTICLE

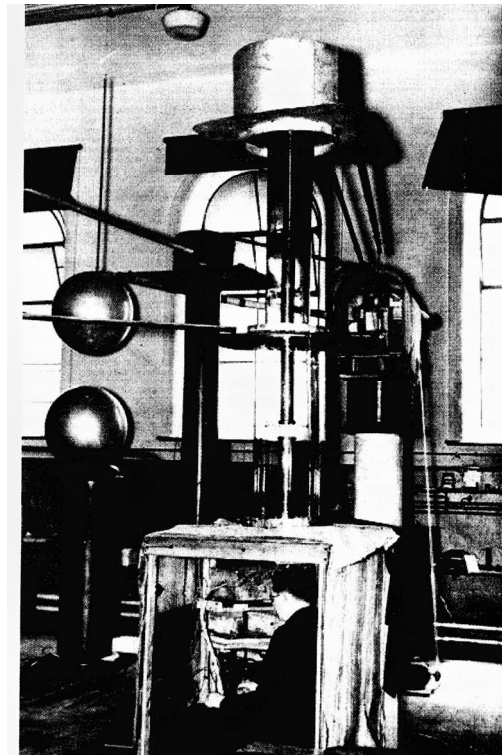
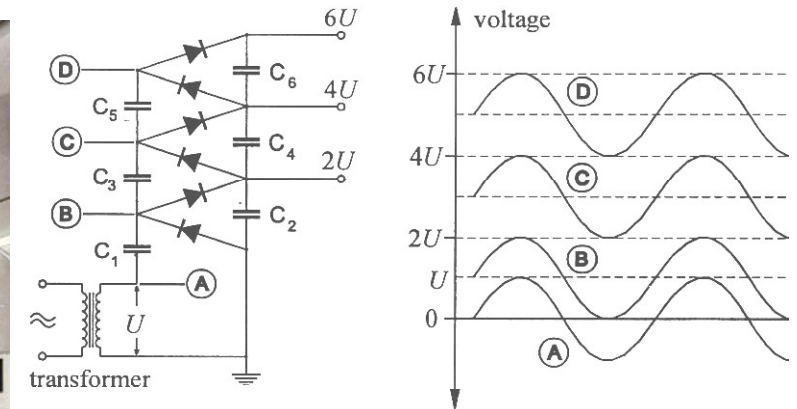
$$E_{Acc} = Fd = qV$$

• TWO SHORTCOMINGS:

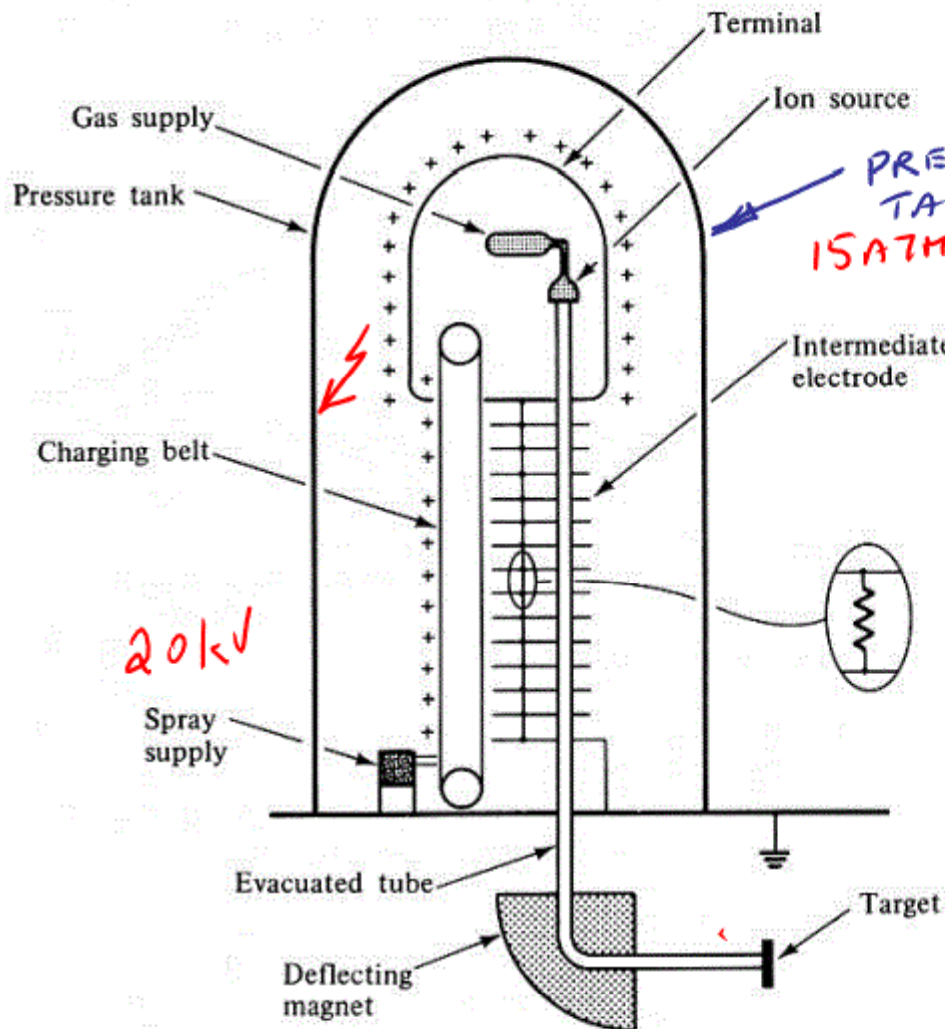
— GENERATING HIGH VOLTAGE

— INSULATING BEYOND $\sim 100 \text{ kV}$ (100 keV)

Cockcroft-Walton Generator



VAN DE GRAAFF



• TRANSPORT CHARGE

Q

TO TERMINAL OF
CAPACITANCE

C

$$V = \frac{Q}{C}$$

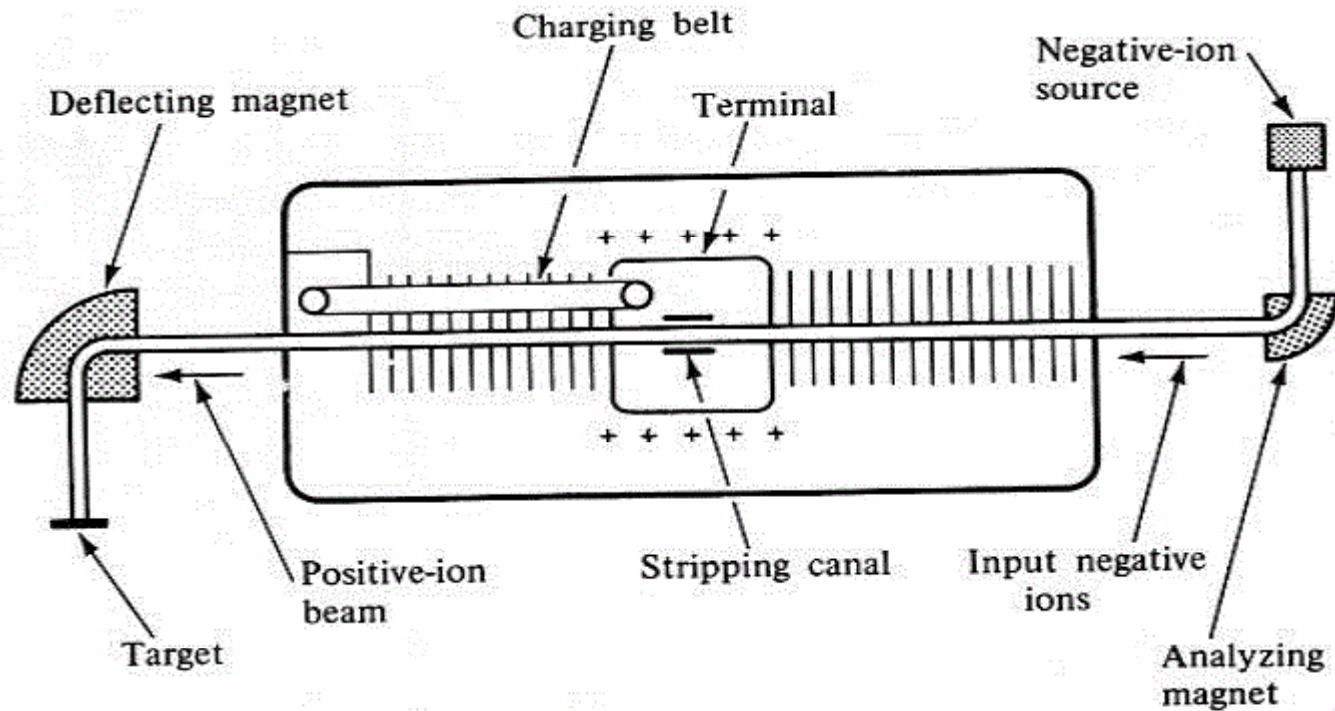
• LIMITATION ~ 12 MV

→ VOLTAGE BREAKDOWN

→ NOT ENOUGH TO
RESOLVE PROTONS
IN THE NUCLEUS

~ 12 MeV

TANDEM VAN DE GRAFF



- USE VOLTAGE ON TERMINAL TWICE
- ACCELERATE -VE IONS UP TO TERMINAL
- STRIP OFF TWO ELECTRONS INSIDE TERMINAL

— ACCELERATE AWAY

- 40 MeV CHALK RIVER HAD LARGE TANDEM



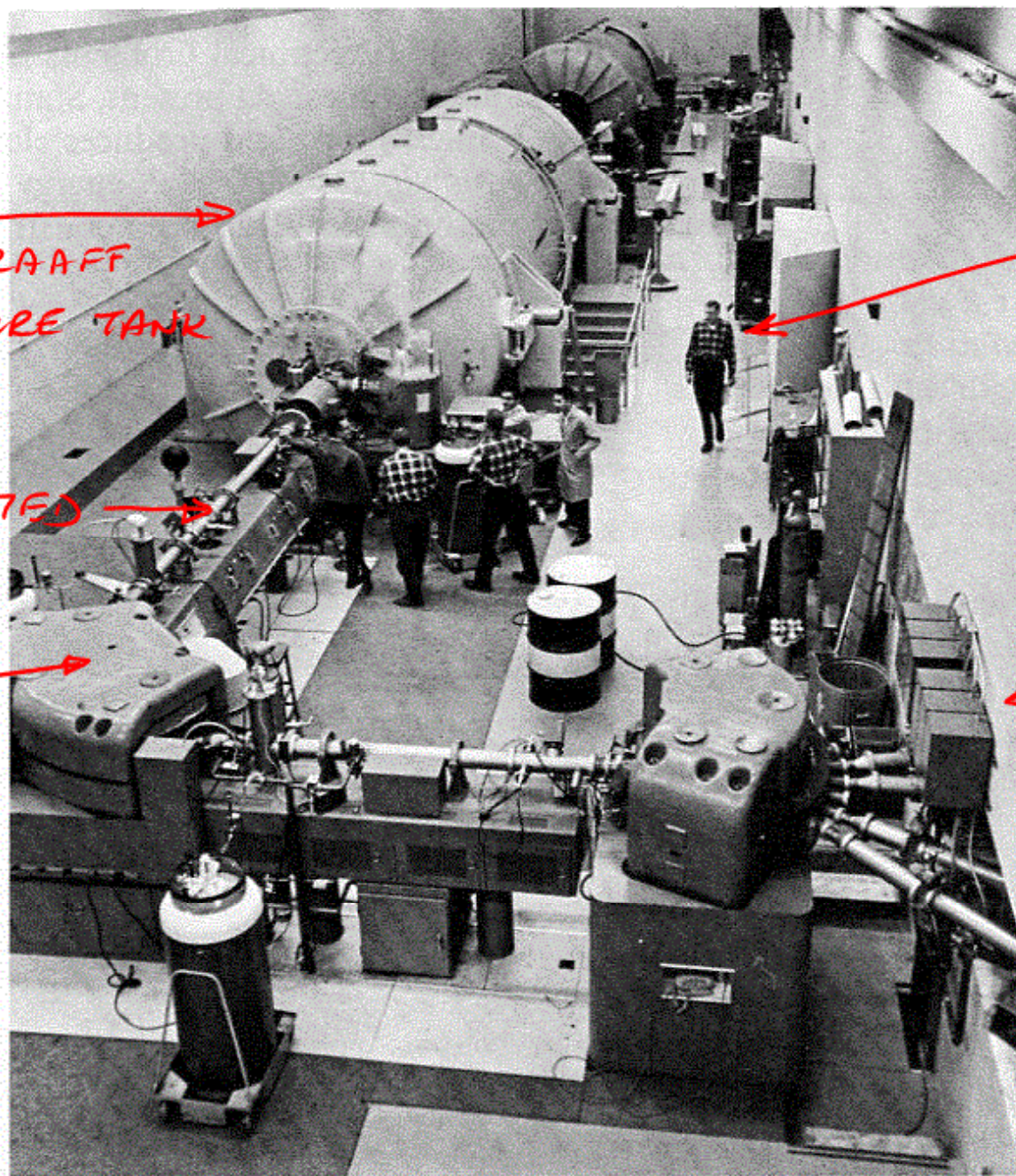
VAN DE GRAAFF
IN PRESSURE TANK

ACCELERATED
BEAM

BENDING
MAGNET

1960'S
PHYSICIST

BEAMS TO
EXPERIMENTS



DC HIGH-VOLTAGE ACCELERATORS – TANDEM VAN DE GRAAFFS



Yale 22-MV tandem.



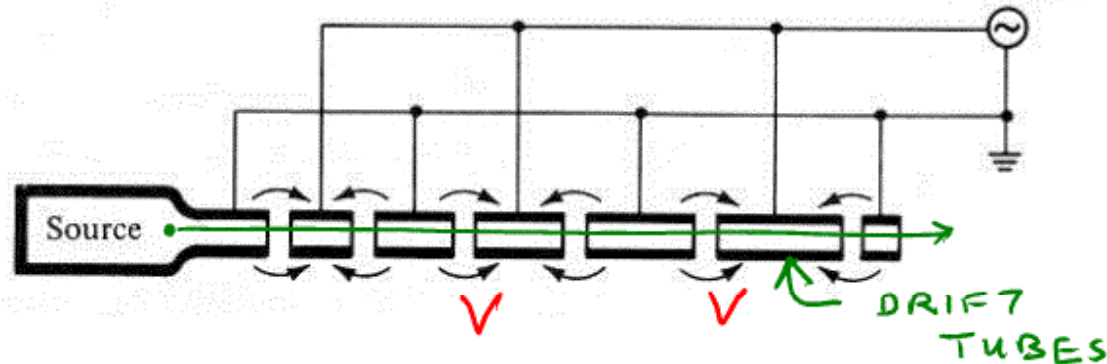
Daresbury folded tandem
(20 MV in a 230-ft tower).

LINEAR ACCELERATOR (LINAC)

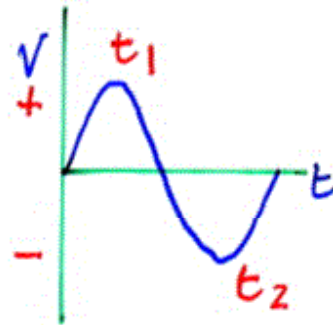
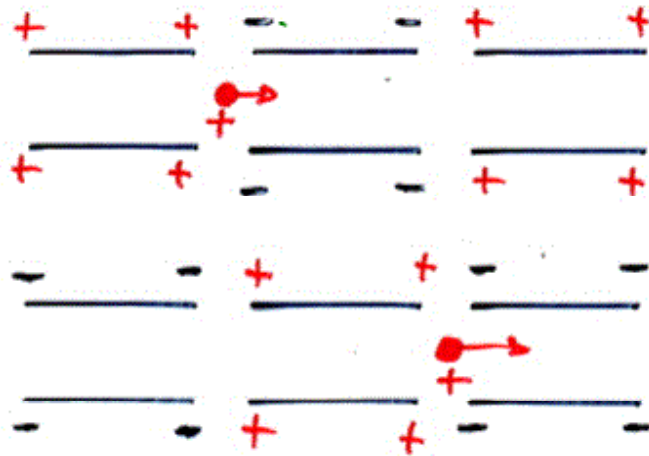
TORONTO USED TO HAVE 40 MeV LINAC

• INVENTED BY WIDEROE

RADIO FREQUENCY ~~2~~
Generator



• USE SAME RELATIVELY SMALL VOLTAGE IN MANY STEPS — REACH EQUIVALENT HIGH VOLTAGE



- FIELD ZERO INSIDE DRIFT TUBES
- PARTICLE MOVES ONE GAP $\rightarrow$ NEXT, IN TIME E-FIELD REVERSES
- PARTICLES ACCELERATING $\rightarrow$ LENGTH OF DRIFT TUBES INCREASES
- $\rightarrow$ NON RELATIVISTIC

- PARTICLE ENTERING DRIFT TUBE n , ENERGY $n \cdot eV$
GAPS TRAVERSED VOLTAGE ACROSS GAP
- NON-RELATIVISTIC KINETIC ENERGY $T = \frac{1}{2} m v^2$

$$v = \left(\frac{2 \cdot n eV}{m} \right)^{\frac{1}{2}} \quad \left(\frac{2T}{m} \right)^{\frac{1}{2}}$$

- THIS VELOCITY TAKES PARTICLE THRU DRIFT TUBE OF LENGTH L_n IN TIME FIELD TAKES TO REVERSE

$$t_n = L_n / v$$

- FREQUENCY OF RADIO FREQUENCY OSCILLATOR f (Hz) HAS REVERSAL TIME $\frac{1}{2f}$

$$L_n = \frac{1}{2f} \left(\frac{2 n eV}{m} \right)^{\frac{1}{2}} \rightarrow L_n \propto \sqrt{n}$$

NUMERICAL VALUES

$$L_n = \frac{1}{2f} \cdot v_n$$

TYPICALLY $v_n = 0.5c$; $f = 7 \text{ MHz} \rightarrow L_n = 10.7 \text{ m}$

- LOW RADIO FREQUENCY LEADS TO VERY LONG STRUCTURES

- PRACTICALLY NEED HIGH RADIO FREQUENCIES

KLYSTRONS $\rightarrow 100 \text{ MHz} \rightarrow 10 \text{ GHz}$

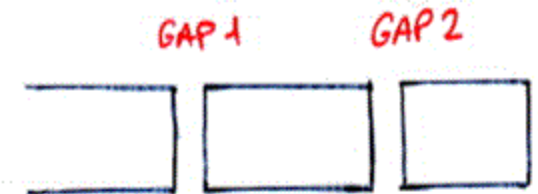
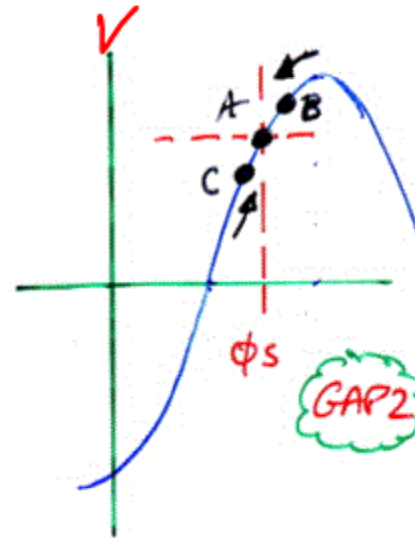
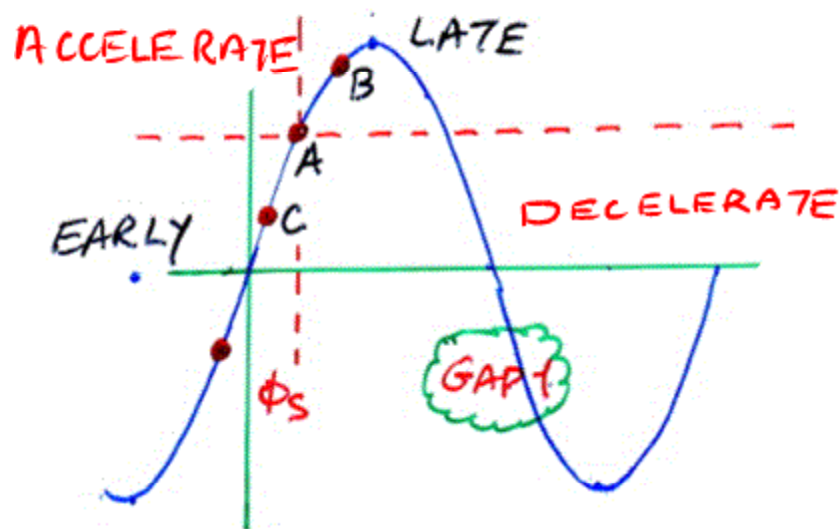
- THIS WIDEROE STRUCTURE IS OBSOLETE

 - $\rightarrow$ VERY INEFFICIENT

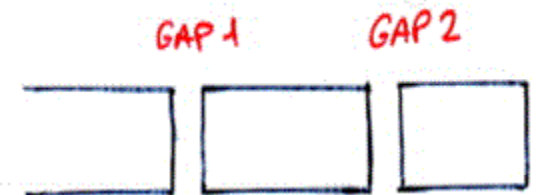
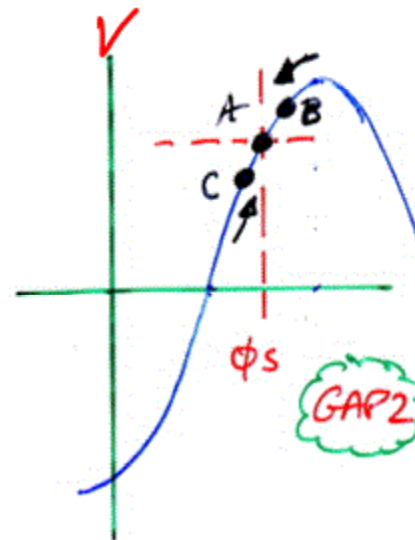
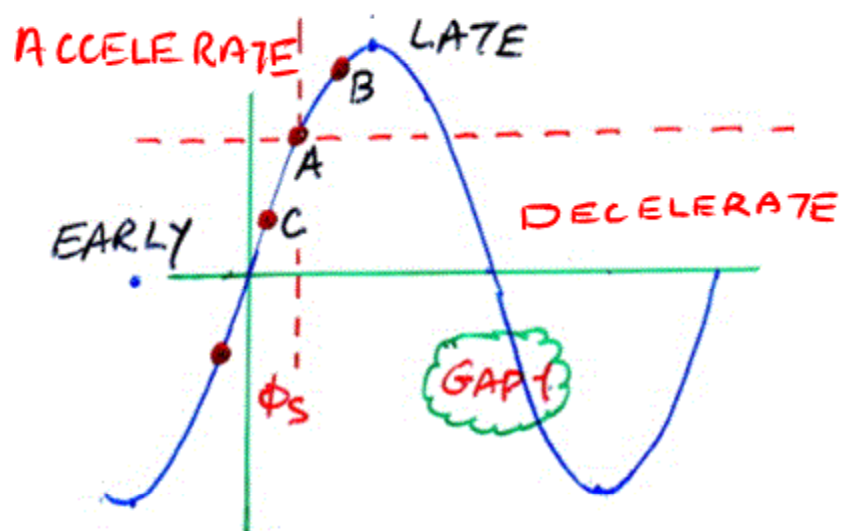
 - $\rightarrow$ RADIATION LOSS

PHASE STABILITY IN LINAC

- TO MAINTAIN PRECISE SYNCHRONISM BETWEEN PARTICLE MOTION & RF OSCILLATOR SEEMS DIFFICULT → NOT SO

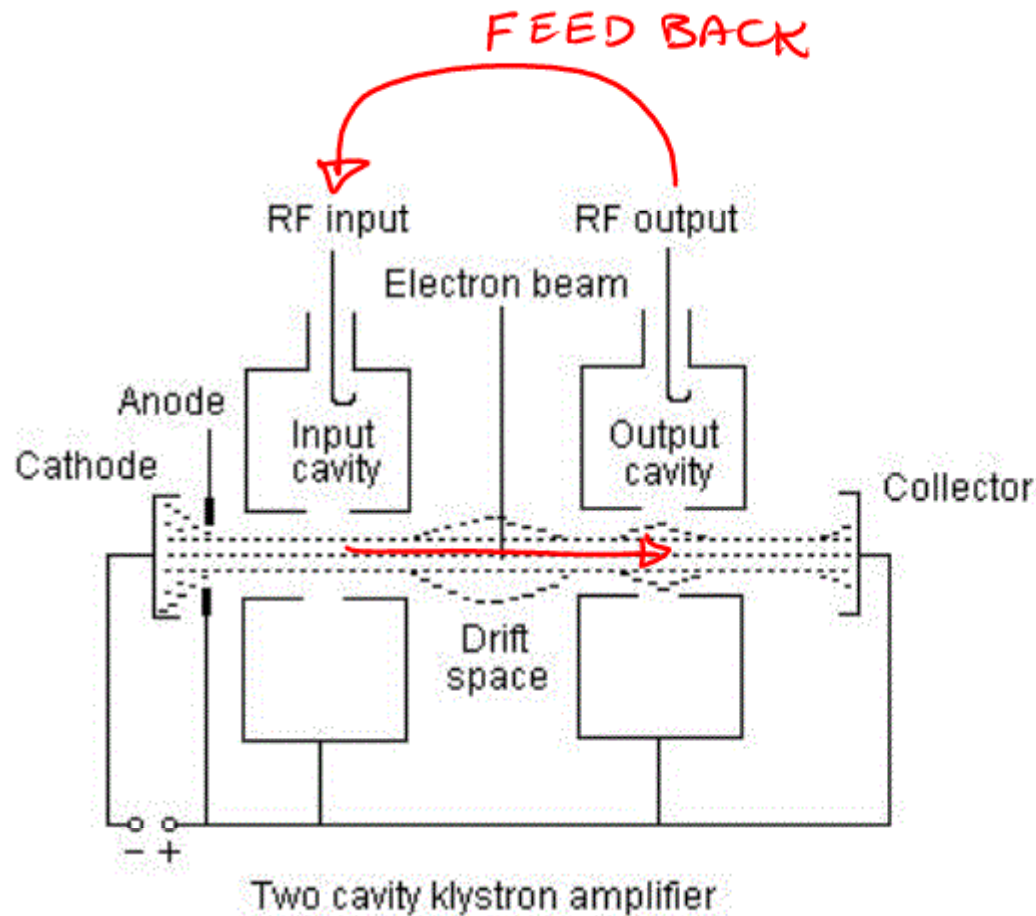


- PARTICLE **A** CROSSES GAP1 PHASE ϕ IN STEP WITH VOLTAGE
- GAP2 - SAME VOLTAGE PHASE - AGAIN ACCELERATED
- PARTICLE **B** ARRIVE **LATE**, VOLTAGE **HIGHER**
ACCELERATED **MORE** ARRIVES AT GAP2 **EARLIER**



- PARTICLE C ARRIVES EARLIER AT GAP 1
 - VOLTAGE LOWER, ACCELERATED LESS
 - ARRIVES LATER IN PHASE AT GAP 2
- B AND C CONVERGE IN PHASE WITH A
- NO NEED TO START WITH PARTICLES ALL IN PHASE WITH RADIO FREQUENCY OSCILLATOR

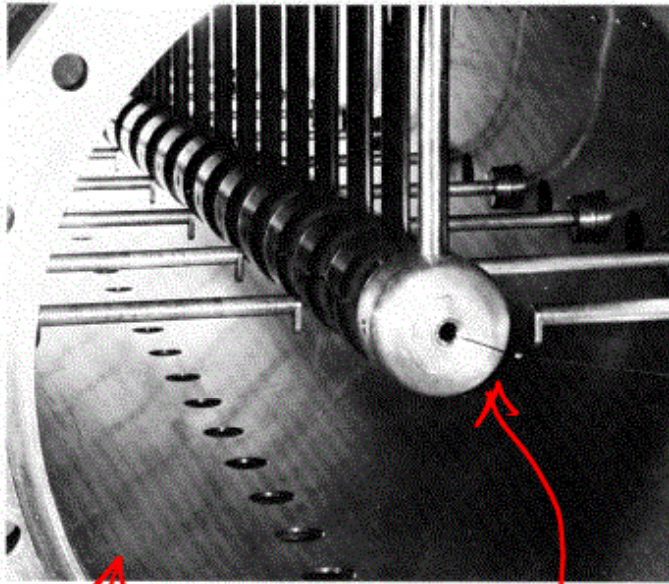
RADIO FREQUENCY POWER GENERATION



2 CAVITY KLYSTRON OSCILLATOR

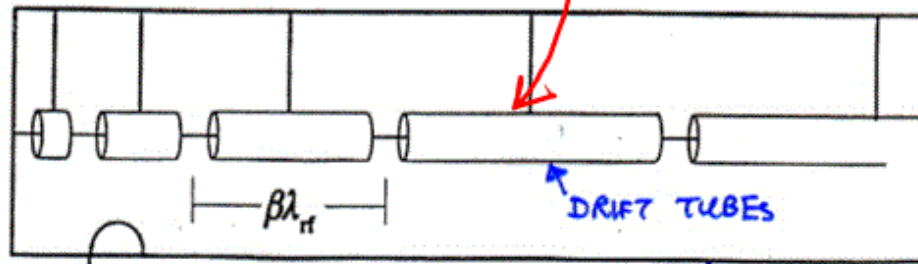


ALVAREZ LINAC STRUCTURE



CONDUCTING ENCLOSURE

DRIFT TUBES



rf generator

RADIO FREQUENCY INPUT

- WIDERDE STRUCTURE VERY INEFFICIENT — RADIO FREQUENCY RADIATION LOSS

- ALVAREZ STRUCTURE — RESONANT CAVITY LIKE KLYSTRON

- USED FOR PROTON SYNCHROTRON INJECTOR
100 MeV $\rightarrow$ 100 MHz

- HIGH ENERGY ELECTRON ACCELERATORS

40 GeV - 500 GeV GHz
RF

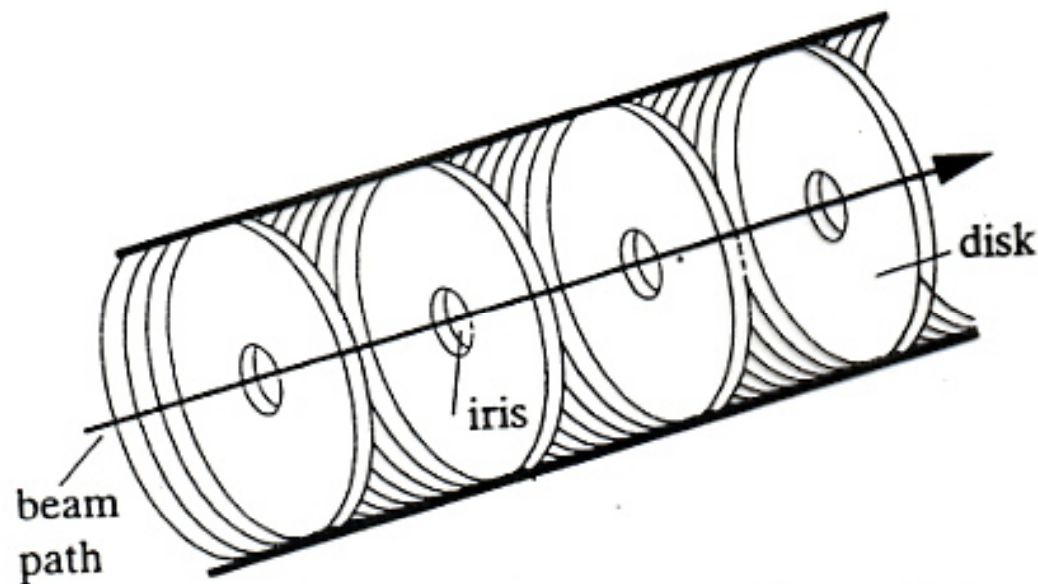


Fig.2.8. Disk loaded accelerating structure for an electron linear accelerator (:

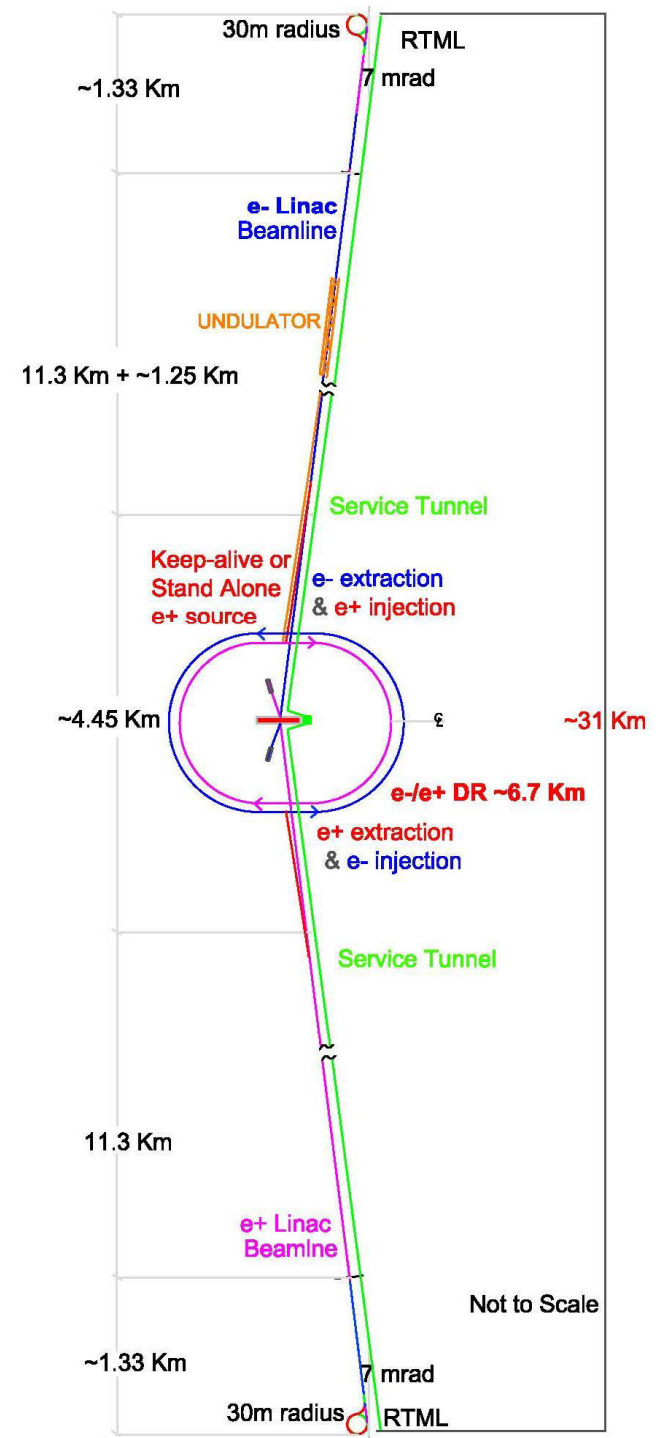
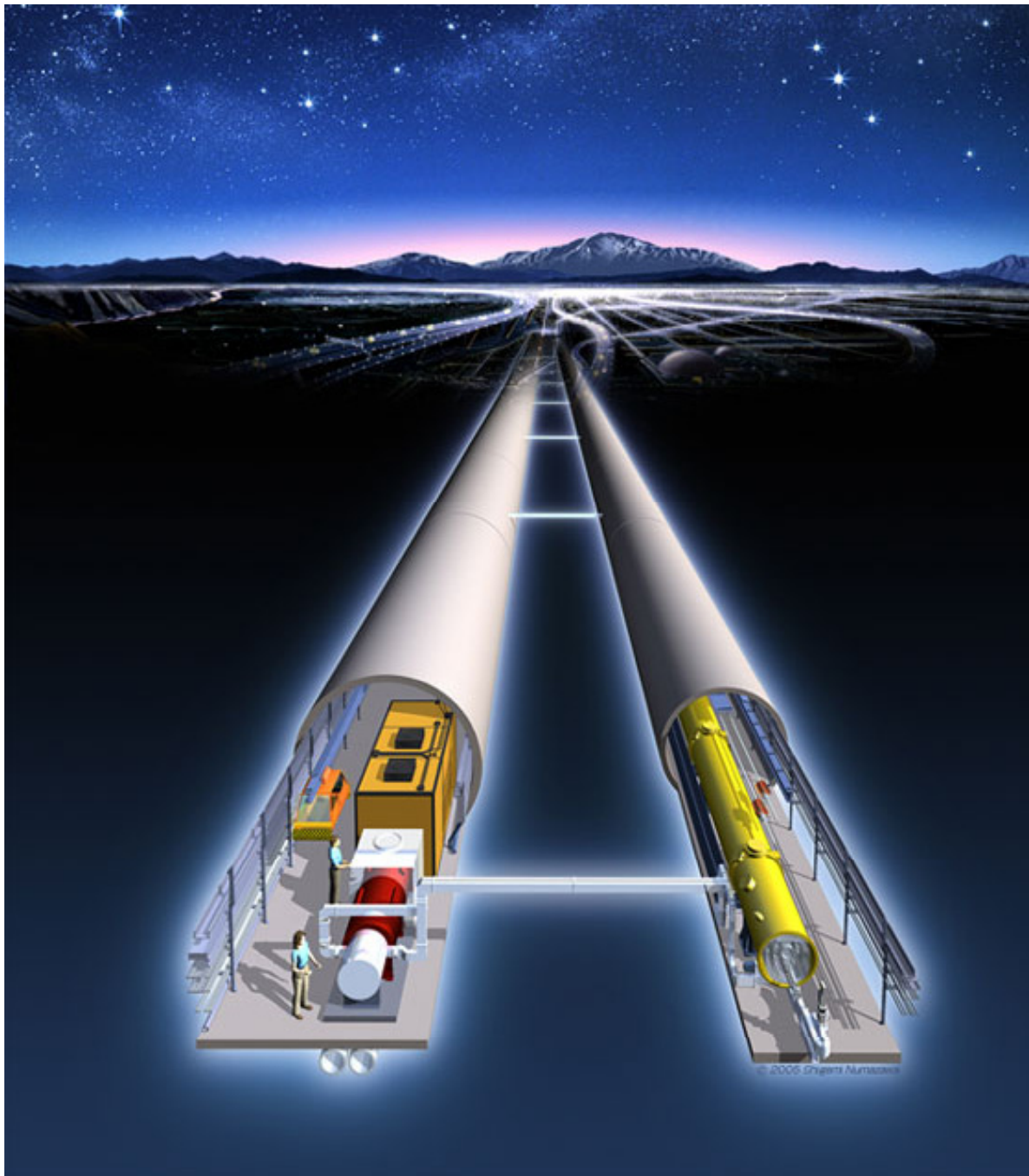
- TRAVELING WAVE LINAC (SLAC)
- IRISES REDUCE v_{gap} OF EM WAVES
↓
- MATCH VELOCITY OF ELECTRONS



SLAC – 50 GeV Electron LINAC

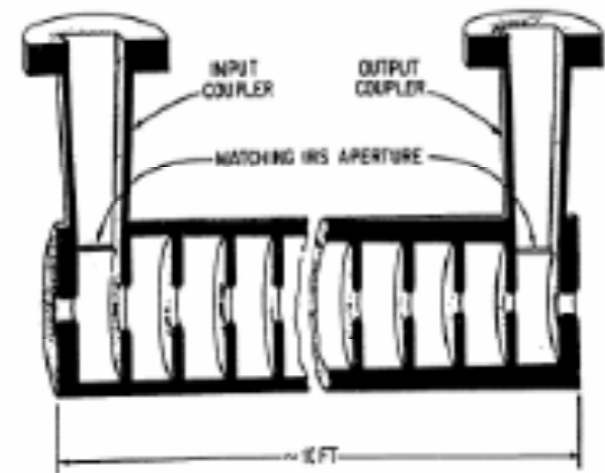


ILC RF CAVITY STRUCTURE





The ISAC 150-keV/u RFQ linac

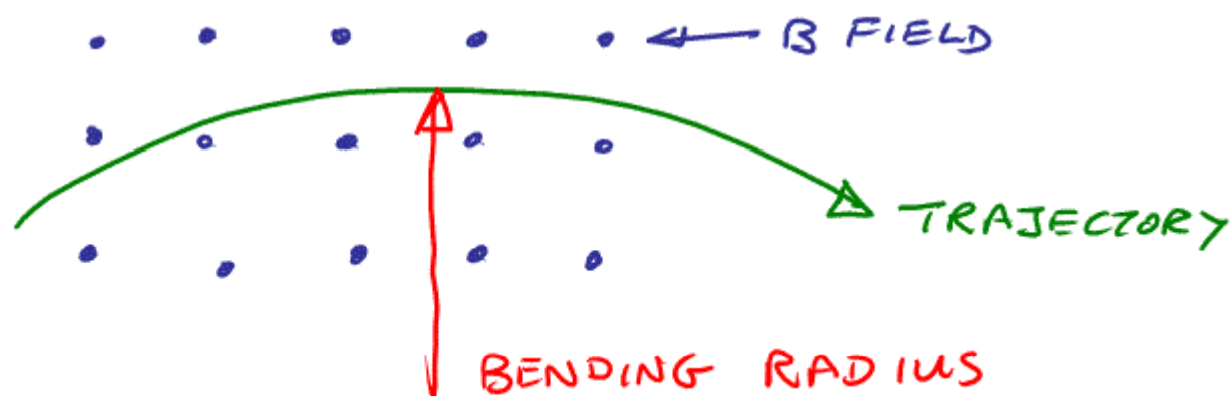


500 keV electron LINAC for Cancer Therapy

PARTICLE BENDING IN MAGNETIC FIELD

$$\vec{F} = q \left(\vec{E} + \frac{1}{c} \vec{v} \times \vec{B} \right) \quad \text{LORENTZ}$$

- FORCE FROM MAGNETIC FIELD NORMAL TO PARTICLE TRAJECTORY



- FOR NO ELECTRIC FIELD & B FIELD NORMAL TO PAGE

$$F = q \frac{v}{c} B \sin \theta \quad \leftarrow 90^\circ = 1 \quad \rightarrow F = q \frac{vB}{c}$$

- FOR A PARTICLE MOVING IN A CIRCLE OF RADIUS p

$$\text{CENTRIPETAL} = \text{LORENTZ} \\ \text{FORCE} \quad \text{FORCE}$$

CIRCULAR ACCELERATORS

- AT PRESENT PARTICLE PHYSICS STILL DOMINATED BY CIRCULAR ACCELERATOR

CESR

PEP II

KEK

JPARC

LEP

SUPER-B

SPS

TEVATRON

AGS / RHIC

LHC

- CIRCULAR ACCELERATORS ARE MOST EFFICIENT & COMPACT WAY TO REACH HIGH ENERGY → UNTIL SYNCHROTRON RADIATION DOMINATES

CENTRIPETAL = LORENTZ
FORCE FORCE

$$\frac{\gamma m v^2}{\rho} = \frac{v B q}{c} \Rightarrow \rho = \frac{p \cdot c}{B q}$$

BENDING RADIUS
IN GAUSSIAN UNITS

• ACCELERATOR BUILDERS USE m, VOLT, TESLA

$$pc = \rho \cdot B \cdot q$$

eV/c m Gauss

$$\text{VOLT} = \text{STATVOLT} / 300$$

$$\text{TESLA} = 10^4 \text{ GAUSS}$$

$$\text{m} = 10^2 \text{ cm}$$

$$pc \left[\frac{\text{V}}{c} \times 300 \right] = \rho \left[\text{m} \times 10^2 \right] B \left[\text{T} \times 10^4 \right] e$$

$$pc [\text{GeV}/c] = 0.3 \rho [\text{m}] B [\text{T}]$$

$$\rho [\text{m}] = \frac{p [\text{GeV}]}{0.3 B [\text{T}]}$$

$$p[\text{sv}] \cdot c = p(\text{cm}) B(\text{g})$$

$$p\left[\frac{\text{V}}{300}\right] \cdot c = p(m \times 10^2) B(T \times 10^4)$$

$$p_c = p[m] B[\tau] \times 10^6 \times 3 \times 10^2$$

$$p[\text{ev}] = p[m] B[\tau] \times 3 \times 10^8$$

$$p[\text{GeV} \times 10^9] = p[m] B[\tau] \times 3 \times 10^8$$

$$p_c[\text{GeV}] = p \underset{200m}{[m]} B[\tau] \underset{87}{\times 0.3}$$

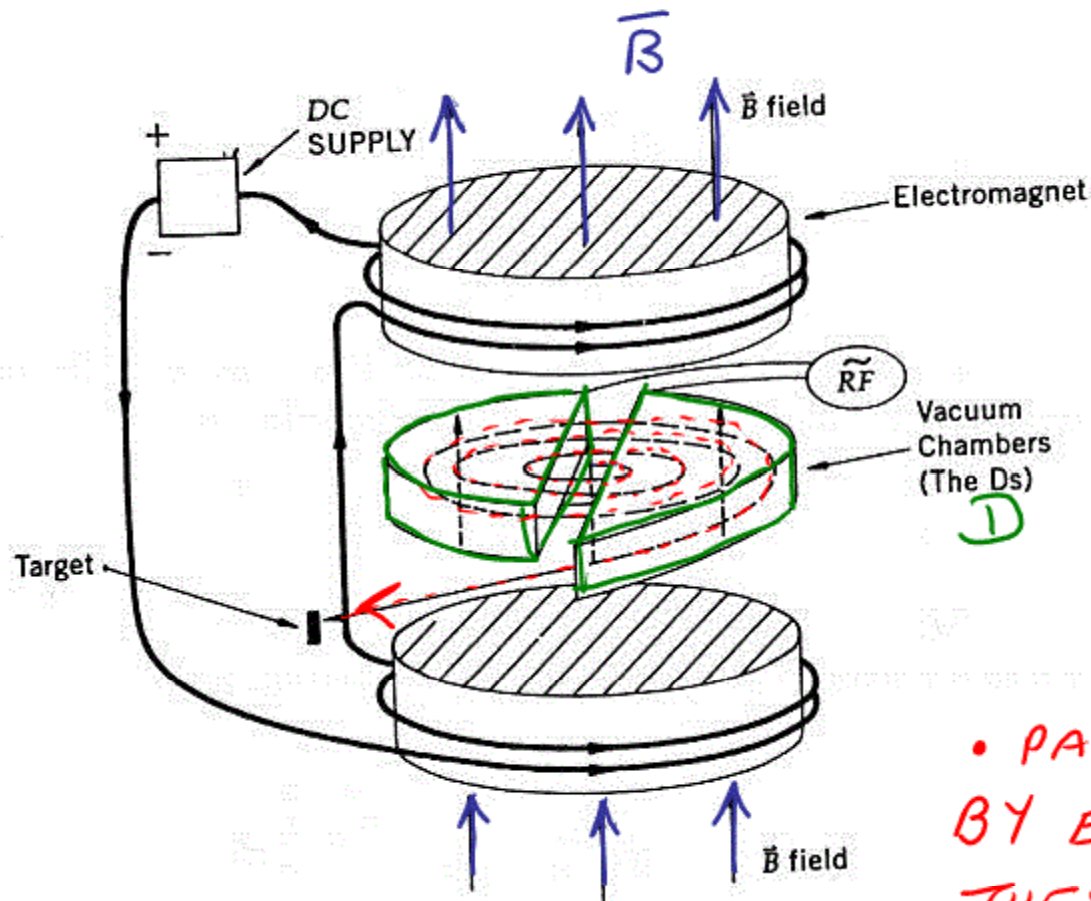
$$p_c = 480 \text{ GeV} = 480 \times 10^9 \text{ eV}$$

$$p_c[\text{ev}] = p(\text{cm}) \times B(\text{g})$$

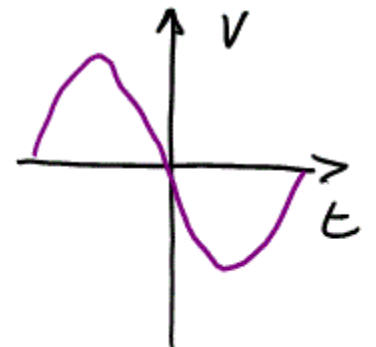
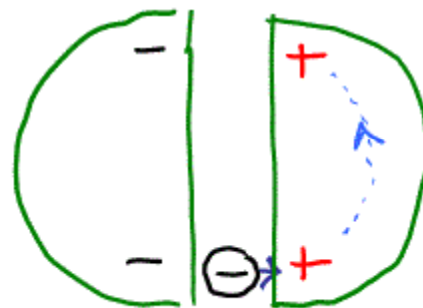
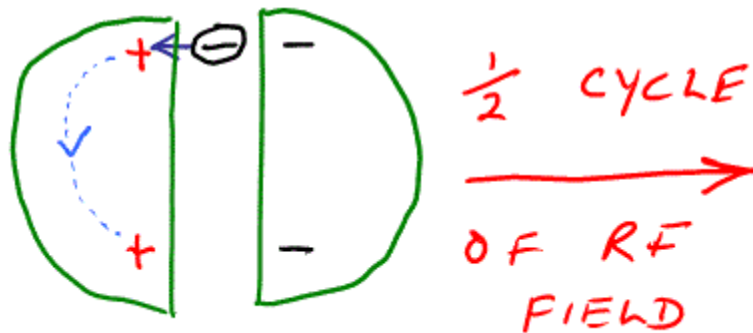
$$= 200 \times 100 \times 8 \times 10000 = 1.6 \times 10^9 \text{ eV}$$

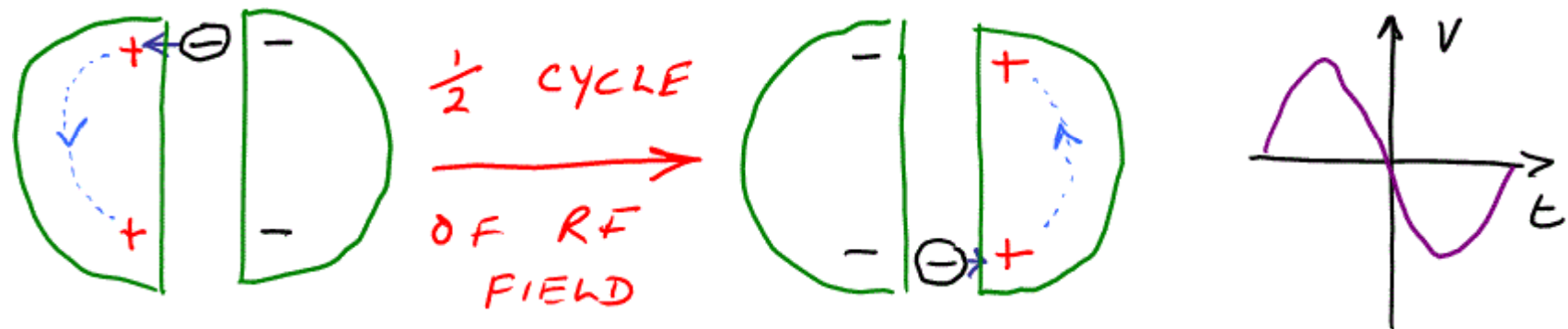
$$= 480 \times 10^9 \text{ eV}$$

THE CYCLOTRON



- TWO HOLLOW D-SHAPED ELECTRODES, EXCITED BY HV RF OSCILLATOR
- NO ELECTRIC FIELD INSIDE "Ds"
- MAGNET FIELD INSIDE Ds
- PARTICLES ACCELERATED BY ELECTRIC FIELD AS THEY CROSS GAP BETWEEN Ds





CENTRIPETAL FORCE = LORENTZ FORCE
FOR AN ORBIT OF RADIUS r

NON-RELATIVISTIC

$$\frac{mv^2}{r} = q \frac{v \cdot B}{c}$$

$$\frac{v}{r} = \frac{qB}{mc} = \text{CONSTANT}$$

$$\text{TIME FOR ORBIT} = 2\pi r / v$$

$$\text{ORBITAL FREQUENCY} = v / 2\pi r$$

$$\text{IF RADIO FREQUENCY } f = \text{ORBITAL FREQUENCY}$$

CONTINUOUS ACCELERATION

CONTINUOUS ACCELERATION

RADIO FREQUENCY = ORBITAL FREQUENCY

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi} \frac{q}{m} \frac{B}{c} = \text{CONSTANT}$$

CYCLOTRON FREQUENCY

→ DOES NOT DEPEND ON RADIUS
OF ORBIT

- PARTICLE STARTS AT SOURCE CLOSE TO CENTRE OF MACHINE
- SPIRALS OUT CONTINUOUSLY GAINING ENERGY FROM RESONANT RF.

THINK AGAIN ABOUT WHY A CYCLOTRON WORKS

$$F_c = F_L$$

$$\frac{mv}{r} = \frac{q \cdot B}{mc} = k$$

$$\frac{v}{r} = \text{CONSTANT} = \text{FREQUENCY}$$

AS r INCREASES, v INCREASES $\rightarrow \frac{v}{r} = \text{CONSTANT}$

FOR A RELATIVISTIC PARTICLE $v = c = \text{CONSTANT}$

$$\therefore \frac{v}{r} = \frac{c}{r} \neq \text{CONSTANT}$$

ELECTRON CYCLOTRON

"MICROTRON"

ELECTRON IS RELATIVISTIC
FOR $E \sim 500 \text{ keV}$



↓ ORBITS INCREASE
IN RADIUS
DURING
ACCELERATION

ANOTHER RELATIVISTIC EFFECT

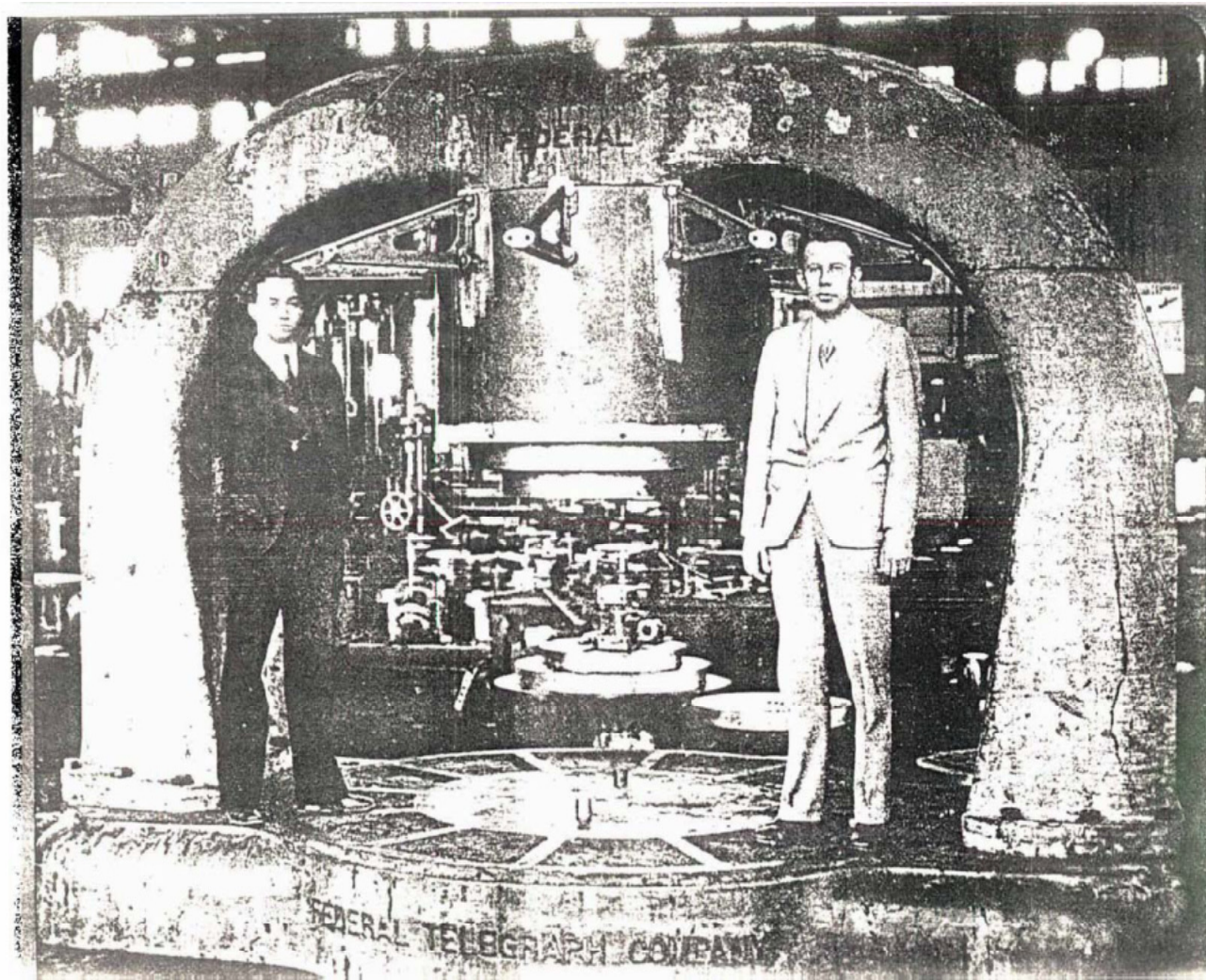
$$f = \frac{1}{2\pi} \frac{q}{m} \frac{B}{c} = \text{RF FREQUENCY} = \text{ORBITAL FREQUENCY}$$

- AS PARTICLES ACCELERATE, TOTAL RELATIVISTIC ENERGY BECOMES $\approx$ MASS ENERGY
- IN THIS SITUATION $m \rightarrow m \gamma$ LORENTZ BOOST

$$f = \frac{1}{2\pi} \frac{q}{\gamma m} \frac{B}{c}$$

DURING ACCELERATION γ INCREASES & RESONANCE CONDITION FAILS

- INCREASE B SYNCHROTRON
- DECREASE RF FREQUENCY SYNCHROCYCLOTRON



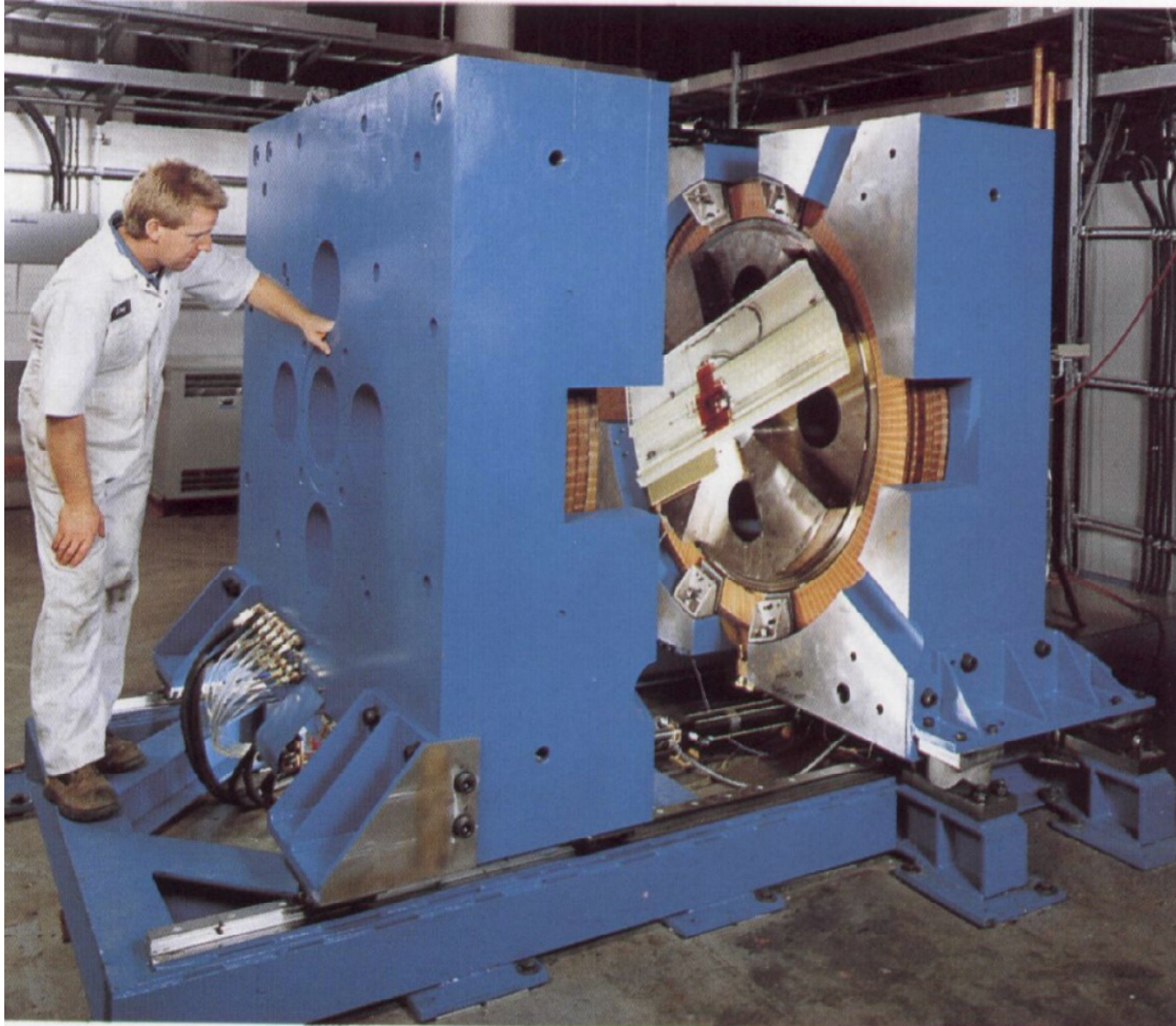
Livingston and Lawrence with the magnet of the “27-inch” (later “37-inch”) cyclotron on which most of Berkeley’s 1930s nuclear physics was performed.
Lab wear was different then!

THE 184-INCH SYNCHROCYCLOTRON



The Berkeley 184" was begun in 1939 as a classical cyclotron, to be operated with $V_{rf} = 1$ MV, but WWII interrupted rf installation and it was used to test mass spectrographic separation of uranium isotopes. FM rf was installed in 1946, yielding 190 MeV d^+ (700 MeV p in 1959).

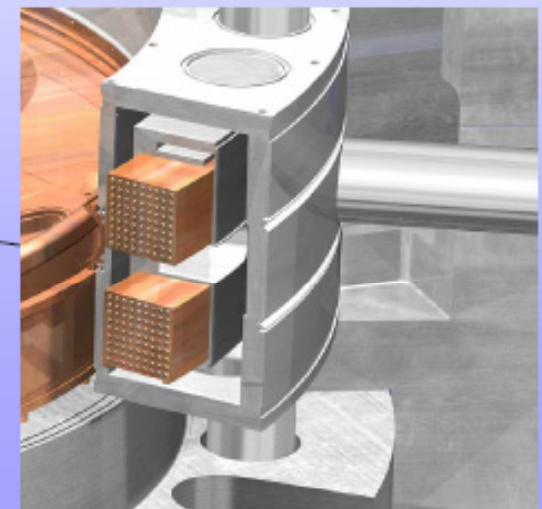
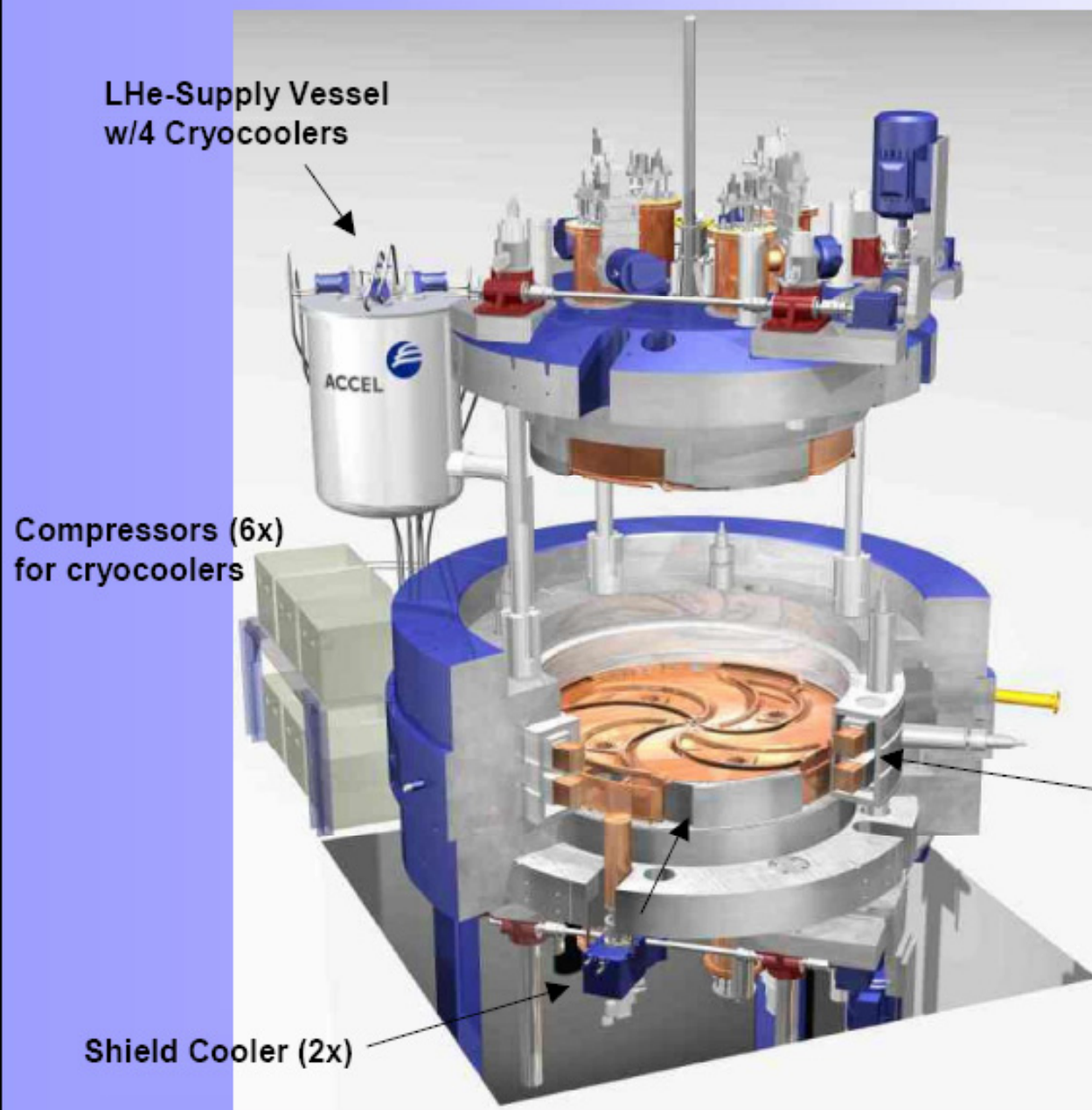
PET Medical Cyclotron



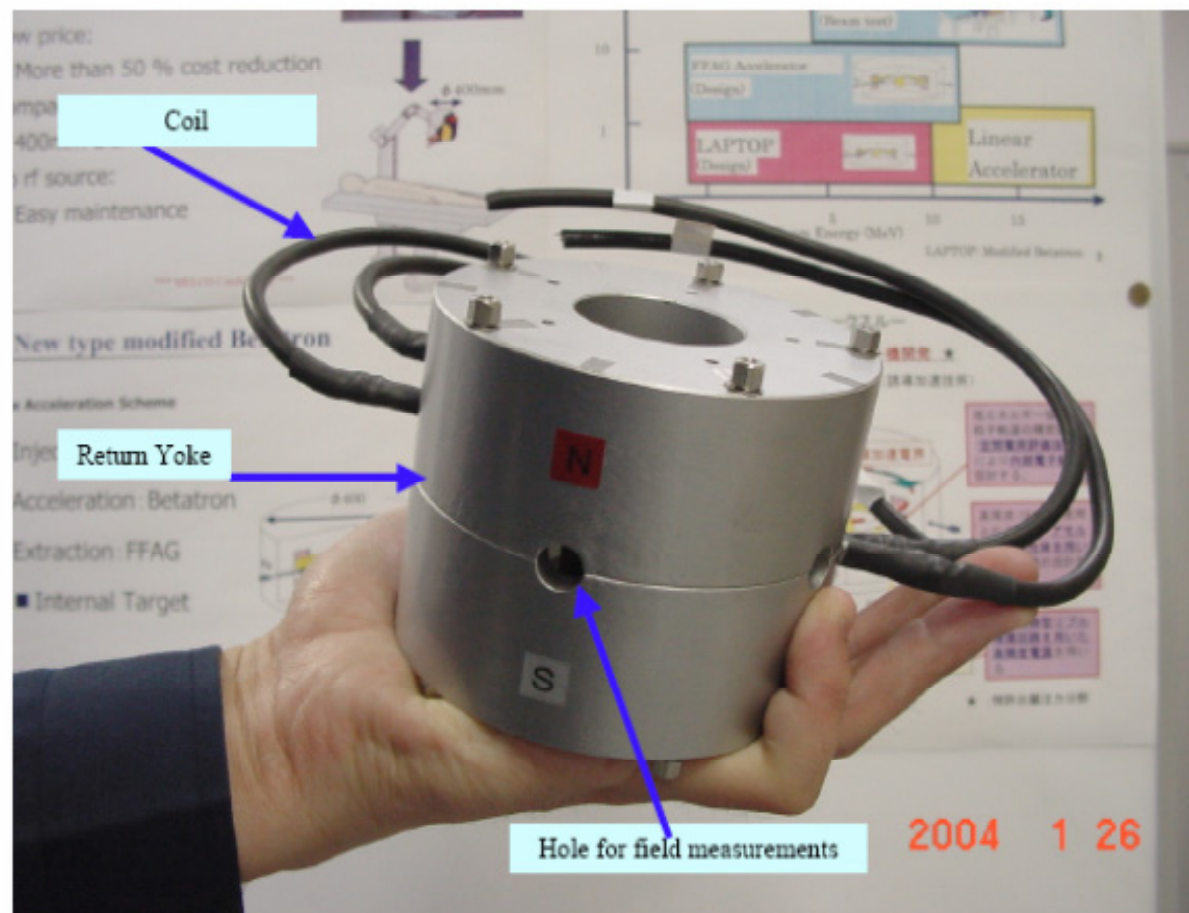
TRIUMF (Vancouver) 500 MeV Cyclotron



250 MeV Superconducting Proton Cyclotron



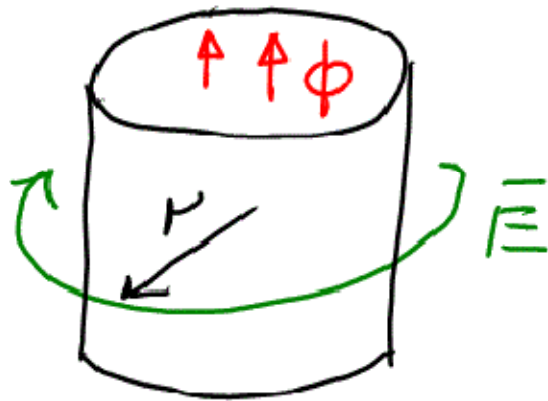
Superconducting Coil



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You can have your own cyclotron – from Mitsubishi

BETA TRON → WIDEROE
FIXED RADIUS ORBIT



• TIME VARYING MAGNETIC FIELD
PRODUCES ELECTRIC FIELD TO
ACCELERATE PARTICLES

↳ TRANSFORMER

↳ AT SAME TIME PROVIDES
MAGNETIC GUIDE FIELD

$$\vec{\nabla} \times \vec{E} = - \frac{d\vec{B}}{dt} \quad \text{← MAXWELL}$$

WANT TO INTEGRATE

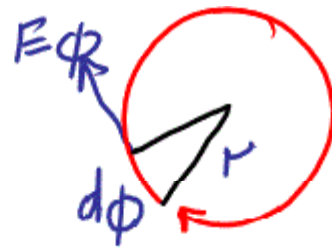
STOKES

$$\int_{\text{LINE}} \vec{v} d\vec{s} = \int_{\text{AREA}} \vec{\nabla} \times \vec{v} d\vec{A}$$

$$\int \vec{E} d\vec{s} = - \int \frac{d\vec{B}}{dt} \cdot d\vec{A} = - \frac{d\Phi}{dt}$$

FLUX
ENCLOSED
BY ORBIT

FROM RIGHT HAND RULE



$\phi \rightarrow$ ANGLE

$$\textcircled{1} \int \vec{E} d\vec{s} = - \int E_{\phi} r d\phi = -2\pi r E_{\phi} \rightarrow E_{\phi} = \frac{1}{2\pi r} \frac{d\Phi}{dt}$$

SINCE PARTICLE IS MOVING
IN A CIRCLE

ACCELERATING
ELECTRIC FIELD

$$\frac{\gamma m v^2}{r} - e(\vec{E} + \vec{v} \times \vec{B})$$

RADIAL = 0

$$\frac{\gamma m v^2}{r} = e v B_{\perp} \rightarrow \frac{p v}{r} = e v B$$

$$\frac{1}{r} = \frac{e B}{p} \quad \textcircled{2}$$

PRACTICAL BETATRON HAS CONSTANT
RADIUS ORBIT

$r = \text{CONSTANT}$ HOW?

USE $\frac{1}{r} = \frac{eB}{p}$ & CONDITION FOR r CONSTANT

$$\frac{d}{dt} \left(\frac{1}{r} \right) = \frac{d}{dt} \left(\frac{eB}{p} \right) = 0 \rightarrow -\frac{1}{r^2} \frac{dr}{dt} = \left(\frac{\dot{B}}{p} - \frac{B\dot{p}}{p^2} \right) = 0$$

ACCELERATION ELECTRIC FIELD GIVES $\dot{\phi}$

$$\dot{\phi} = e E_{\phi} = \frac{e}{2\pi r} \dot{\Phi} \rightarrow \frac{\dot{B}}{p} = \frac{B}{p^2} \frac{e}{2\pi r} \dot{\Phi}$$

FROM TOP OF PAGE

$$B = P/e r$$

$$\frac{\dot{B}}{p} = \frac{P}{e r p^2} \cdot \frac{e}{2\pi r} \cdot \dot{\Phi} \rightarrow \dot{\Phi} = 2\pi r^2 \dot{B}$$

REWRITE $\frac{d\Phi}{dt} = 2\pi r^2 \frac{dB(r)}{dt}$ ORBIT RADIUS

COMPLETE FLUX ENCLOSED BY ORBIT $\Phi = \pi r^2 B_A$
 AVERAGE FIELD ENCLOSED BY ORBIT

SO $\frac{d\Phi}{dt} = \pi r^2 \frac{dB_A}{dt}$

AT ORBIT $\frac{dB(r)}{dt} = \frac{1}{2} \frac{dB_A}{dt}$ AVERAGE ENCLOSED
 $\int dt \rightarrow B(r) = \frac{1}{2} B_A + B_0$

FOR STABLE ORBIT

WIDEROE $\frac{1}{2}$ CONDITION

FIELD AT ORBIT $= \frac{1}{2} \left(\begin{array}{l} \text{AVERAGE FLUX} \\ \text{DENSITY THRU} \\ \text{ORBIT} \end{array} \right)$

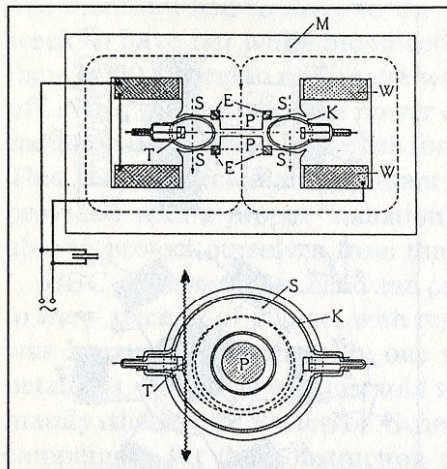


Fig. 10.3: Diagram of BBC's double-beam betatron.
 M = Magnet yoke
 P = Central magnet poles
 S = Steering poles
 W = Exciting coils
 E = Expansion coils
 K = Ring tube
 T = Anticathode (target)
 [Wi62].

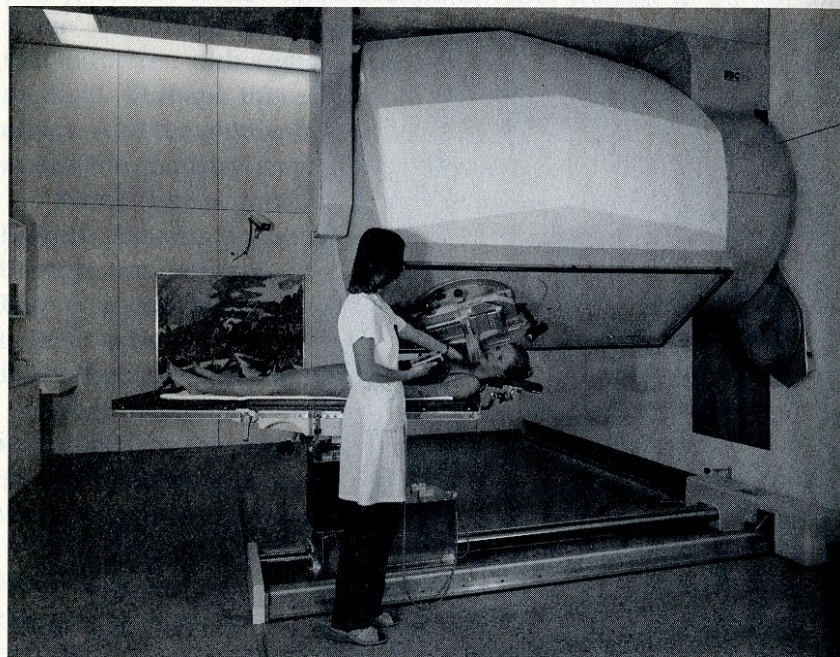


Fig. 10.4: Betatron radiation therapy, Inselspital Berne (phot.: BBC).

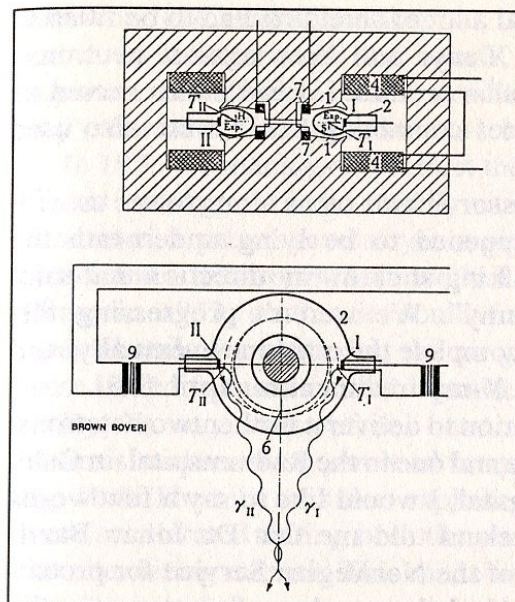
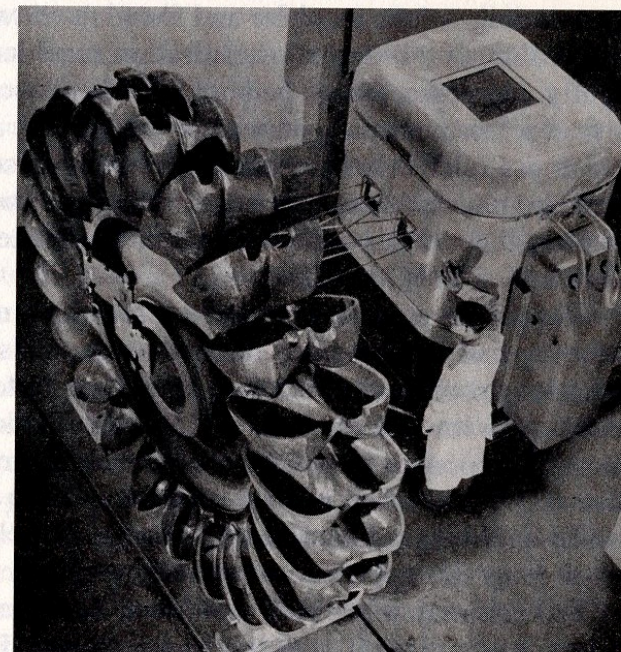


Fig. 10.5: BBC stereo two-beam betatron for materials testing.
 1 = Magnet pole
 2 = Ring tube
 4 = Coil
 6 = Orbit
 7 = Expansion coil
 9 = Impulse transformer
 I+II = Electron sources
 $T_I + T_{II}$ = Targets
 $Y_I + Y_{II}$ = X-rays
 [Se58].

Fig. 10.6: A betatron being used to test a Pelton-wheel at Georg Fischer AG, Schaffhausen (photograph: BBC).



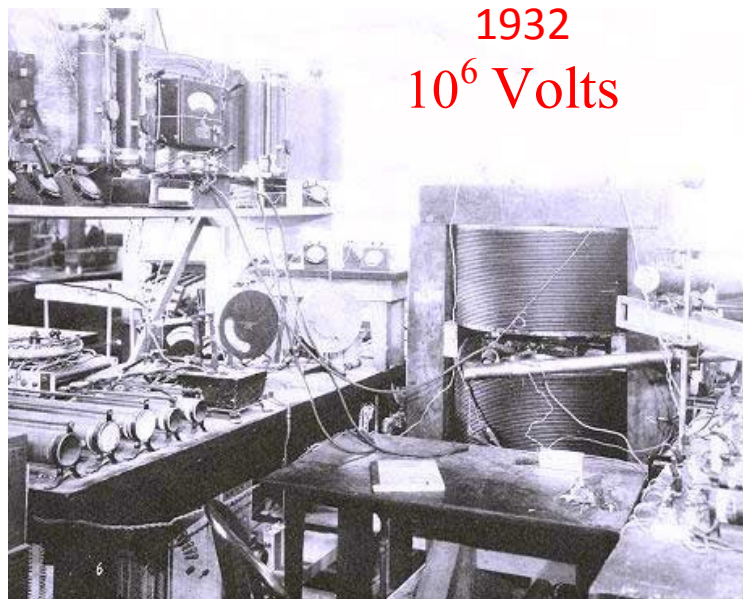


This machine is just
a model for a bigger
one, of course

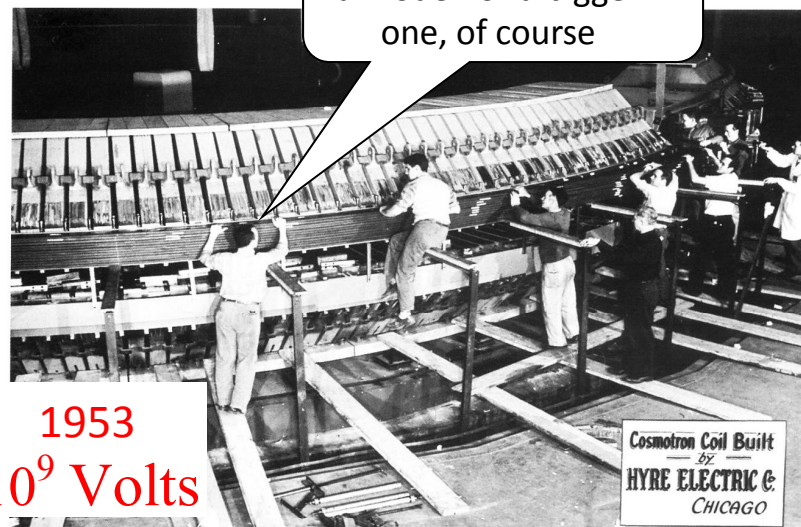
1931
 10^4 Volts



Scanned at the American
Institute of Physics

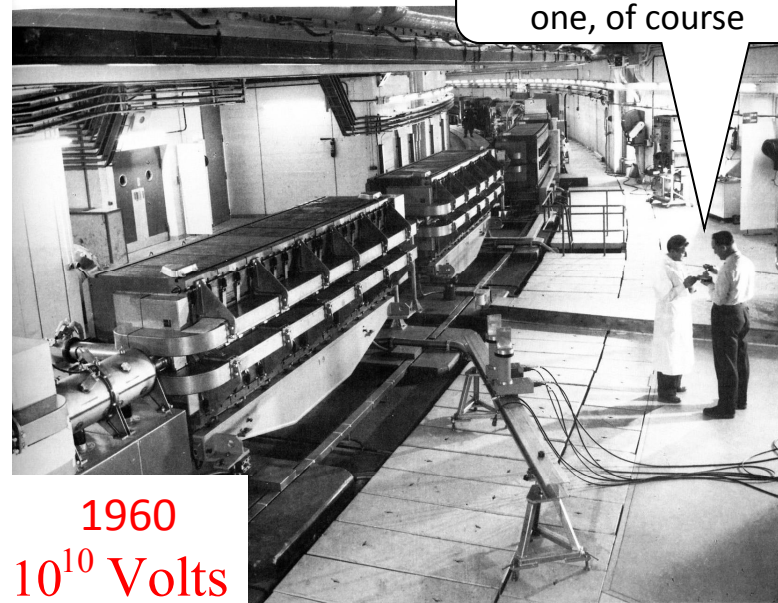


1932
 10^6 Volts



This machine is just
a model for a bigger
one, of course

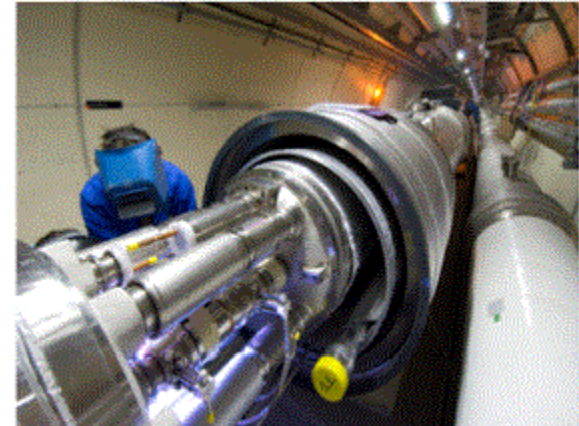
1953
 10^9 Volts



This machine is just
a model for a bigger
one, of course

1960
 10^{10} Volts

BUILDING THE LHC



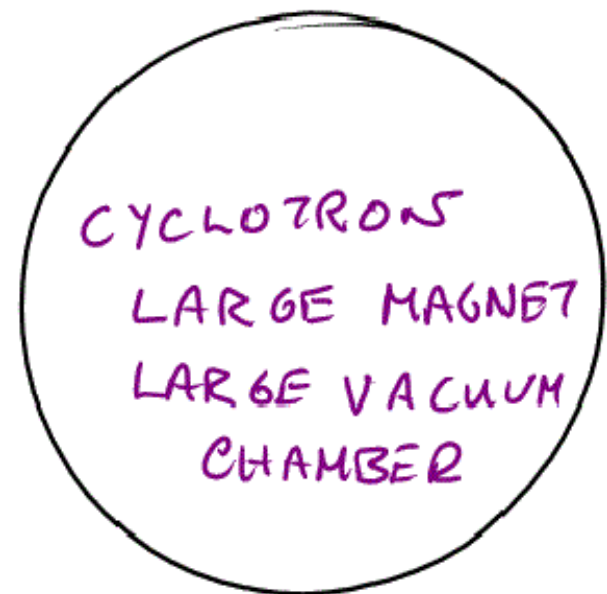
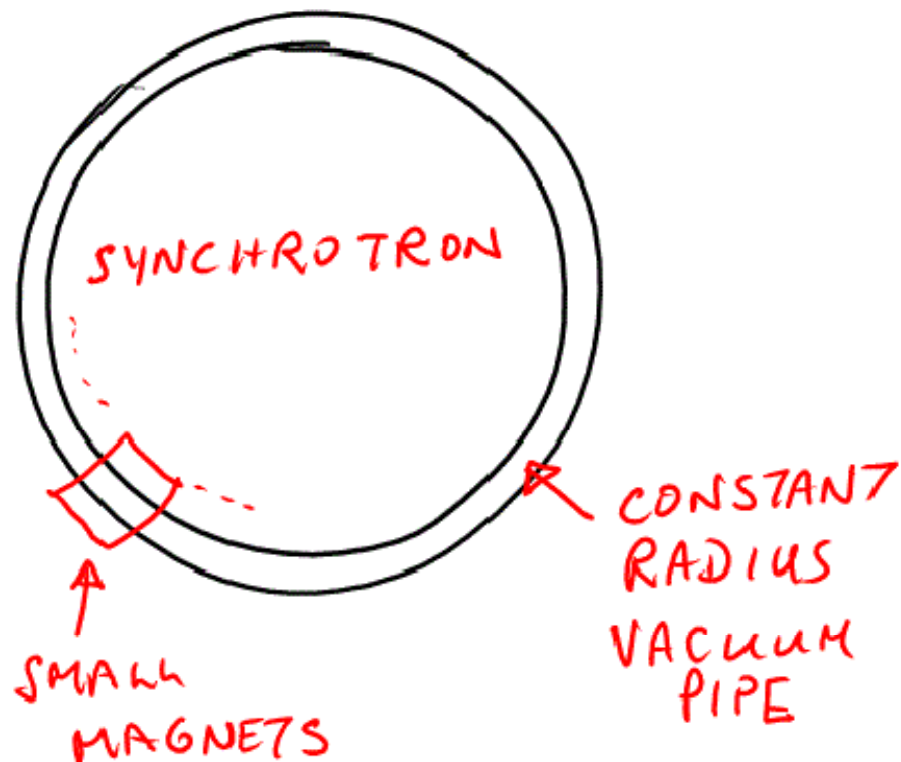


Alors, c'est fini!
Et maintenant?

FOR HIGH ENERGY, ONLY MACHINES WITH
CONSTANT ORBIT RADIUS ARE PRACTICAL

$$\frac{1}{r} = \frac{eB}{p} \rightarrow p = eB \cdot r \rightarrow r \propto p$$

$p \rightarrow \text{HIGH}$ $r \rightarrow \text{HIGH}$



Synchrotron Ring Schematic

