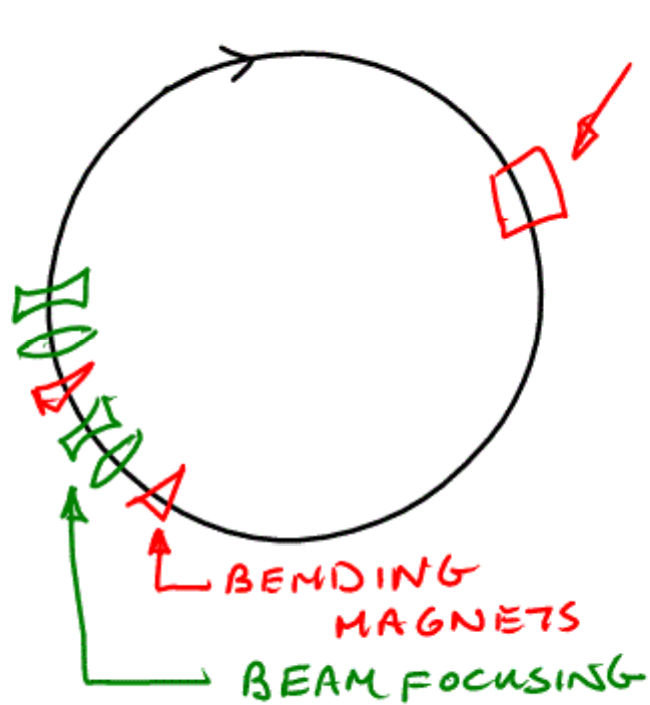


RADIO FREQUENCY CAVITIES

- IN THE ELECTROSTATIC ACCELERATOR THE ACCELERATING VOLTAGE IS CONSTANT
- IN THE CYCLOTRON WE HAVE A RADIO FREQUENCY ACCELERATING VOLTAGE.
- WHAT ABOUT SYNCHROTRON?
- USE A RADIO FREQUENCY CAVITY WITH A RESONANT FREQUENCY EQUAL TO ORBITAL FREQUENCY (OR SOME MULTIPLE)
- IF Q -VALUE OF CAVITY IS LARGE, AND ENERGY TRANSFERRED TO BEAM SMALL



EFFICIENT USE OF
RADIO FREQUENCY POWER



ACCELERATING STRUCTURE

- TYPICALLY ENERGY TRANSFERRED FROM ACCELERATING FIELD TO BEAM IS SMALL
- SAY A PARTICLES RECEIVES 1 MeV ON TRANSIT THROUGH CAVITY - 10^{10} PARTICLES / BUNCH
— ENERGY EXTRACTED
~ mJoule

• ENERGY STORED IN OSCILLATING FIELDS IN CAVITY MANY ORDERS OF MAGNITUDE GREATER

→ USE HIGH Q

RESONANT CAVITIES

— LET'S HAVE A LOOK AT RF CAVITIES

SIMPLE RF CAVITY SYSTEM

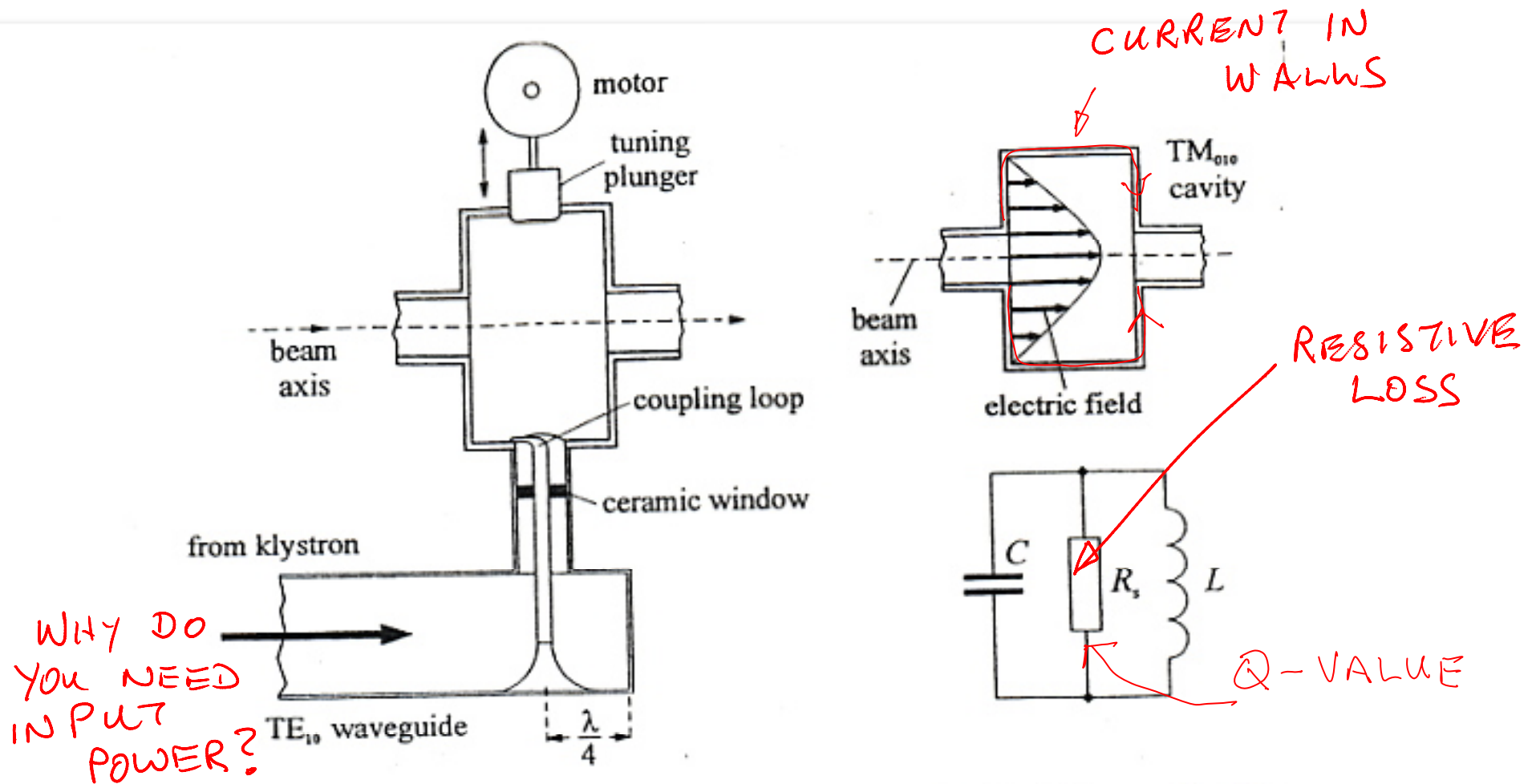


Fig. 5.4 Design of a single-cell accelerating structure using the TM_{010} mode. The exact resonant frequency is adjusted using a tuning plunger. The resonator is excited by an inductive coupling loop.

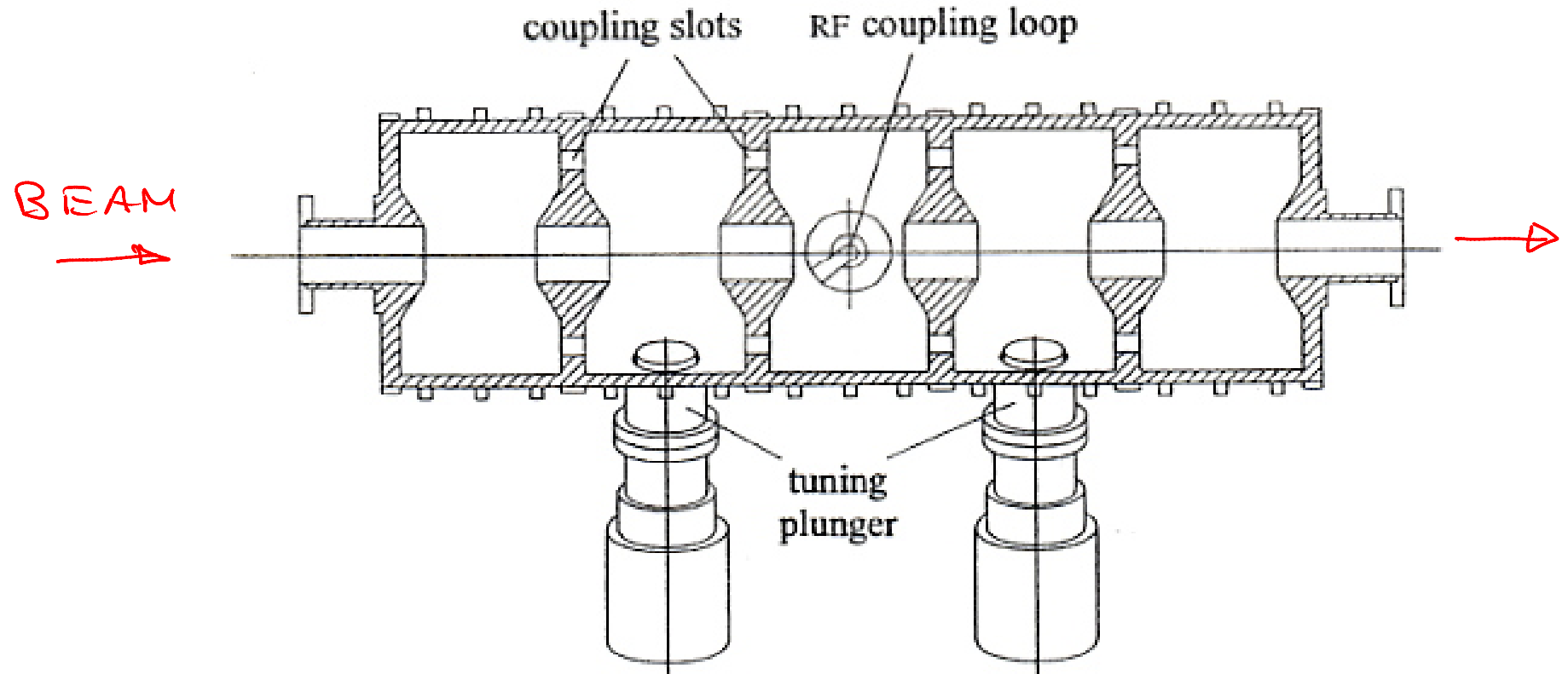


Fig. 5.5 Layout of a five-cell accelerating structure. The power feed is coupled to the middle cell and two tuning plungers are sufficient for the entire structure.

$$f_{\text{RF}} = 500 \text{ MHz}$$

$$R_s = 3.0 \times 10^6 \Omega$$

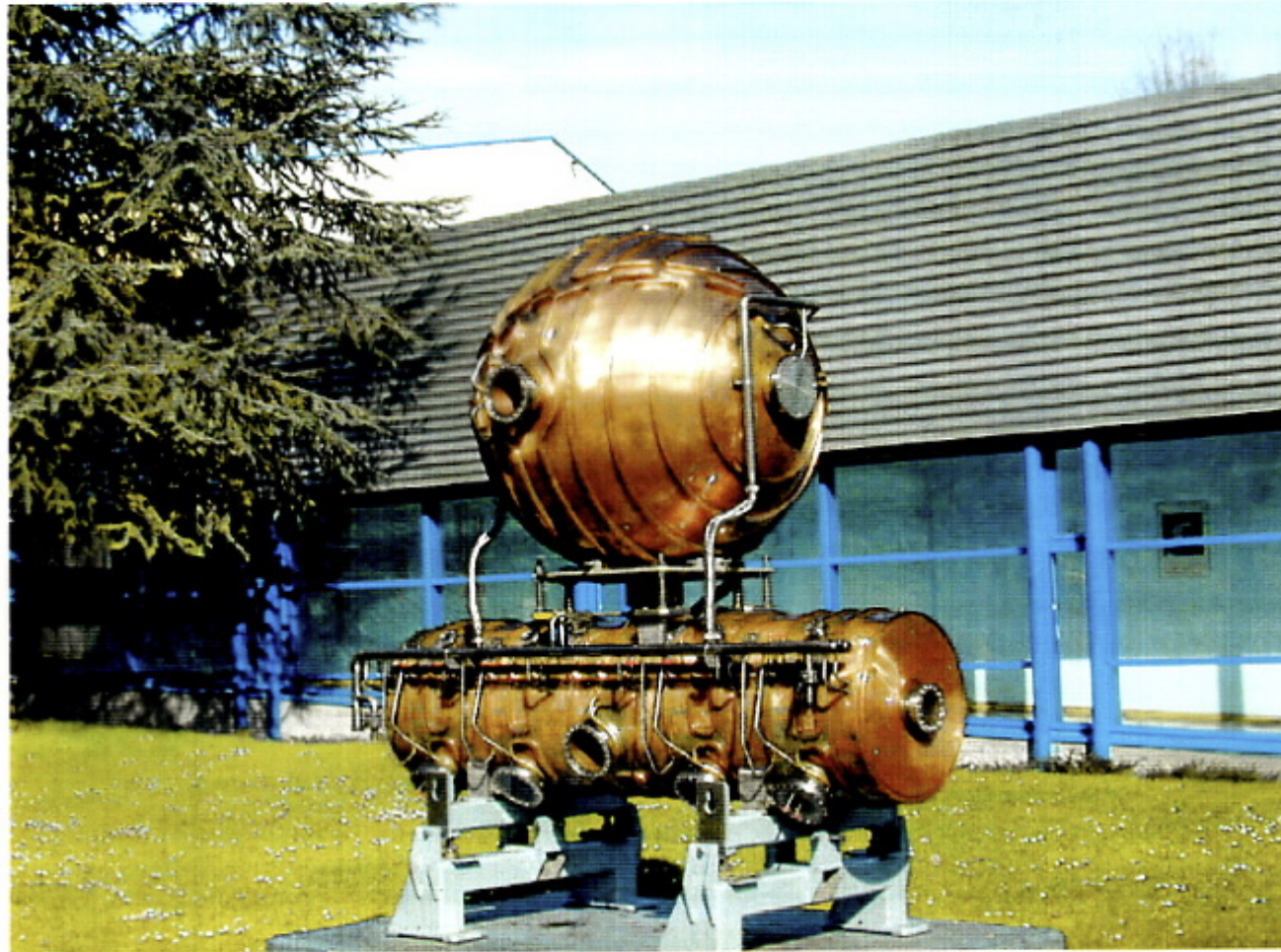
$$Z = 80 \Omega$$

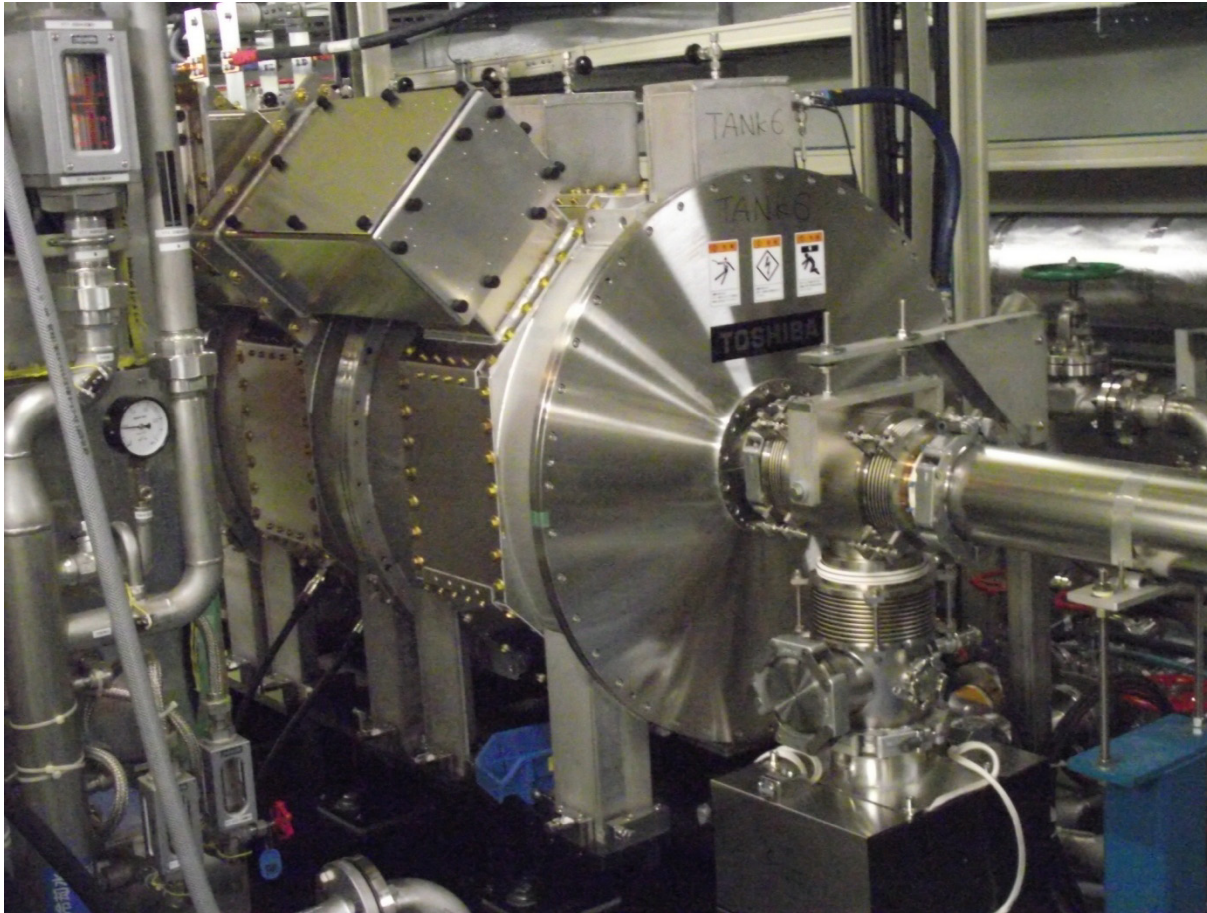
$$Q = 38\,000$$

$$P_{\text{RF}} = 50 \text{ kW}$$

$$U_{\text{cav}} = 548 \text{ kV}$$

Copper RF Cavity

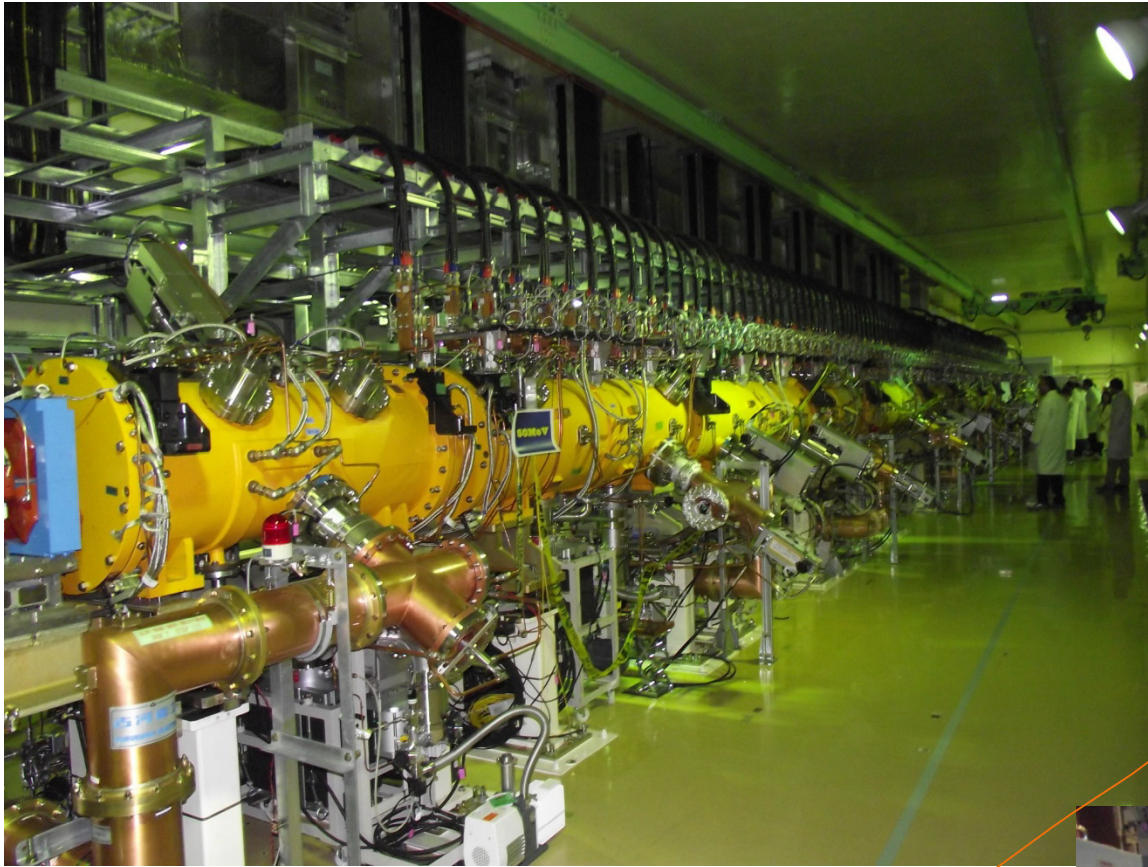




RF CAVITY IN JPARC

KLYSTRONS



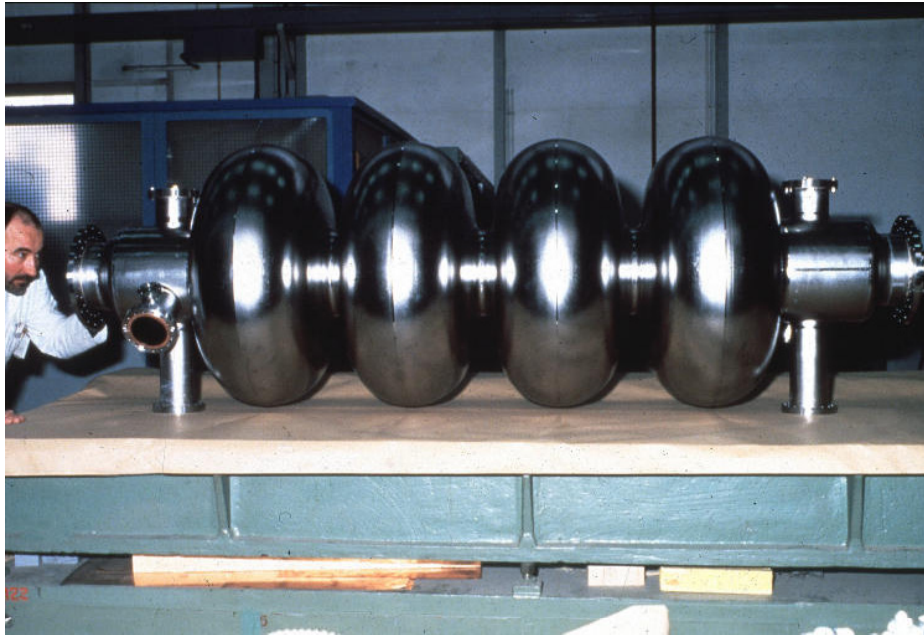


JPARC LINAC

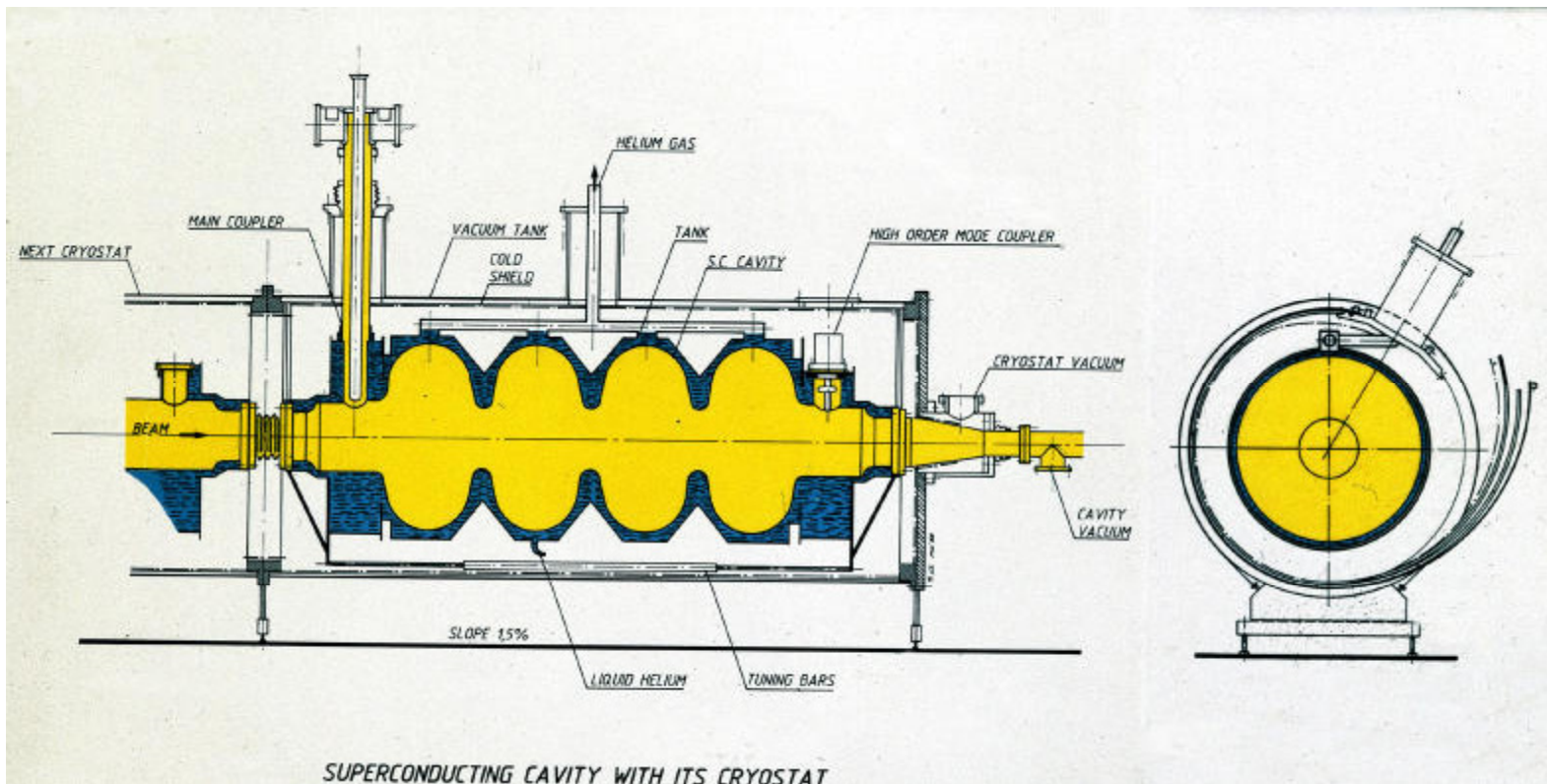
KLYSTRON

KLYSTRON GALLERY →

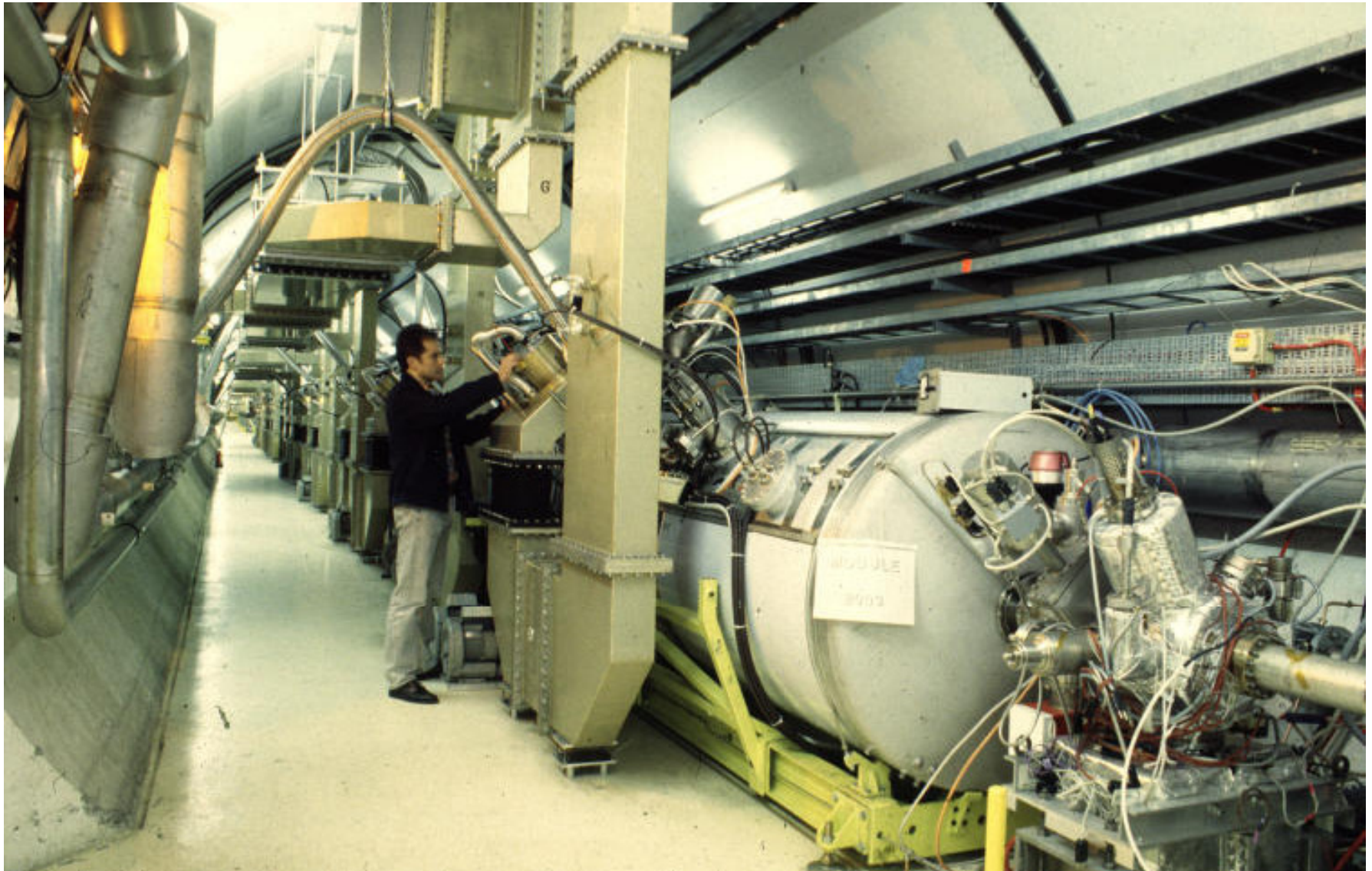




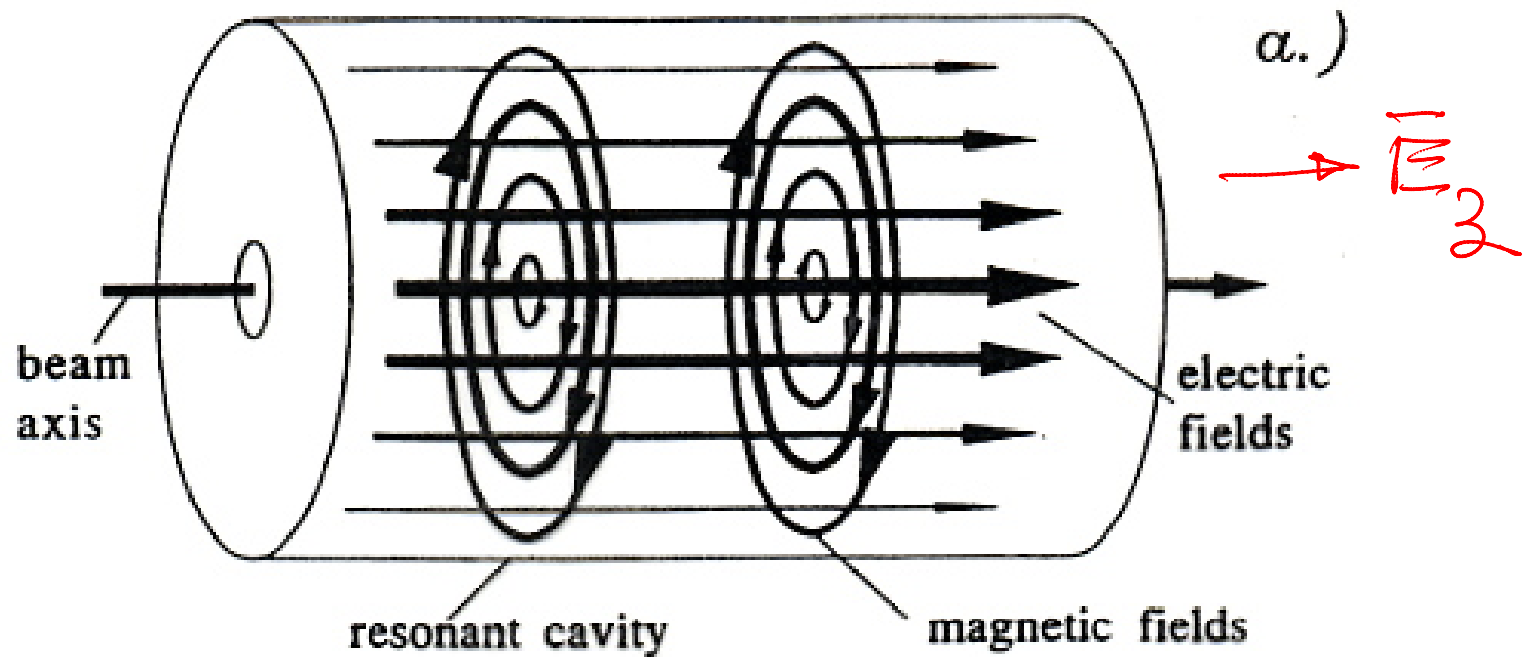
LEP SUPER CONDUCTING CAVITY



LEP SUPER CONDUCTING CAVITY

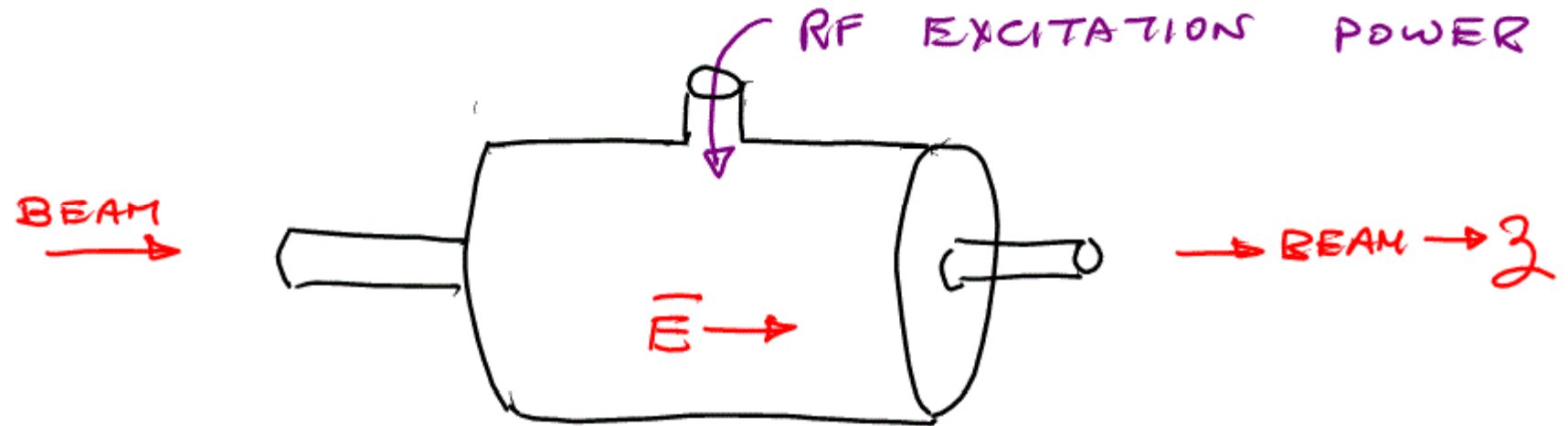


SOLVE LAPLACE EQUATION
WITH CYLINDRICAL BOUNDARY CONDITIONS

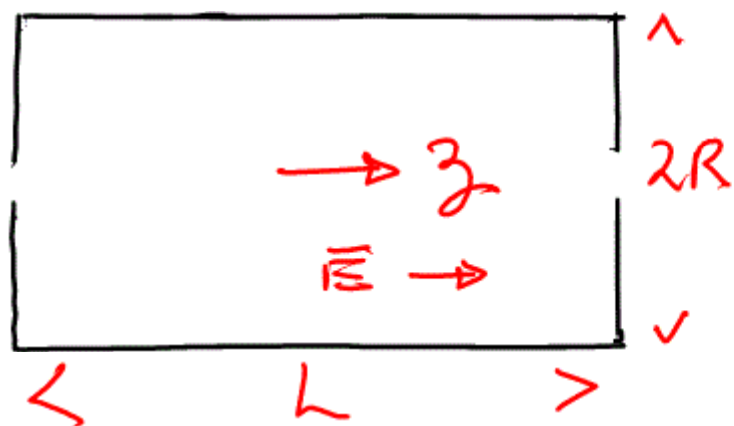


FREE ELECTROMAGNETIC
WAVES HAVE $E_z = 0$

RADIO FREQUENCY ACCELERATING CAVITIES



- IDEALIZATION OF A CAVITY — PILL BOX
- BEAM ENTERS/EXITS THROUGH HOLES FOR BEAM PIPE.
- ANTENNA COUPLES RF POWER FROM KLYSTRON INTO CAVITY.



• WE WANT TO INVESTIGATE
SPATIAL VARIATION OF
ACCELERATING $\vec{E}$

• NEED E_z FREE EM WAVE
DOES NOT
HAVE THIS

• BUT SOLVING LAPLACE EQUATION WITH
BOUNDARY CONDITIONS

• THERE ARE VARIOUS MODES OF ELECTRIC
AND MAGNETIC OSCILLATIONS

$TM_{010} \rightarrow$ ONLY E_z AND B_θ

MAXWELL

FOR $j \equiv 0$ $\vec{\nabla} \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} \rightarrow \frac{1}{r} \frac{\partial (r B_\theta)}{\partial r} = \frac{1}{c^2} \frac{\partial E_z}{\partial t}$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow \frac{\partial E_z}{\partial r} = -\frac{\partial B_\theta}{\partial t}$

I'M USING CYLINDRICAL COORDINATES!

$$(\vec{\nabla} \times \vec{A})_r = \left(\frac{1}{r} \frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z} \right) \rightarrow = \frac{\partial B_\theta}{\partial t}$$

$$(\vec{\nabla} \times \vec{A})_\theta = \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) = \frac{1}{c^2} \frac{\partial E_z}{\partial t}$$

$$(\vec{\nabla} \times \vec{A})_z = \frac{1}{r} \left(\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right)$$

ONLY $B_\theta \neq 0$

AND $\frac{\partial B_\theta}{\partial z} = 0$

ONLY $E_z = 0$

$$\frac{\partial E_z}{\partial \theta} = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) = \frac{1}{c^2} \frac{\partial E_z}{\partial t}$$

$$\frac{1}{r} \frac{\partial r}{\partial r} B_\theta + \frac{r}{r} \frac{\partial B_\theta}{\partial r} = \frac{1}{c^2} \frac{\partial E_z}{\partial t}$$

DIFFERENTIATE W.R.T TIME

$$\frac{1}{r} \frac{\partial B_\theta}{\partial t} + \frac{\partial^2 B_\theta}{\partial t \partial r} = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}$$

$\frac{\partial E_z}{\partial r}$ 

$\frac{\partial^2 E_z}{\partial r^2}$ 

 MAXWELL

$$\frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{\partial^2 E_z}{\partial r^2} = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2} \quad \text{--- ①}$$

ONLY E_z APPEARS — ONLY E_z ACCELERATES PARTICLES

TIME VARIATION OF E_z IN IDEAL CAVITY
WILL JUST BE SINUSOIDAL.

$$E_z(r, t) = E(r) e^{i\omega t}$$

$$\frac{\partial}{\partial t} E_z = i\omega E(r) e^{i\omega t} = i\omega E_z$$

$$\frac{\partial^2}{\partial t^2} E_z = -\omega^2 E(r) e^{i\omega t} = -\omega^2 E_z$$

WRITE $\frac{\partial^2 E_z}{\partial r^2} = \frac{\partial^2 E(r)}{\partial r^2} e^{i\omega t}$

$$\frac{\partial E_z}{\partial r} = \frac{\partial E(r)}{\partial r} e^{i\omega t}$$

AND $\frac{\partial^2 E_z}{\partial t^2} = -\omega^2 E(r) e^{i\omega t}$

THEN FOR $\frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{\partial^2 E_z}{\partial r^2} = \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2}$

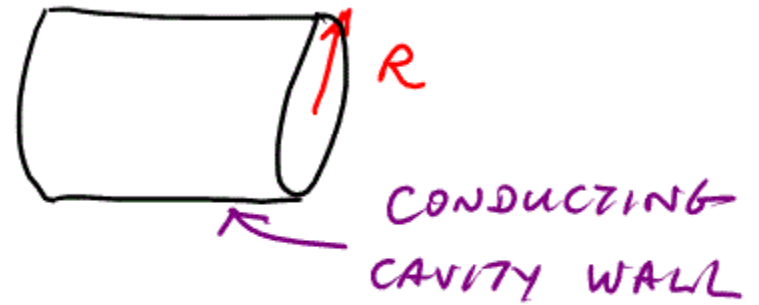
HAVE: $\frac{\partial^2 E(r)}{\partial r^2} + \frac{1}{r} \frac{\partial E(r)}{\partial r} + \frac{\omega^2}{c^2} E(r) = 0$

THIS IS BESSEL'S EQUATION OF ORDER ZERO
IT DESCRIBES THE RADIAL VARIATION OF
 E_z WHICH HAS TIME VARIATION $e^{i\omega t}$
IT'S SOLUTION IS:

$$E(r) = E_0 J_0\left(\frac{\omega}{c} \cdot r\right)$$

$$E(r) = E_0 J_0\left(\frac{\omega}{c} \cdot r\right)$$

AT $r=R$ - CAVITY WALL
ELECTRIC FIELD VANISHES



$$\frac{\omega}{c} \cdot R \rightarrow 1^{st} \text{ ZERO OF } J_0 \rightarrow \frac{\omega}{c} \cdot R = 2.4048$$

$$\frac{2\pi f}{c} \cdot R = 2.405; \quad \text{FOR } R \sim 30 \text{ cm}$$

$$f = \frac{c}{2\pi} \cdot \frac{2.4}{30} \sim 4 \times 10^8$$

RESONANT FREQUENCY

TYPICAL CAVITY HAS RESONANT FREQUENCY $\sim 500 \text{ MHz}$

ILC 1.36 GHz

CLIC 306 GHz

Q VALUE $\Rightarrow$ LOSSES IN CAVITY MATERIAL?

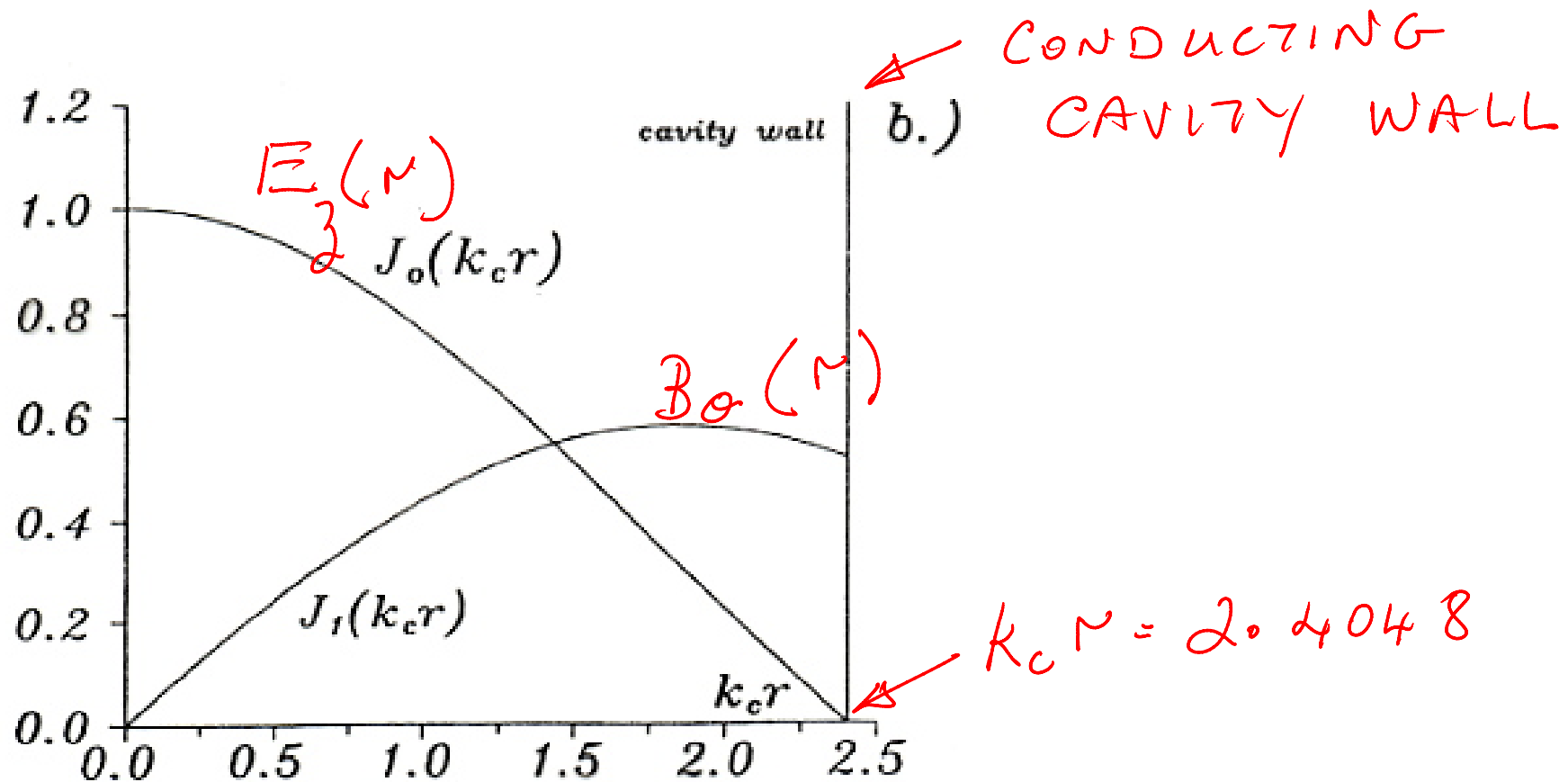
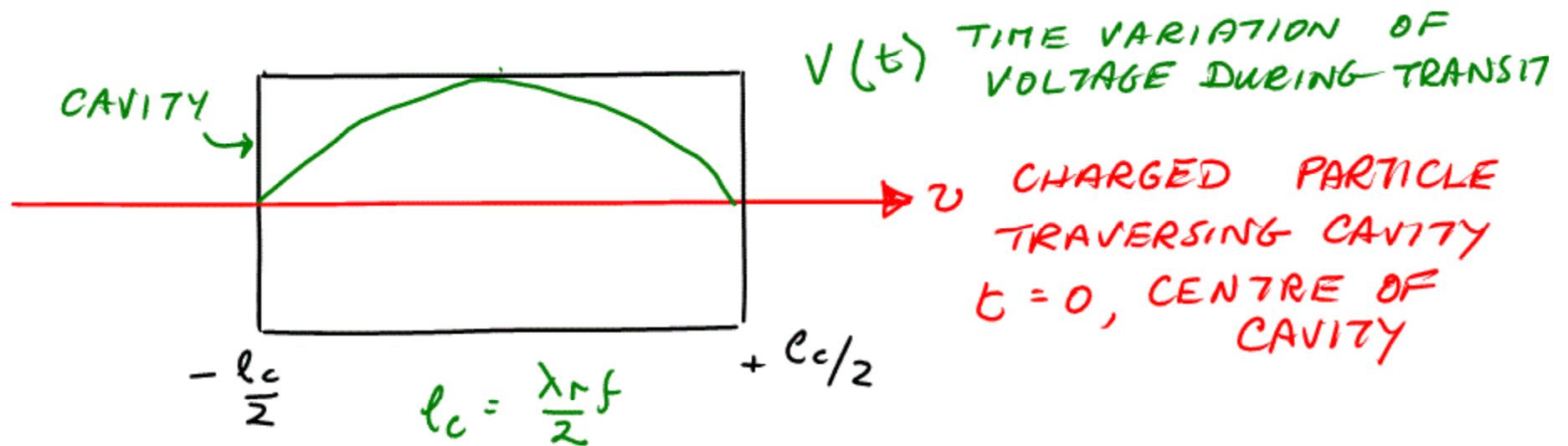


Fig. 2.7. Electromagnetic field pattern for a TM_{010} mode in a circular waveguide a.) three dimensional field configuration, b.) radial dependence of fields

BEAM ENERGY GAIN IN OSCILLATING $\vec{E}$ FIELD



$$e V(t) = e V_0 \cos \omega t = e V_0 \cos \left(\omega \frac{s}{v} \right)$$

$$\Delta E_{KIN} = F \cdot d$$

$$= e E d$$

$$\Delta E_{KIN} = \int_{-l_c/2}^{+l_c/2} e E_0 \cos \left(\omega \frac{s}{v} \right) ds$$

$$\Delta E_{KIN} = \int e \frac{V_0}{l_c} \cos \left(\omega \cdot \frac{s}{v} ds \right) = \frac{\sin u}{u} e V_0 ; u = \frac{\omega l_c}{2v}$$

TRANSIT TIME FACTOR PEAK VOLTAGE

TRANSIT TIME FACTOR $\sin u / u = T$

{ ENERGY GAINED BY PARTICLE PASSING
THROUGH CENTRE OF CAVITY AT PEAK FIELD }

{ ENERGY GAIN FROM CONSTANT FIELD }

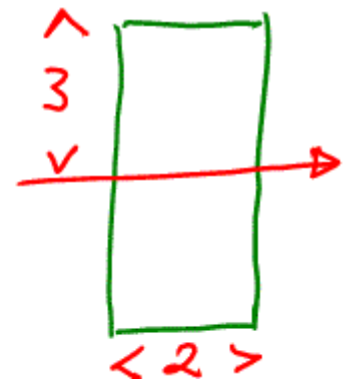
WANT CLOSE TO 1

FOR $u = 0.8$ $\frac{\sin u}{u} = 0.9$

$u = \frac{\omega l}{2v} = \frac{\omega l}{2c}$; $\frac{\omega l}{c} = 1.6$ ← CAVITY LENGTH

BUT $\frac{\omega R}{c} = 2.405$ ← CAVITY RADIUS

$\frac{\omega/c \cdot R}{\omega/c \cdot l} = \frac{2.405}{1.60} \approx \frac{3}{2}$ ← TYPICAL CAVITY ASPECT RATIO



CAVITY DIMENSIONS

• SAW THAT A CAVITY OF RADIUS $\sim 30\text{cm}$
RESONATES AT $\sim 500\text{ MHz}$

• FOR $\frac{\omega L}{2C} = 0.8 \rightarrow L = \frac{1.6 \times C}{2\pi \cdot 500 \times 10^6} = 15\text{ cm}$

• WHY IS A PRACTICAL $T \sim 0.8$?

FOR $T = 0.998 (\approx 1)$ $T = \frac{\sin 0.1}{0.1}$

FROM ABOVE

$$L = \frac{0.2 \cdot C}{2\pi \cdot 500 \times 10^6} = 2\text{ cm}$$



- ELECTRICAL BREAK DOWN
- SMALL ΔE ON TRANSIT
- HARD TO MANUFACTURE

CAVITY Q-VALUE

$$Q = \frac{\text{STORED ENERGY}}{(\text{ENERGY LOST IN ONE})}$$

(RADIAN OF OSCILLATION)

WANT THIS TO
BE LARGE !

ENERGY CONSTANTLY OSCILLATING $\vec{E} \leftrightarrow \vec{B}$
ONLY NEED TO CALC ENERGY IN E_{MAX}

$$U = \epsilon_0 \frac{E^2}{2} \text{ VOLUME } U = \frac{\epsilon_0}{2} \int E^2 dV \leftarrow \text{VOLUME OF CAVITY}$$

$E(r) = E_0 J_0\left(\frac{\omega}{c} \cdot r\right)$

$$= \frac{\epsilon_0}{2} \int E_0^2 J_0^2\left(\frac{\omega}{c} \cdot r\right) dV \rightarrow E_0^2 \frac{\epsilon_0}{2} \int J_0^2\left(\frac{\omega}{c} \cdot r\right) \int_0^{2\pi} d\phi \int_0^L dz$$

$$= \frac{\epsilon_0}{2} E_0^2 \frac{2\pi}{\phi} \frac{L}{z} \int_0^R J_0^2\left(\frac{\omega}{c} \cdot r\right) r dr$$

$$\text{put } \rho = \frac{r}{R} \quad \left. \begin{array}{l} \rho = 0 \rightarrow R \\ \rho = 0 \rightarrow 1 \end{array} \right\}$$

$$= \epsilon_0 E_0^2 \pi L \int_0^1 J_0^2 \left(\frac{\omega}{c} \cdot \rho R \right) \rho R \cdot R d\rho$$

$$= \epsilon_0 E_0^2 \pi L R^2 \int_0^1 J_0^2 \left(\frac{\omega R}{c} \cdot \rho \right) \rho d\rho$$

$$\int_0^1 J_0^2(\alpha x) x dx = \frac{1}{2} [J_0'(\alpha)]^2 = \frac{1}{2} [J_1(\alpha)]^2; J_0' = -J_1$$

$$= \epsilon_0 E_0^2 V \cdot \frac{1}{2} [J_1(\alpha)]^2; \alpha = \frac{\omega R}{c}$$

$$U = \frac{1}{2} \epsilon_0 E_0^2 V \underbrace{J_1^2(2.405)}_{(0.52)^2}$$



STORED ENERGY $\sim 27\%$ OF UNIFORM FIELD

NOW NEED TO LOOK AT

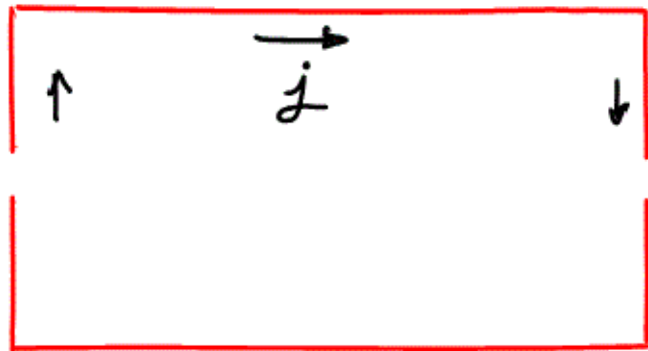
LOSSES TO GET Q

FILLING CAVITY

$$\frac{1}{2} \epsilon_0 E_0^2 V$$

OHMIC HEATING IN CAVITY WALL

OSCILLATING FIELD $\rightarrow$ INDUCED CURRENT $\rightarrow$ RESISTIVE LOSS



j IS SURFACE CURRENT DENSITY
AMPERE $\rightarrow \int B ds = \mu_0 i$

$$j = B_\theta / \mu_0$$

IN CAVITY $B_\theta(r) = e^{i\omega t}$

$$E_z = E_z(r) e^{i\omega t} = E_0 J_0\left(\frac{\omega}{c} \cdot r\right) e^{i\omega t}$$

WANT B_θ IN ORDER TO GET j

USE MAXWELL

$$\frac{\partial E_z}{\partial r} = \frac{\partial B_\theta}{\partial t}$$

$$\begin{cases}
 \frac{\partial E_z}{\partial r} = E_0 \frac{\partial}{\partial r} \left[J_0 \left(\frac{\omega}{c} \cdot r \right) \right] e^{i\omega t} \\
 \frac{\partial B_\theta}{\partial t} = i\omega B_\theta(r) e^{i\omega t}
 \end{cases}$$

$$\frac{\partial E_z}{\partial r} = \frac{\partial B_\theta}{\partial t}$$

$$i\omega B_\theta(r) e^{i\omega t} = E_0 \underbrace{\frac{\partial}{\partial r} \left[J_0 \left(\frac{\omega}{c} \cdot r \right) \right]}_{\text{}} e^{i\omega t}$$

WHITTAKER

&

WATSON

"MODERN ANALYSIS"

1902

Page 359

$$\frac{d}{dz} \left\{ z^{-n} J_n(z) \right\} = -z^n J_{n+1}(z)$$

$$n \rightarrow 0 \quad ; \quad z \rightarrow \frac{\omega}{c} \cdot r$$

$$\frac{c}{\omega} \frac{d}{dr} \left[J_0 \left(\frac{\omega}{c} \cdot r \right) \right] = -J_1 \left(\frac{\omega}{c} \cdot r \right)$$

so $i\cancel{\omega} B_\theta(r) e^{i\omega t} = -\cancel{\omega} \frac{E_0}{c} J_1 \left(\frac{\omega}{c} \cdot r \right)$

$$|X|^2 = XX^*$$

$$|B_\theta(r)|^2 = \left| \frac{E_0}{c} J_1 \right|^2$$

$$B_\theta(r) = \frac{E_0}{c} J_1 \left(\frac{\omega}{c} \cdot r \right)$$

USE THIS TO CALCULATE THE CURRENT
DENSITY IN THE CAVITY WALLS

$$B_{\theta}(r) = \frac{E_0}{c} J_1\left(\frac{\omega}{c} \cdot r\right)$$

- INSTANTANEOUS POWER LOSS INTO RESISTOR

$$P(t) = \operatorname{Re} I(t) \operatorname{Re} V(t)$$

- TIME AVERAGED POWER LOSS

$$\begin{aligned} \overline{P} &= \int_{\text{CYCLE}} \operatorname{Re} I_0 e^{i\omega t} \operatorname{Re} V_0 e^{i\omega t} = \frac{1}{2} I_0 V_0 \\ &= \frac{1}{2} I_0^2 R \end{aligned}$$

CAVITY POWER LOSS

$$\int_{\text{CAVITY SURFACE}} \frac{1}{2} \rho_s \overset{\text{CURRENT DENSITY}}{J^2} = \frac{1}{2} \int \rho_s \left[\frac{B_{\theta}}{\mu_0} \right]^2 ds = \frac{1}{2} \rho_s \left(\frac{E_0}{\mu_0 c} \right)^2 \int J_1^2\left(\frac{\omega}{c} \cdot r\right) ds$$

SURFACE RESISTIVITY

$$\begin{aligned}
 P &= \frac{1}{2} \rho_s \left(\frac{E_0}{\mu_0 c} \right)^2 \int J_1^2 \left(\frac{\omega}{c} \cdot r \right) ds \\
 &= \frac{1}{2} \rho_s \left(\frac{E_0}{\mu_0 c} \right)^2 \left\{ \overset{\substack{\uparrow \\ 2 \text{ ENDS}}}{2 \times 2\pi} \int_0^R J_1^2 \left(\frac{\omega}{c} \cdot r \right) r dr \right. \\
 &\quad \left. + \underbrace{2\pi R L J_1^2 \left(\frac{\omega}{c} \cdot R \right)}_{\substack{\uparrow \\ \int \text{ OVER SURFACE} \\ \text{ OF CYLINDER} \\ \text{ (RADIUS CONST)}}} \right\}
 \end{aligned}$$

$\int \text{ OVER SURFACE OF CYLINDER (RADIUS CONST)}$

$$\int_0^\alpha J_1^2(u) u du = \frac{\alpha^2}{2} \left\{ J_1^2(\alpha) - J_0(\alpha) J_2(\alpha) \right\}$$

$$\int_0^{\frac{\omega}{c} R} J_1^2 \left(\frac{\omega}{c} r \right) \left(\frac{\omega}{c} \right)^2 r dr = \left(\frac{\omega}{c} R \right)^2 \left\{ J_1^2 \left(\frac{\omega}{c} R \right) - \underbrace{J_0 \left(\frac{\omega}{c} R \right)}_{\text{ZERO ON CAVITY WALL}} J_2 \left(\frac{\omega}{c} R \right) \right\}$$

$$P = \frac{1}{2} \rho_s \left(\frac{E_0}{\mu_0 c} \right)^2 \left\{ 4\pi \frac{R^2 J_1^2}{2} + 2\pi R L J_1^2 \right\}$$

$$P = \frac{1}{2} \rho_s \left(\frac{E_0}{\mu_0 c} \right)^2 2\pi R L \left(1 + \frac{R}{L} \right) J_1^2$$

DEFINITION OF $Q = \frac{\omega U}{P}$

STORED ENERGY

$$= \frac{1}{2} \epsilon_0 E_0^2 V J_1^2$$

$$= \frac{1}{2} \epsilon_0 E_0^2 \underbrace{\pi R^2 L}_{\text{VOLUME}} J_1^2$$

$\frac{P}{\omega} \rightarrow$ POWER LOSS PER
RADIAN OF OSCILLATION

$$Q = \frac{\omega u}{P} \quad \omega \text{ is RESONANT FREQUENCY OF THE CAVITY}$$

$$\omega = 2\pi f = \frac{2.405 \cdot c}{R} \quad \frac{\omega}{c} \cdot R = 2.405$$

So $Q = \frac{2.405 c}{R} \frac{\frac{1}{2} \epsilon_0 E_0^2 \pi R^2 L J_1^2}{\frac{1}{2} \rho_s \frac{E_0^2}{Z_0^2} 2\pi R L \left(1 + \frac{R}{L}\right) J_1^2}$

Q VALUE
OF CAVITY

$$= \frac{2.405 \mu_0 c}{2 \rho_s \left(1 + \frac{R}{L}\right)}$$

WANT Q LARGE

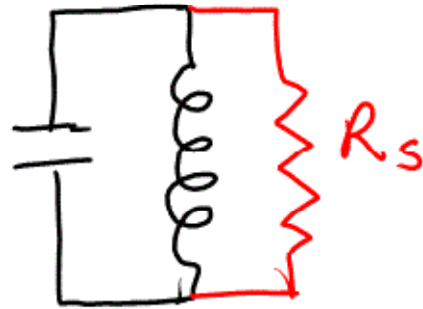
→ ρ_s SMALL

→ $\frac{R}{L}$ SMALL → c of T

FOR Cu $Q \sim 10^4$ in 400 MHz REGION

FOR SUPERCONDUCTING CAVITIES $Q \sim 10^{11}$

SHUNT IMPEDANCE



THE ENTIRE RF POWER IS
CONVERTED TO HEAT IN CAVITY
WALL $\rightarrow$ SHUNT IMPEDANCE
THIS IS WHAT GENERATES THE
ACCELERATING VOLTAGE

$$P_{RF} = \frac{1}{2} \frac{V^2}{R_s} \rightarrow R_s = \frac{V^2}{2 P_{RF}}$$

CAN ALSO THINK OF SHUNT IMPEDANCE AS

$$R_s = \frac{(\text{ENERGY GAIN/UNIT CHARGE})^2}{2P}$$

$2P \leftarrow$ JUST CALCULATED

$$\Delta E_{KIN} = e V T$$

$$= e E_0 l \cdot T \leftarrow \text{TRANSIT FACTOR}$$

$\uparrow$
CAVITY LENGTH

$$R_s = \frac{E_0^2 L^2 T^2}{2} \cdot \frac{2 M_0^2 c^2}{2 \pi f_s E_0^2 R L \left(1 + \frac{R}{L}\right) J_1^2(2.405)}$$

$$R_s = \frac{Z_0^2}{\pi f_s} \cdot \frac{L}{2R} \cdot \frac{T^2}{\left(1 + \frac{R}{L} J_1^2[2.405]\right)}$$

DORIS - 5 GeV

$$R_s = 3.6 \times 10^6 \Omega$$

$$P_{RF} = 50 \text{ kW}$$

$$V_{CAV} = 548 \text{ kV}$$

PIETRA - 30 GeV

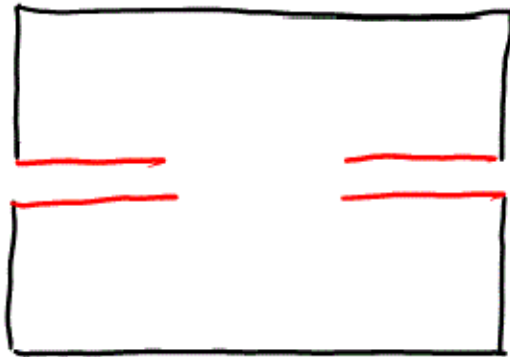
$$R_s = 1.8 \times 10^6 \Omega$$

$$P_{RF} = 125 \text{ kW}$$

$$V_{CAV} = 2.12 \text{ MV}$$

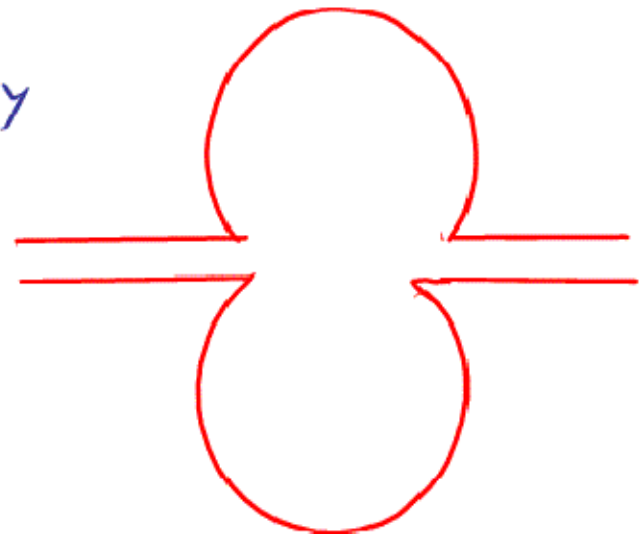
CAVITY GEOMETRY

- CAN INCREASE ENERGY GAIN OF PILL BOX BY ADDING RE-entrant SURFACES



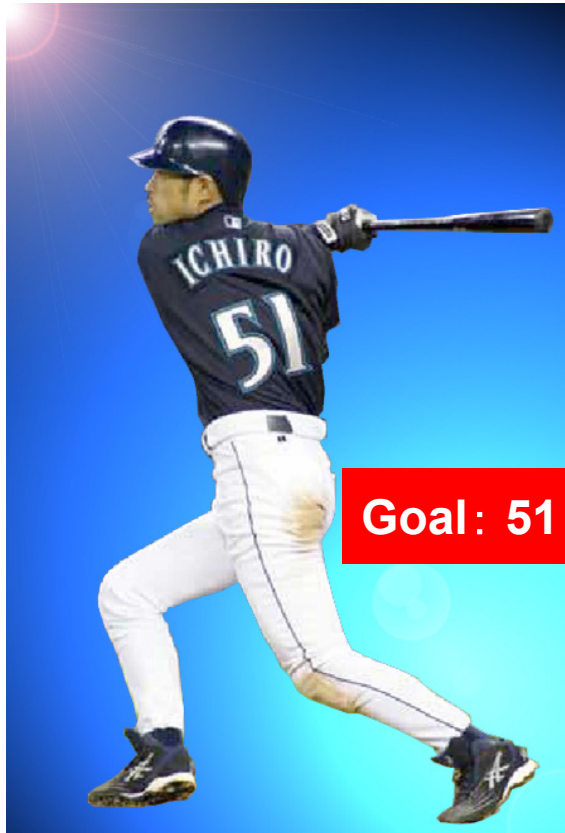
$$E_{acc} = V/d \quad \uparrow \text{ DECREASE}$$

- RESISTIVE LOSSES MAINLY ON CYLINDRICAL WALLS CAN RAISE Q BY $\rightarrow$





ICHIRO Cavity



Goal: 51 MV/m

