High Energy Physics experiments?

1. Collide Particles
2. Detect Final State
3. Understand Connection of 1 + 2

Accelerators & Beams

E_{CM} and $\mathcal{L}$

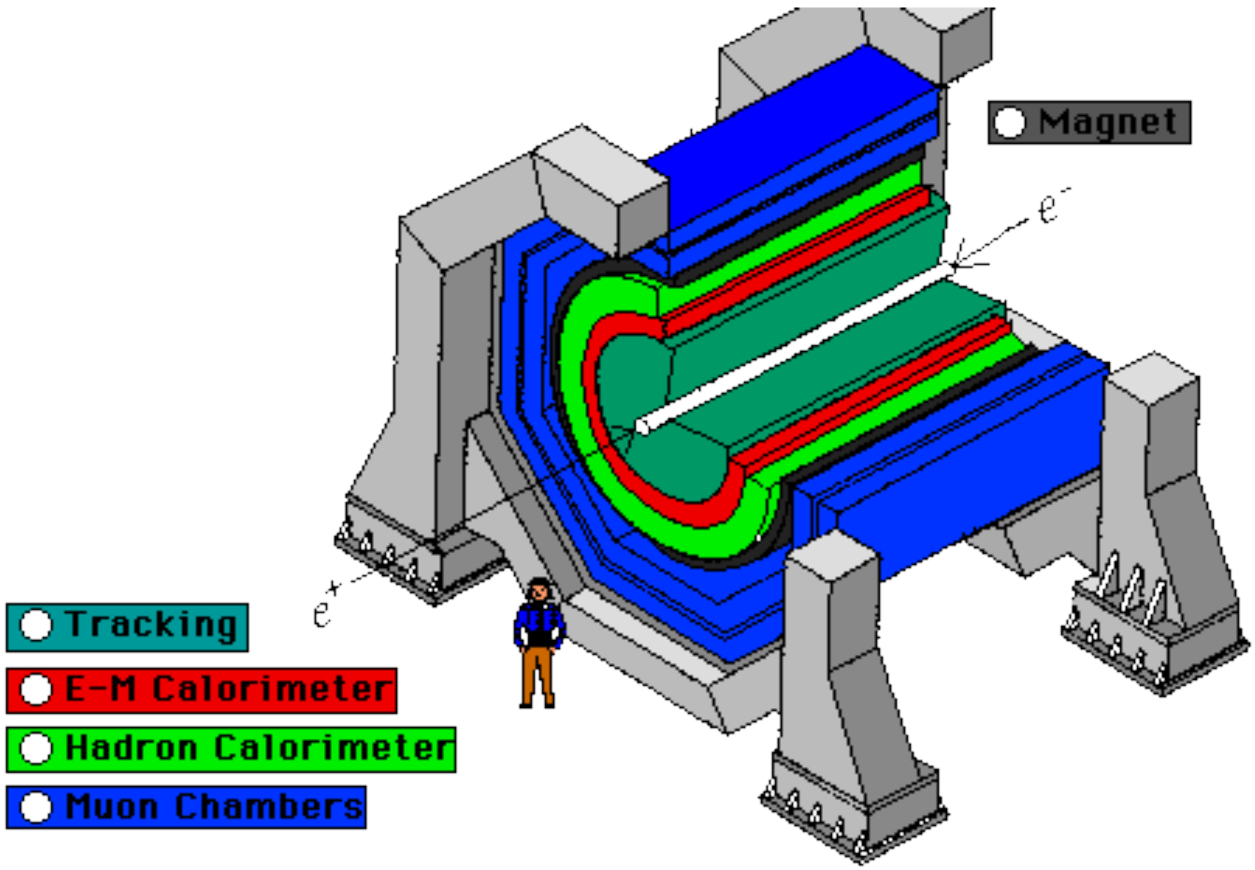
Detectors

$$\frac{\sigma_\mu}{p_\mu}$$

Analysis

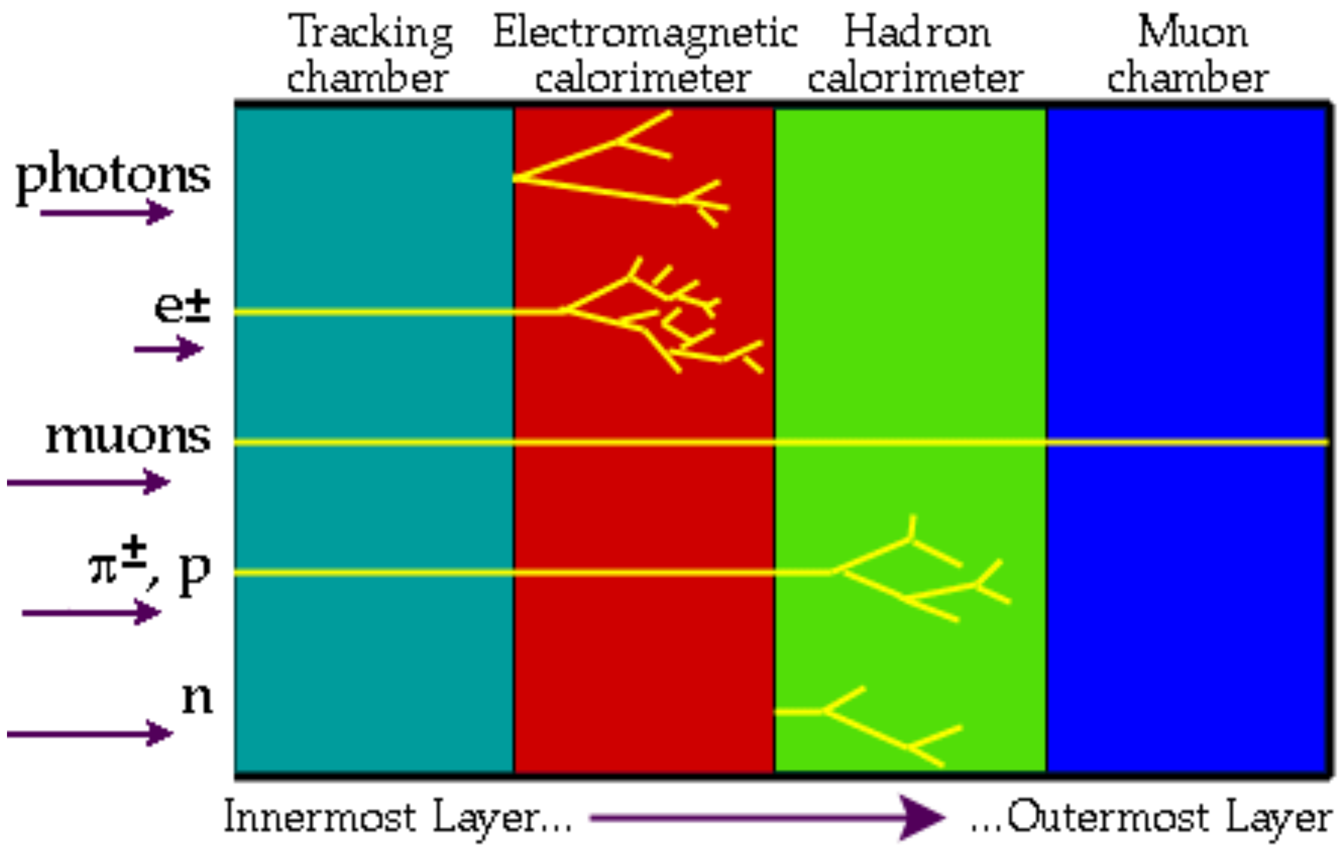
$$\frac{S}{B}$$

Generic Detector



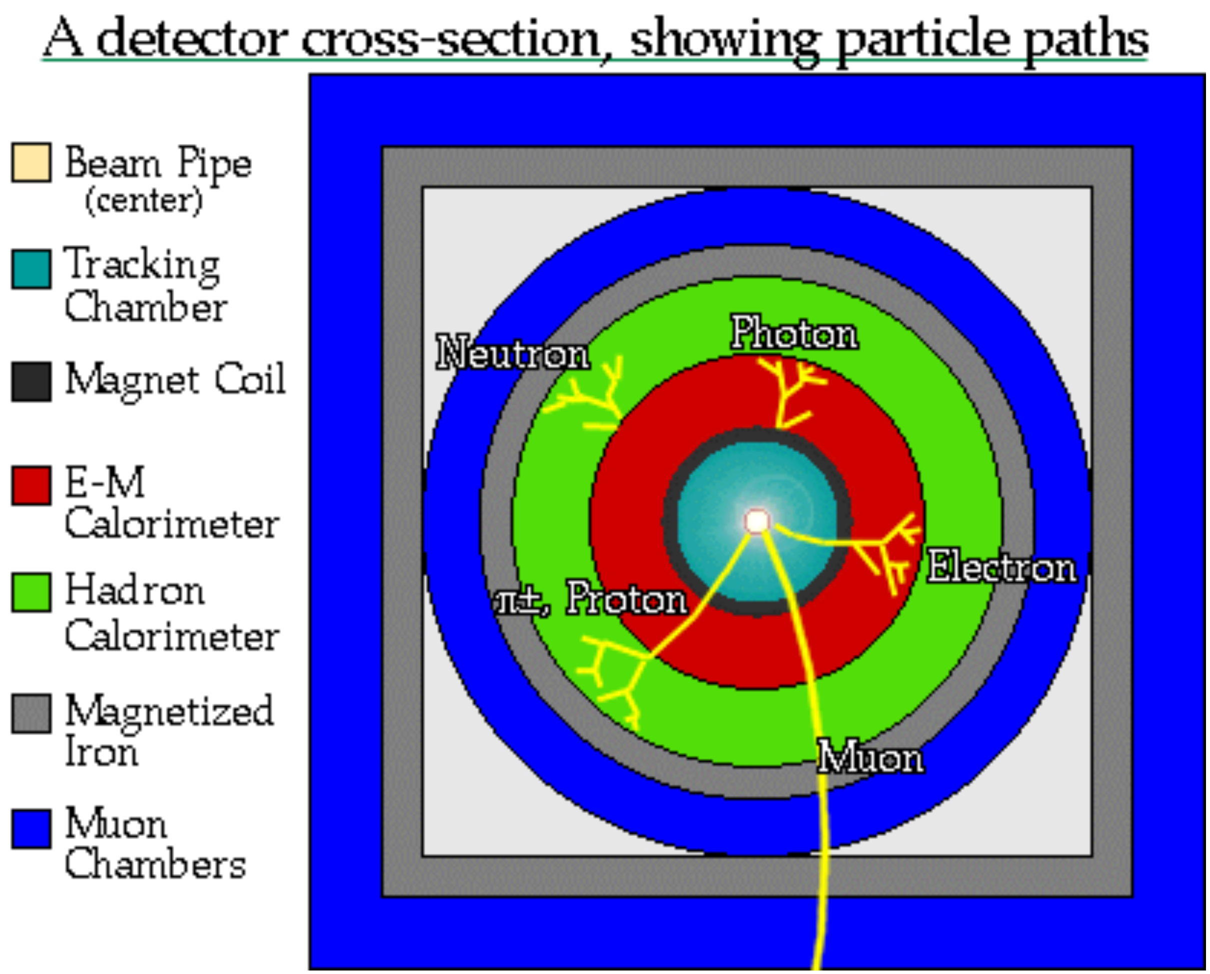
- Layers of Detector Systems around Collision Point

Generic Detector



- Different Particles detected by different techniques.
  - Tracks of Ionization – Tracking Detectors
  - Showers of Secondary particles – Calorimeters

Generic Detector



- Different Particles detected by different techniques.
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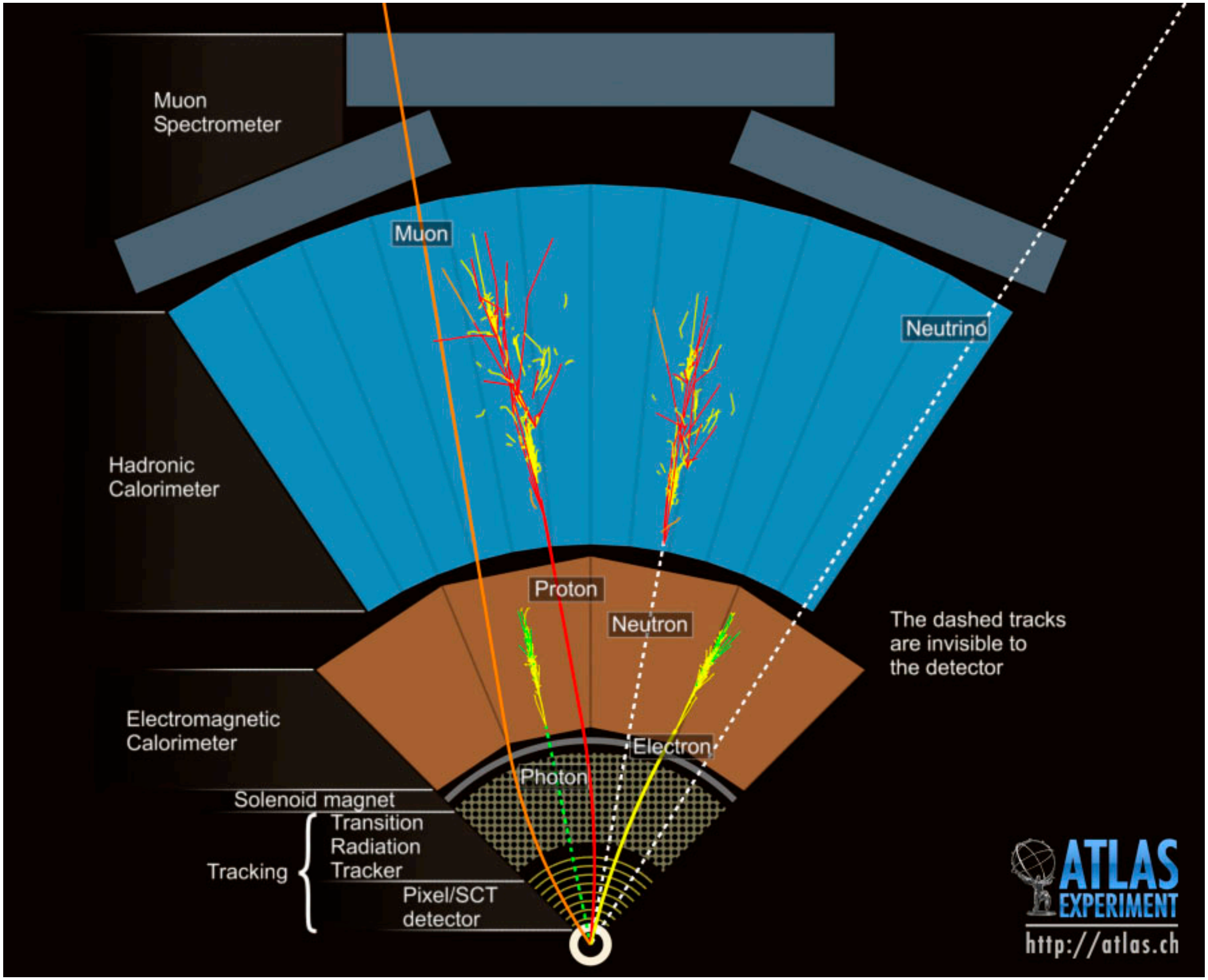
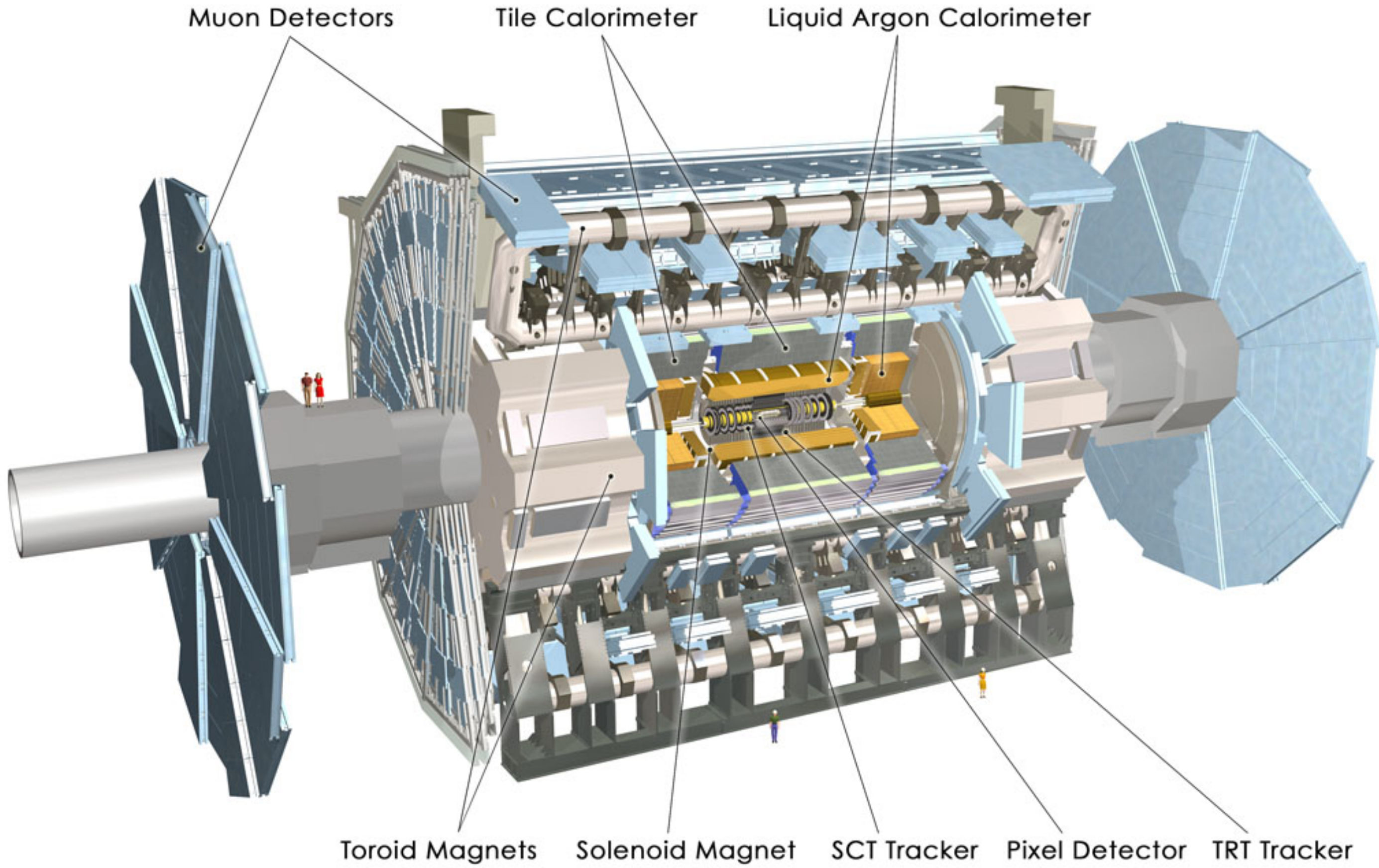
ATLAS Detector



ATLAS Detector



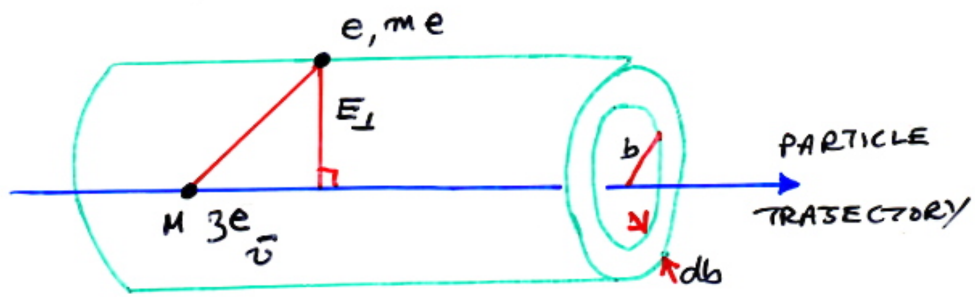
- Different Particles detected by different techniques.
  - Tracks of Ionization – Tracking Detectors
  - Showers of Secondary particles – Calorimeters

Interaction of Charged Particles with Matter

- All particle detectors ultimately use interaction of electric charge with matter
  - Track Chambers
  - Calorimeters
  - Even Neutral particle detectors $n \, \gamma \, \pi^0 \, \nu$

- Ionization
  - Average energy loss
  - Landau tail
- Multiple Scattering
- Cerenkov
- Transition Radiation
  - Electron's small mass - radiation

Energy Loss to Ionization



- Heavy charged particle interacting with atomic electrons
- All electrons with shell at impact parameter b
- Energy loss $\Delta p = \Delta p_T$ - symmetry

$$\Delta p_T = \int_{-\infty}^{+\infty} F dt = e \int E_T dt = e \int E_T \frac{dt}{dx} dx = e \int E_T \frac{dx}{v}$$

- Gauss $\int \bar{E} \bullet \bar{n}\, dA = 4\pi Q_{ENCLOSED}$

$$\int E_T dA = 4\pi z e$$

$$\int E_T 2\pi b dx = 4\pi z e$$

$$\int E_T dx = \frac{2ze}{b}$$

$$\Delta p_T = \frac{2ze^2}{bv}$$

$$\Delta E = \frac{(\Delta p_T)^2}{2m_e}$$

$$\Delta E(b) = \frac{2z^2 e^4}{m_e v^2 b^2}$$

- Density of electrons N_e

$$-dE(b) = \Delta E(b) N_e dV$$

$$-dE(b) = \Delta E(b) N_e 2\pi b db dx$$

$$-dE(b) = \frac{4\pi z^2 e^4}{m_e v^2} N_e \frac{db}{b} dx$$

Physical limits of integration

$$\int_{b=0}^{\infty} \rightarrow \int_{b\min}^{b\max}$$

$$\frac{-dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln\left(\frac{b_{MAX}}{b_{MIN}}\right)$$

- Maximum ΔE ⇨ minimum b
- In a classical head-on collision $\Delta E_{MAX} = \frac{1}{2} m_e (2v)^2$
- Relativistically $\Delta E_{MAX} = \frac{1}{2}\gamma^2 m_e (2v)^2 = 2\gamma^2 m_e (v)^2$

$$\Delta E_{MAX} = \frac{2z^2 e^4}{m_e v^2 b_{MIN}^2}$$

$$b_{MIN}^2 = \frac{2z^2 e^4}{m_e v^2} \frac{1}{2\gamma^2 m_e (v)^2}$$

$$b_{MIN} = \frac{z\ e^2}{\gamma m_e v^2}$$

Physical limits of integration

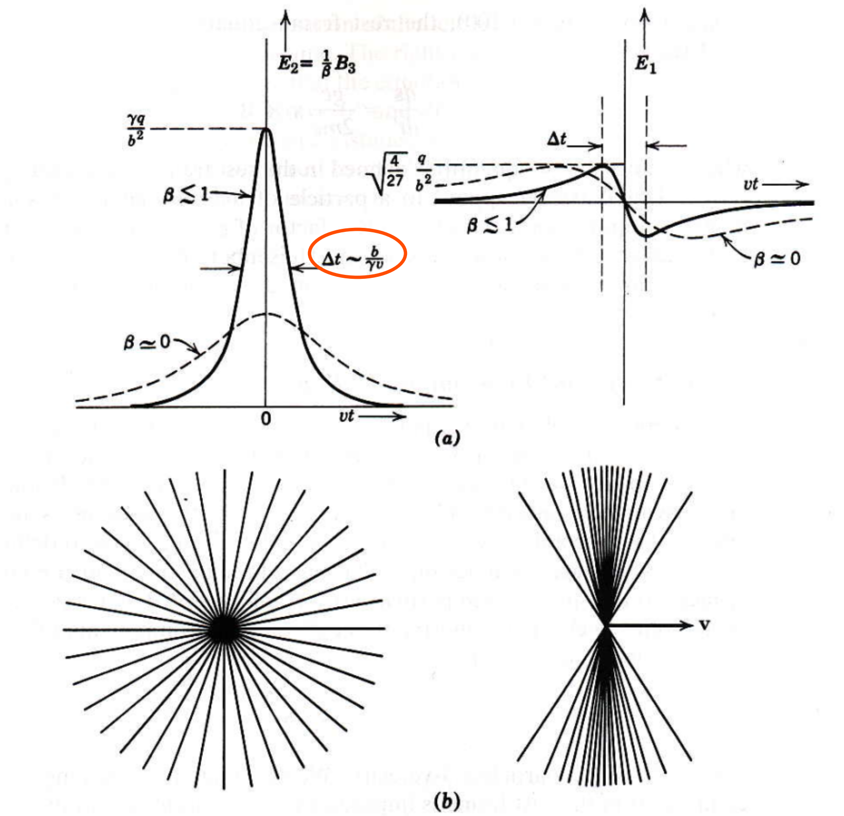


Figure 11.9 Fields of a uniformly moving charged particle. (*a*) Fields at the observation point *P* in Fig. 11.8 as a function of time. (*b*) Lines of electric force for a particle at rest and in motion ($\gamma = 3$). The field lines emanate from the *present* position

- Time for EM interaction $\Delta t \sim \dfrac{b}{\gamma v}$

- Electrons bound in atoms
- Time of interaction must be small, compared to orbital period, else energy transfer averages to zero
- Orbital period $\tau \sim \dfrac{1}{\bar{\omega}}$
- Collision time $t \sim \dfrac{b}{\gamma v}$

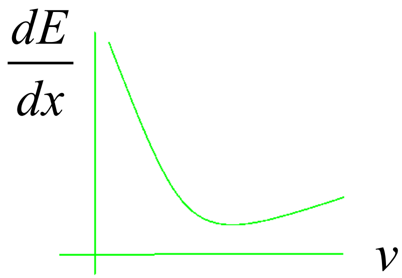
$$\frac{b}{\gamma v} \leq \tau \sim \frac{1}{\bar{\omega}}$$

$$b_{MAX} = \frac{\gamma v}{\bar{\omega}}$$

- Put in integration limits

$$\frac{-dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln\left(\frac{\gamma^2 m_e v^3}{z e^2 \bar{\omega}}\right)$$

Ionization Loss



$$\frac{-dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln\left(\frac{\gamma^2 m_e v^3}{z e^2 \overline{\omega}}\right)$$

$$-\frac{dE}{dx} : \frac{1}{v^2} \ln\left(\frac{\gamma^2 m v^3}{z e^2 \overline{\omega}}\right)$$

- This works for heavy particles like α
- Breaks down for $M \le M_{PROTON}$

- Correct QED treatment gives Bethe – Bloch equation

Maximum energy transfer in single collision

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2 m_e \gamma^2 \beta^2 T_{MAX}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right]$$

Mean excitation potential of material

Density correction

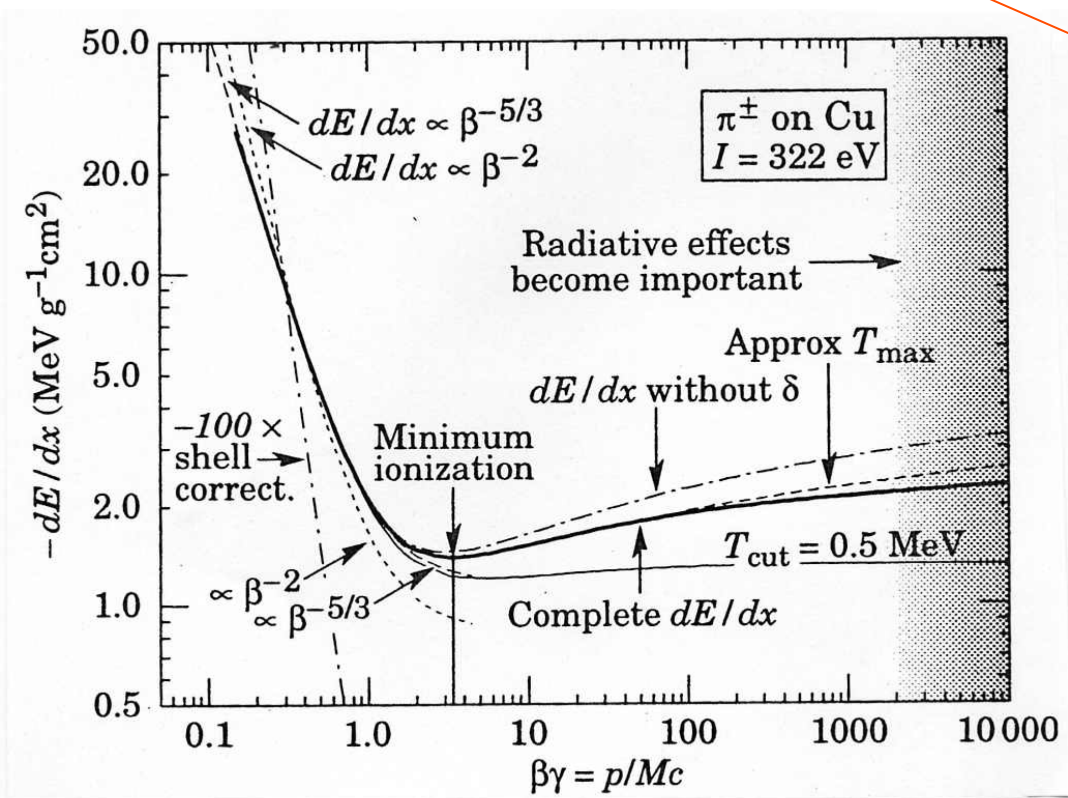
Shell correction

Bethe – Bloch Equation

$$2\pi N_A r_e^2 m_e c^2 = 0.1535\, MeV \frac{cm^2}{g}$$

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e\gamma^2\beta^2 T_{MAX}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$

$M ? \; m_e \quad T_{MAX} \to 2m_e c^2 \beta^2 \gamma^2$



$2\pi N_A r_e^2 m_e^2 = 0.1535\, MeV \frac{cm^2}{g}$

- Mean excitation potential
  This is main parameter in B –B
  $\sim \bar{\omega} h$

Hard to calculate

measure $\frac{dE}{dx}$ infer I

- Empirically

$$\frac{I}{Z} = 12 + \frac{7}{Z} eV \quad Z < 13$$

$$\frac{I}{Z} = 9.76 + 58.8 \cdot Z^{-1.19} eV \quad Z > 13$$

CM

$$P_R^* = m\gamma\beta^* \quad , \quad E_R = m\gamma \quad , \quad M \gg m$$

$$\beta^* = \beta^{LAB}$$

$$E_R^{LAB} = \gamma\,(E_R^* + \beta\, p_R^*)$$

$$= \gamma\,(m\gamma + m\gamma\beta^2)$$

ORIGINAL $E^{LAB} = m$

$$\Delta E = m\gamma^2 + m\gamma^2\beta^2 - m$$

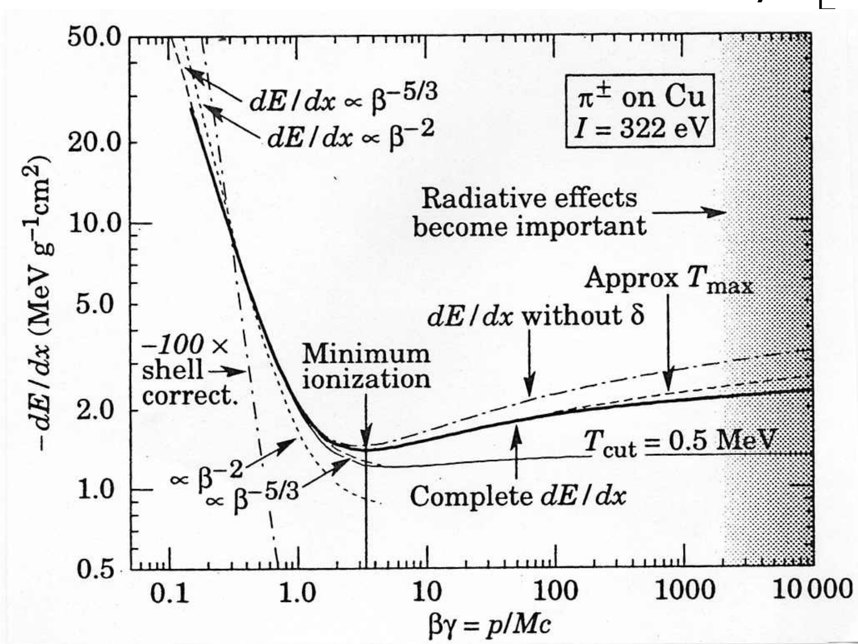
$$= m(\gamma^2 - 1) + m\gamma^2\beta^2$$

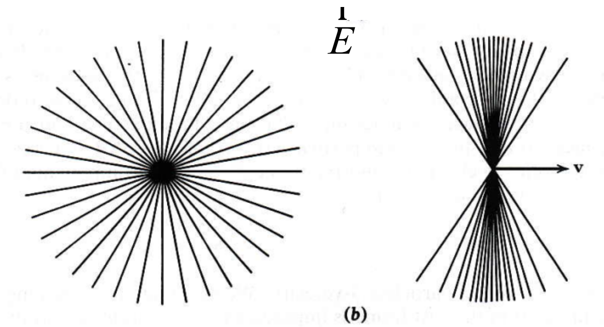
BUT $(\gamma^2 - 1) = \beta^2\gamma^2$

$$\Delta E = 2m\gamma^2\beta^2$$

Relativistic rise & Density Correction

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{2m_e\gamma^2\beta^2 T_{MAX}}{I^2}\right) - 2\beta^2 - \delta - 2\frac{C}{Z}\right]$$





$$b_{\max} = \frac{\gamma v}{\varpi}$$ Increases with energy

- Electric field polarizes material along path
- Far off electrons shielded from field and contribute less

$$\frac{dE}{dx} \to \frac{dE}{dx} - \delta$$

- Polarization greater in condensed materials, hence density correction

Particle Identification

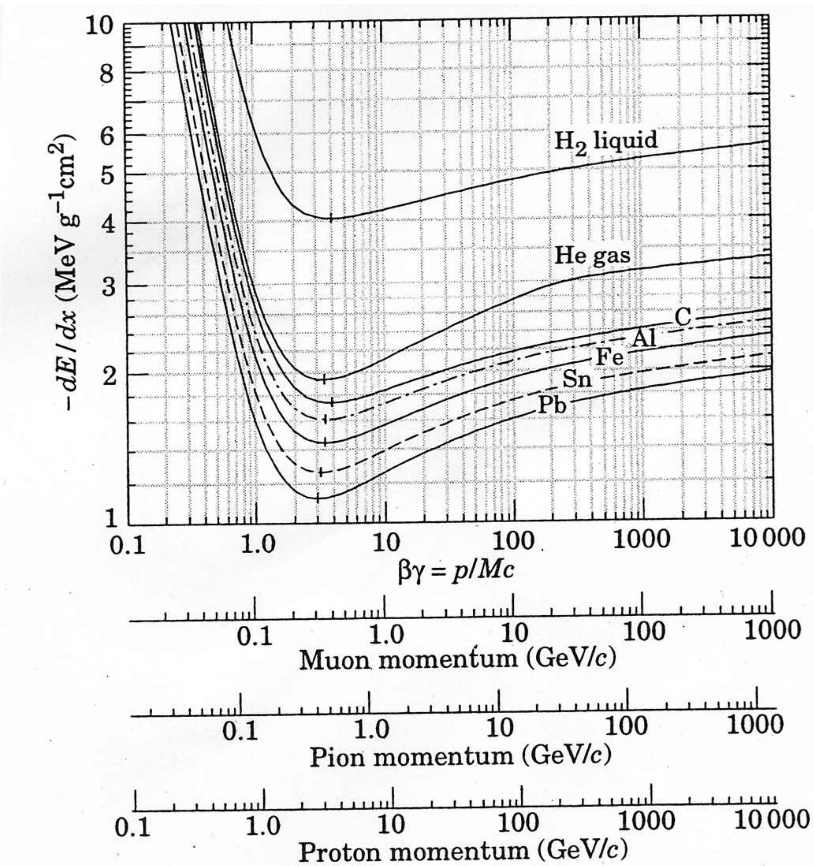
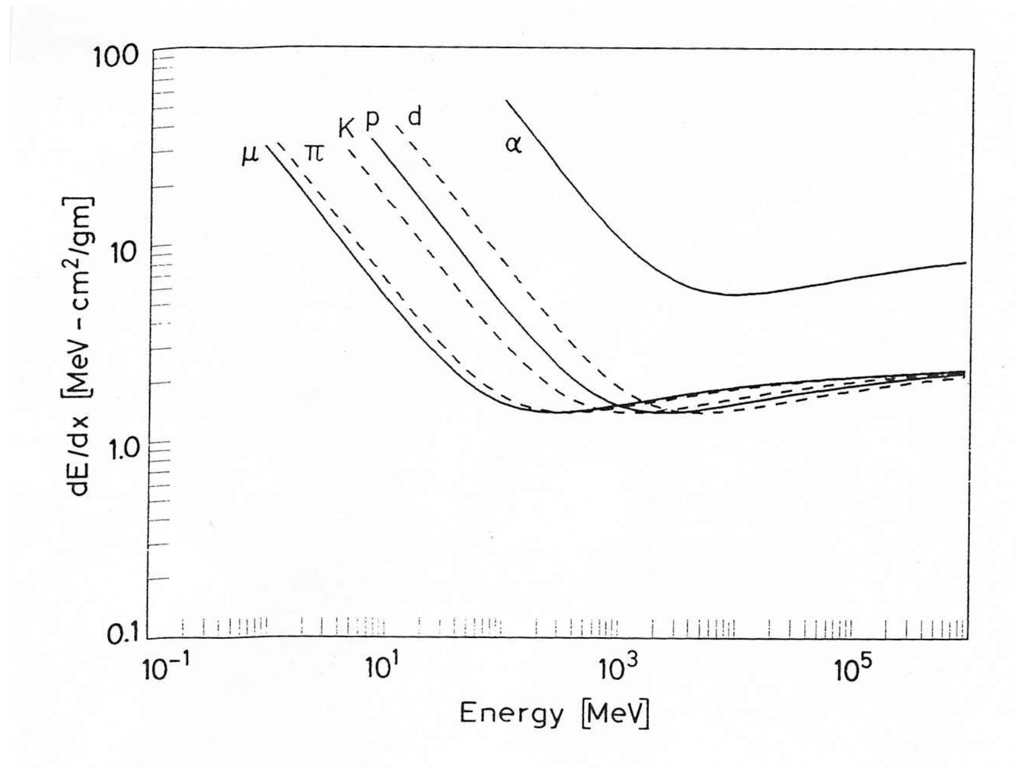


Figure 23.2: Energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, tin, and lead.

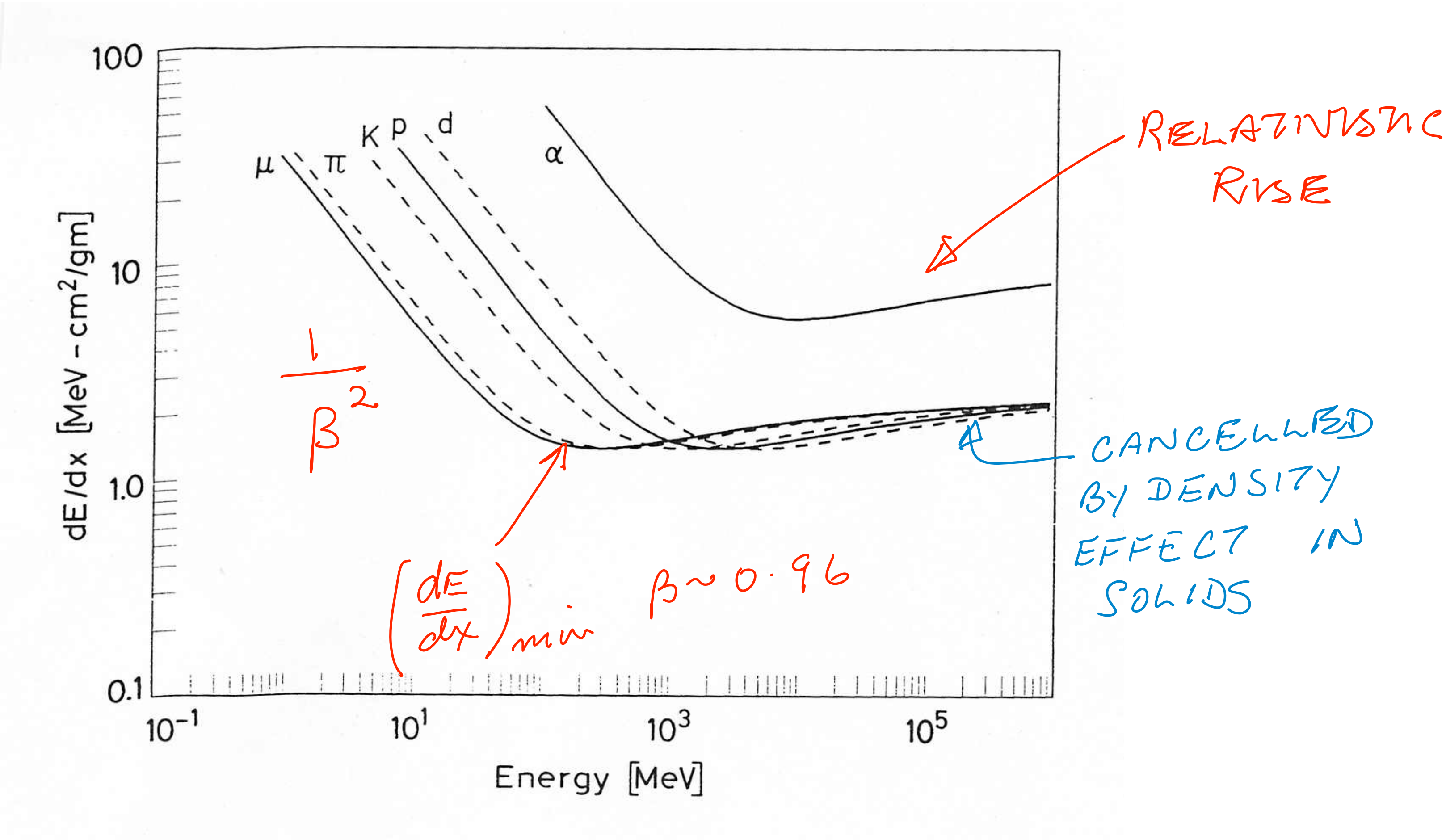


$\frac{dE}{dx}$ depends on velocity

Usually measure $p = \gamma\beta M$

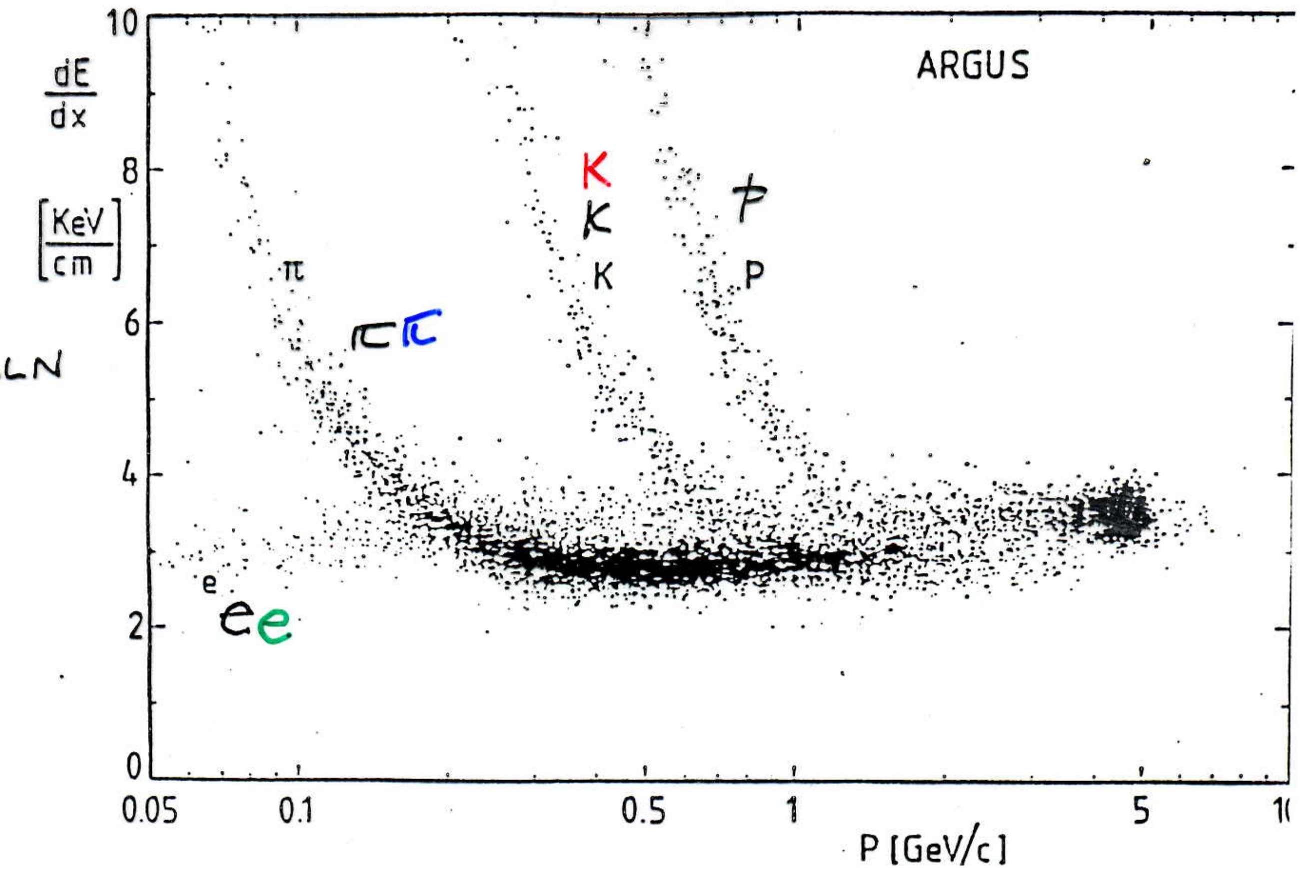
$\frac{dE}{dx}$ determines mass

Particle Identification



$\frac{dE}{dx}$ RESLN

$\sim 5\%$



SCALING LAW FOR dE/dx

FOR PARTICLES IN THE SAME MEDIUM, BETHE-BLOCH HAS THE FORM

ONLY A FUNCTION OF VELOCITY

$$-\frac{dE}{dx} = z^2 f(\beta) \rightarrow \frac{dE}{dx} \equiv \frac{dE}{dx}(z^2, \beta)$$

KINETIC ENERGY $T = (\gamma - 1) mc^2 \rightarrow \beta \equiv \beta(T/m)$

SO $-\frac{dE}{dx} = z^2 f(T/m)$

SCALING LAW

$$-\frac{dE_2}{dx}(T_2) \simeq -\frac{z_2^2}{z_1^2} \frac{dE_1}{dx}\left(T_2 \frac{m_1}{m_2}\right)$$

Mass Stopping Power

$\frac{dE}{dx}$ expressed as (mass)x(thickness) is relatively constant over a wide range of materials

$$\varepsilon \frac{[Mass]}{[Area]} \quad -\frac{dE}{d\varepsilon} = -\frac{1}{\rho}\frac{dE}{dx} = z^2 \frac{Z}{A} f(\beta, I)$$

density (ρ)

Roughly constant over periodic table ($\frac{Z}{A}$)

In variation (I)

$\frac{dE}{d\varepsilon} \longrightarrow$ Independent of material

10 MeV proton loses same energy in

$\frac{1gm}{cm^2}$ Cu or $\frac{1gm}{cm^2}$ Fe, Al, ….

Mixtures of Materials

Bragg's Rule

$$\frac{1}{\rho}\frac{dE}{dx} = \frac{\omega_1}{\rho_1}\frac{dE}{dx_1} + \frac{\omega_2}{\rho_2}\frac{dE}{dx_2} + \ldots$$

fraction by weight $\omega_i = \frac{a_i A_i}{A_{mixture}}$

No of atoms of i element molecule (a_i)

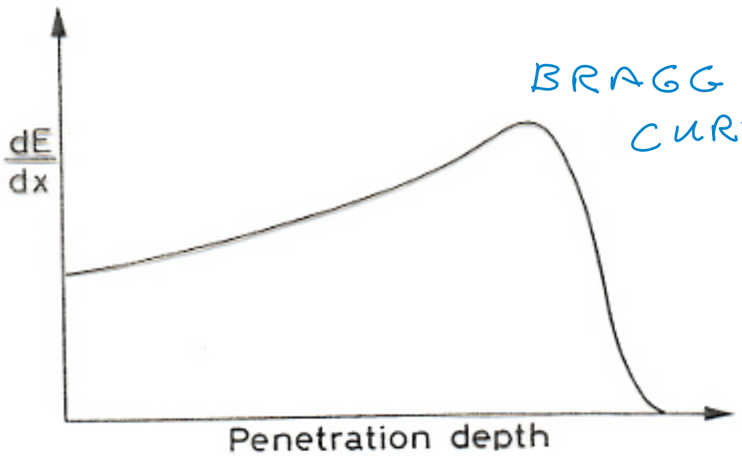
$$A_{mixture} = \sum a_i A_i$$

$$Z_{mixture} = \sum a_i Z_i$$

$$\ln I_{mixture} = \sum \frac{a_i Z_i}{Z_i} \ln I_i$$

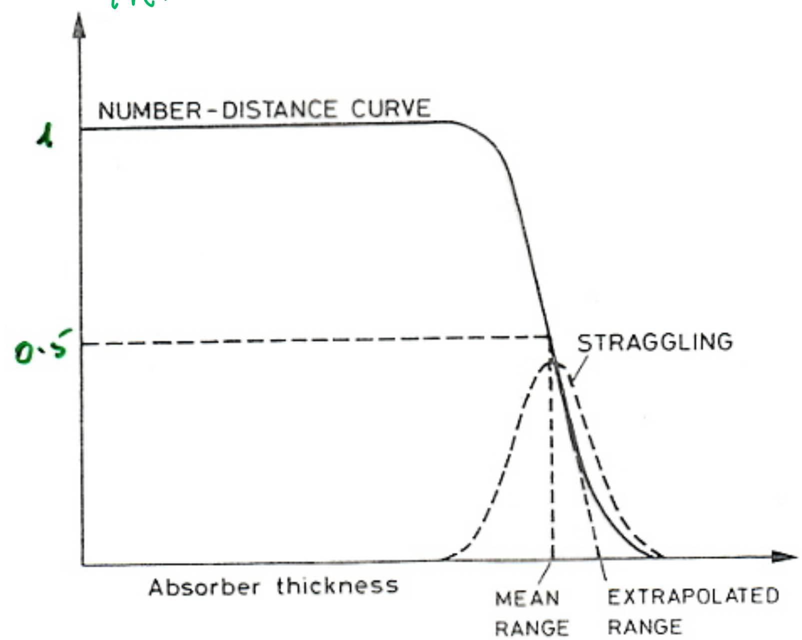
BETHE-BLOCH BREAKS DOWN AS $\beta \to 0$

BRAGG CURVE



• HADRON THERAPY FOR TUMORS USING π, p

TRANSMISSION



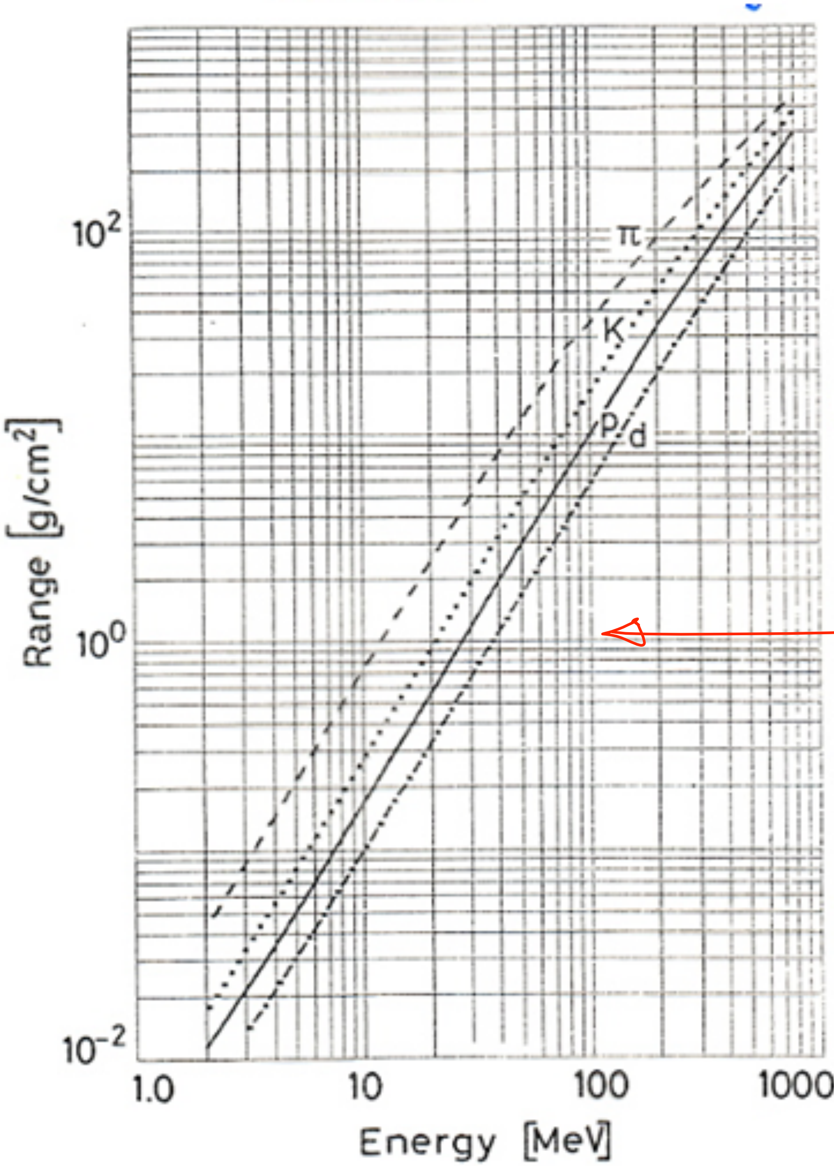
RANGE STATISTICALLY DISTRIBUTED FOR A BEAM OF PARTICLES.

CALCULATED RANGE CURVES IN ALUMINUM

$$R(T) = R_0(T_{MIN}) + \int_{T_{MIN}}^{T} \left(\frac{dE}{dx}\right)^{-1} dE$$

EMPIRICALLY DETERMINED

MINIMUM ENERGY AT WHICH B-B STILL VALID



NUMERICAL INTEGRATION OF B-B

Electron Energy Loss

- More complicated than heavy particles discussed so far
- Small mass $\longrightarrow$ radiation (bremsstrahlung) dominates
- Above critical energy, radiation dominates
- Below critical energy, ionization dominates

$$\left(\frac{dE}{dx}\right)_{TOTAL} = \left(\frac{dE}{dx}\right)_{IONIZATION} + \left(\frac{dE}{dx}\right)_{RADIATION}$$

- What constitutes a heavy particle, depends on energy scale

Bethe Bloch for electrons

- Projectile deflected
- Projectile and atomic electrons have equal masses
- Also identical particles – statistics
- Equal masses ⟶ $T_{MAX} = \frac{T_E}{2}$

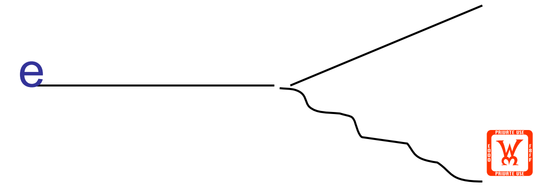
KINETIC ENERGY IN UNITS OF $m_e c^2$

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A}\frac{z^2}{\beta^2}\left[\ln\left(\frac{\tau^2(\tau+2)}{2(I/m_e c^2)}\right) - F(\tau) - \delta - 2\frac{C}{Z}\right]$$

$$F(\tau)^{Electron} \neq F(\tau)^{Positron}$$

identical *non-identical*

Bremsstrahlung



- Below ~100 GeV/c only important for electrons
- > 100 GeV/c becomes important for muons $\sigma \propto \left(\frac{e^2}{mc^2}\right)^2 : \frac{1}{m^2}$
- $E_B(\mu) \approx \frac{E_B(e)}{40{,}000}$ in the GeV range

$$: \frac{1}{\nu}$$

$$-\left(\frac{dE}{dx}\right)_{RAD} = N \int_0^{\nu_0 = E_o/h} h\nu \frac{d\sigma}{d\nu}(E_0,\nu) \qquad N = \frac{\rho N_A}{A}$$

atoms /cc

$$-\left(\frac{dE}{dx}\right)_{RAD} = NE_0\Phi\left(Z^2\right)$$

independent of
function of material

$$\Phi = 4Z^2 r_e^2 \alpha \left[\ln\left(183Z^{-1/3} - \frac{1}{18} - f(Z)\right)\right]$$

$$\left(\frac{dE}{dx}\right)_{RAD} \propto E, Z^2$$

can emit all energy in a few photons -> large fluctuations

$$\left(\frac{dE}{dx}\right)_{ION} \propto \ln(E), Z$$

Radiation Length

$$-\left(\frac{dE}{dx}\right)_{RAD} = NE_0\Phi\left(Z^2\right)$$

assume indep of E

$$-\frac{dE}{E_0} = N\Phi\left(Z^2\right)$$

$$\int \rightarrow \ln E - \ln E_0$$

$$E = E_0 \exp\left(-\frac{x}{\chi_0}\right)$$

$$\chi_0 = \frac{1}{N\Phi}$$

- χ_0 distance over which the electron energy is reduced by1/e on average
- Radiation Length

$$\frac{1}{\chi_0} \approx \left[4Z(Z+1)\frac{\rho N_A}{A}\right] r_e^2 \alpha \left[\ln\left(\frac{183}{Z^{1/3}}\right) - f(z)\right]$$

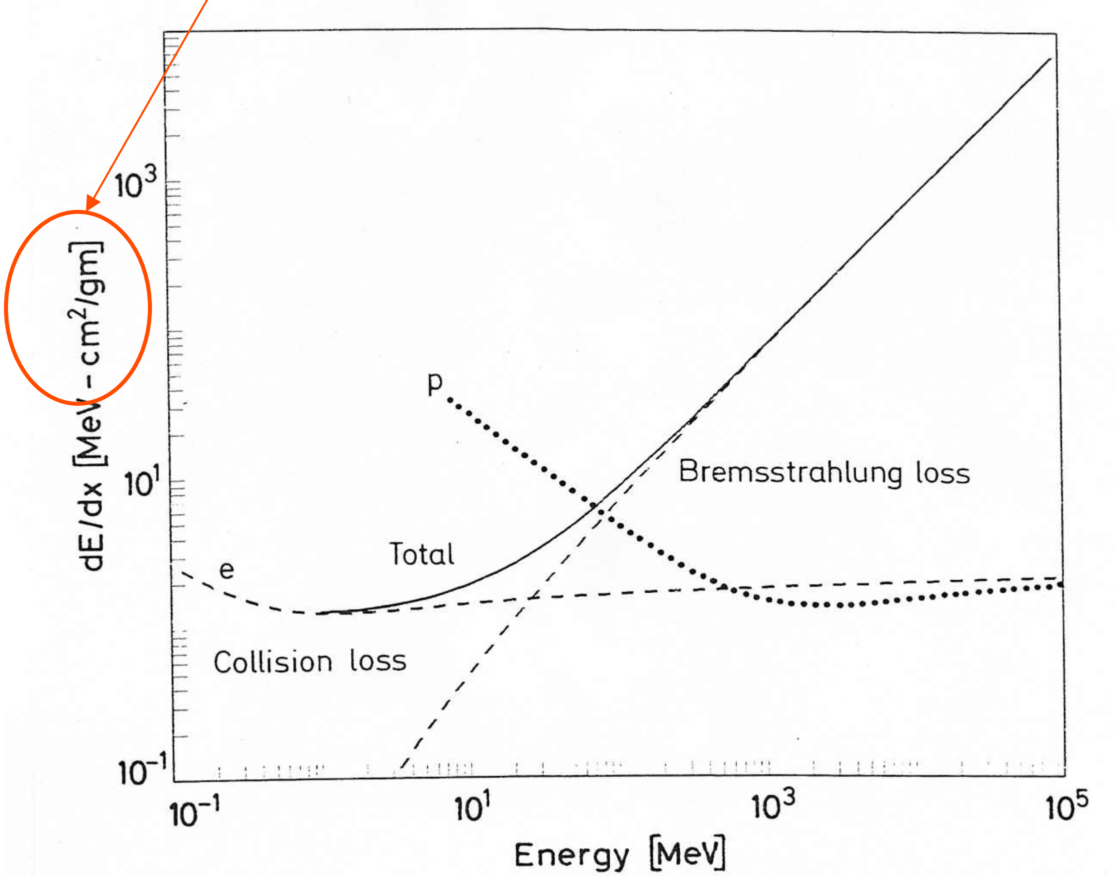
- for *x* expressed in units of χ_0

$$-\frac{dE}{dt} = E_0$$

← ROUGHLY INDEPENDENT OF MATERIAL

Electron Energy Loss

approx valid for any material



electrons in Cu

Table 2.3. Radiation lengths for various absorbers

Material	[gm/cm^2]	[cm]
Air	36.20	30050
H_2O	36.08	36.1
NaI	9.49	2.59
Polystyrene	43.80	42.9
Pb	6.37	0.56
Cu	12.86	1.43
Al	24.01	8.9
Fe	13.84	1.76
BGO	7.98	1.12
BaF_2	9.91	2.05
Scint.	43.8	42.4

$$X_0 = \frac{716.4 \text{ g cm}^{-2} A}{Z(Z+1)\ln(287/\sqrt{Z})}$$

good approximation (3%) except for He

CRITICAL ENERGY FOR VARIOUS MATERIALS

	Ec (MeV)
Pb	9.51
Cu	24.8
Fe	27.4
Al	52
Water	92
Air	102

$$\left(\frac{dE}{dx}\right)_{RAD} = \left(\frac{dE}{dx}\right)_{ION}$$

High Energy Muons

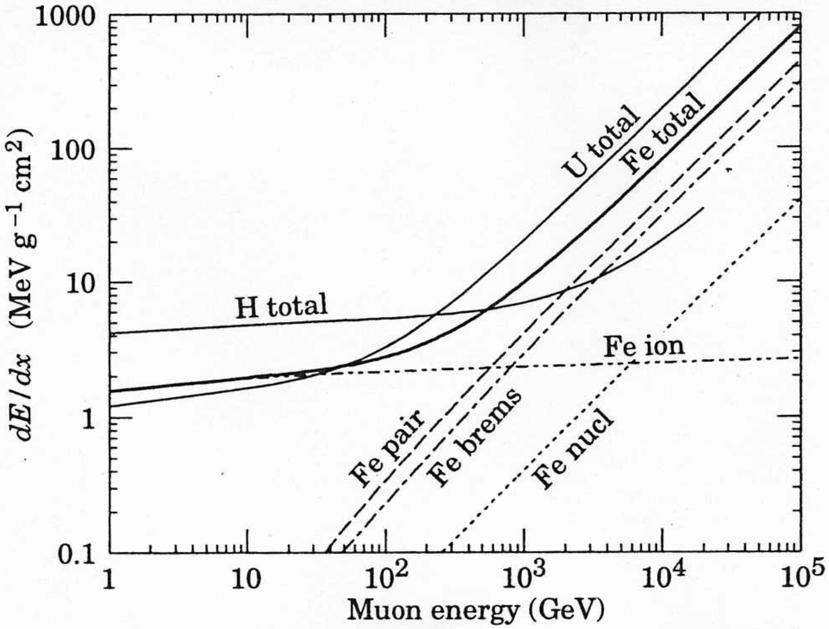
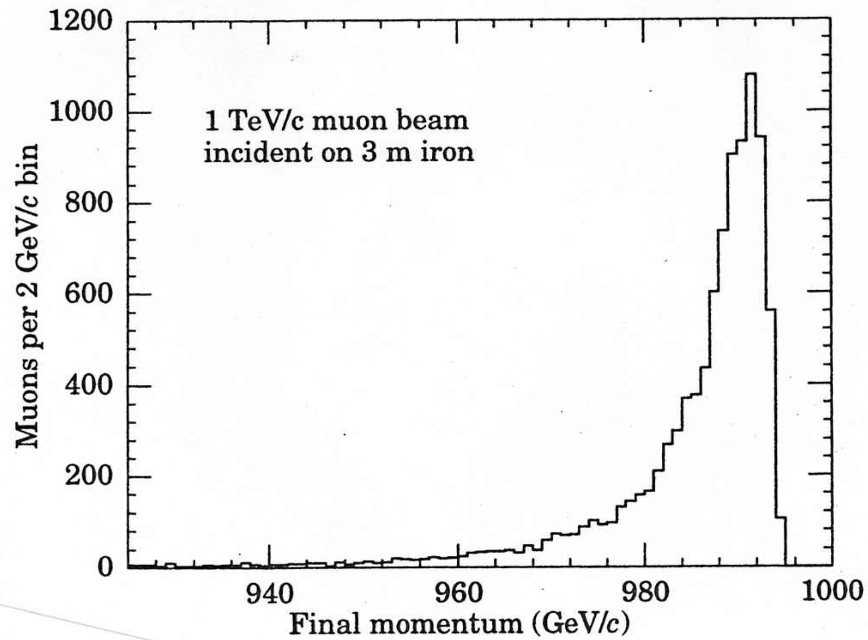
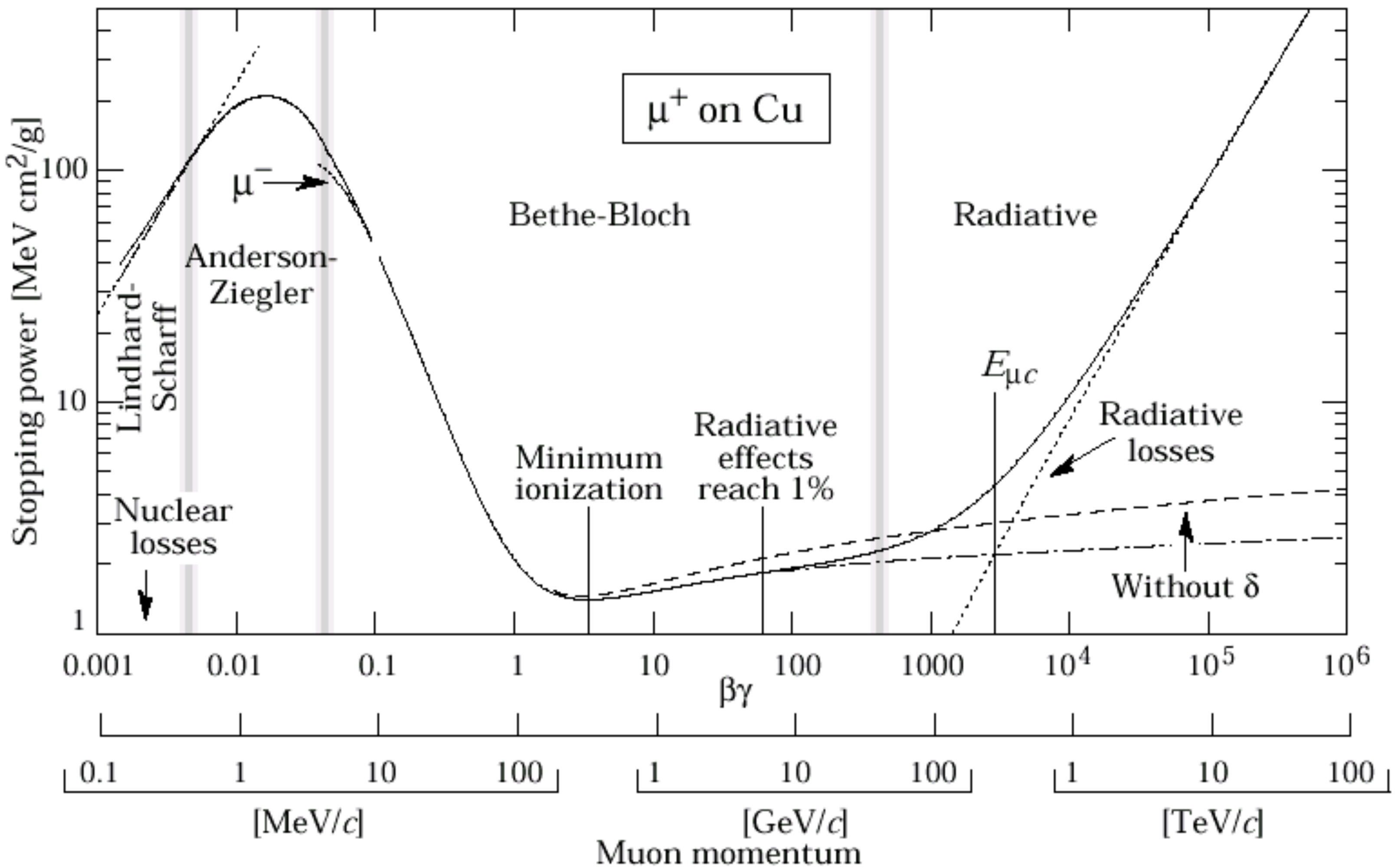


Figure 23.11: The average energy loss of a muon in hydrogen, iron, and uranium as a function of muon energy. Contributions to dE/dx in iron from ionization and the processes shown in Fig. 23.10 are also shown.



Muons in Cu



μ+ on Cu
Stopping power [MeV cm2/g]
Lindhard-Scharff
Anderson-Ziegler
μ−
Bethe-Bloch
Radiative
Eμc
Radiative losses
Minimum ionization
Radiative effects reach 1%
Nuclear losses
Without δ
0.001
0.01
0.1
1
10
100
1000
10^4
10^5
10^6
βγ
0.1
1
10
100
[MeV/c]
1
10
100
[GeV/c]
1
10
100
[TeV/c]
Muon momentum

Energy Loss Distribution

- So far have discussed $\left(\frac{dE}{dx}\right)_{MEAN}$
- In general energy loss for a given particle $\Delta E \neq (\Delta E)_{MEAN}$
- For a mono-energetic beam
    - distribution of energy losses

- Thick Absorber – Gaussian Energy Loss

- Thin Absorber – Possibility of low probability, high fractional energy transfers

LANDAU DISTRIBUTION

THE IONIZATION LOSS dE/dx IN BETHE-BLOCH IS MEAN ENERGY LOSS

THERE IS ACTUALLY A LONG TAIL TO HIGH ENERGY LOSSES — THEORY DUE TO LANDAU

FOR MATERIAL DEPTH L, MOST PROBABLE ENERGY LOSS $\langle \Delta E \rangle \sim \langle dE/dx \rangle L$

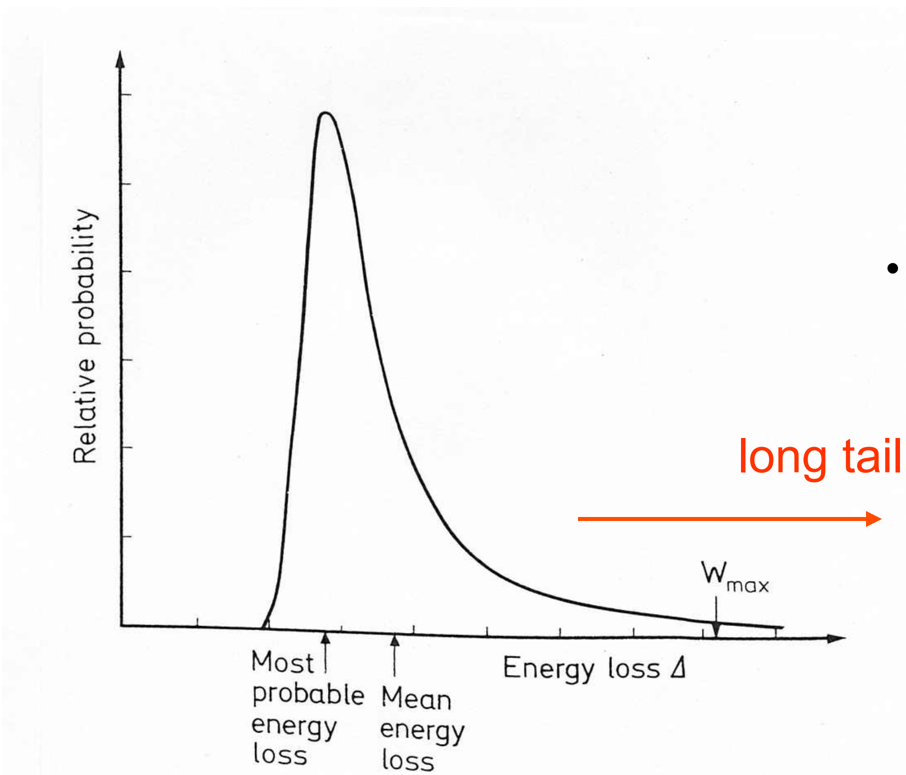
LANDAU DEVIATIONS $\Delta E - \langle \Delta E \rangle$

PROBABILITY DISTRIBUTION ABOUT MOST PROBABLE ENERGY LOSS $\longrightarrow$

$$P(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\lambda + e^{-\lambda}\right)\right\}$$

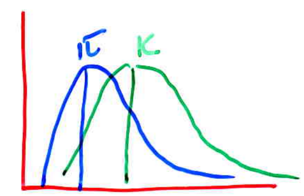
$$\lambda \equiv \frac{\Delta E - \langle \Delta E \rangle}{\langle \Delta E \rangle}$$

Typical Energy Loss in Thin Absorber



- Scintillator
- Wire Chamber Cell
- Si tracker wafer

- Practical Implications
  - Use of dE/dx for particle ident
  - Landau tails cause limitation in separation



- Position in tracking chamber
  - Landau tails smear resolution

- Separation of 1 from 2 particles in an ionization/scintillator counter
  - Landau tails smear ionization

- Various Calculations
- Landau – most commonly used
- Vavilov - "improved" Landau

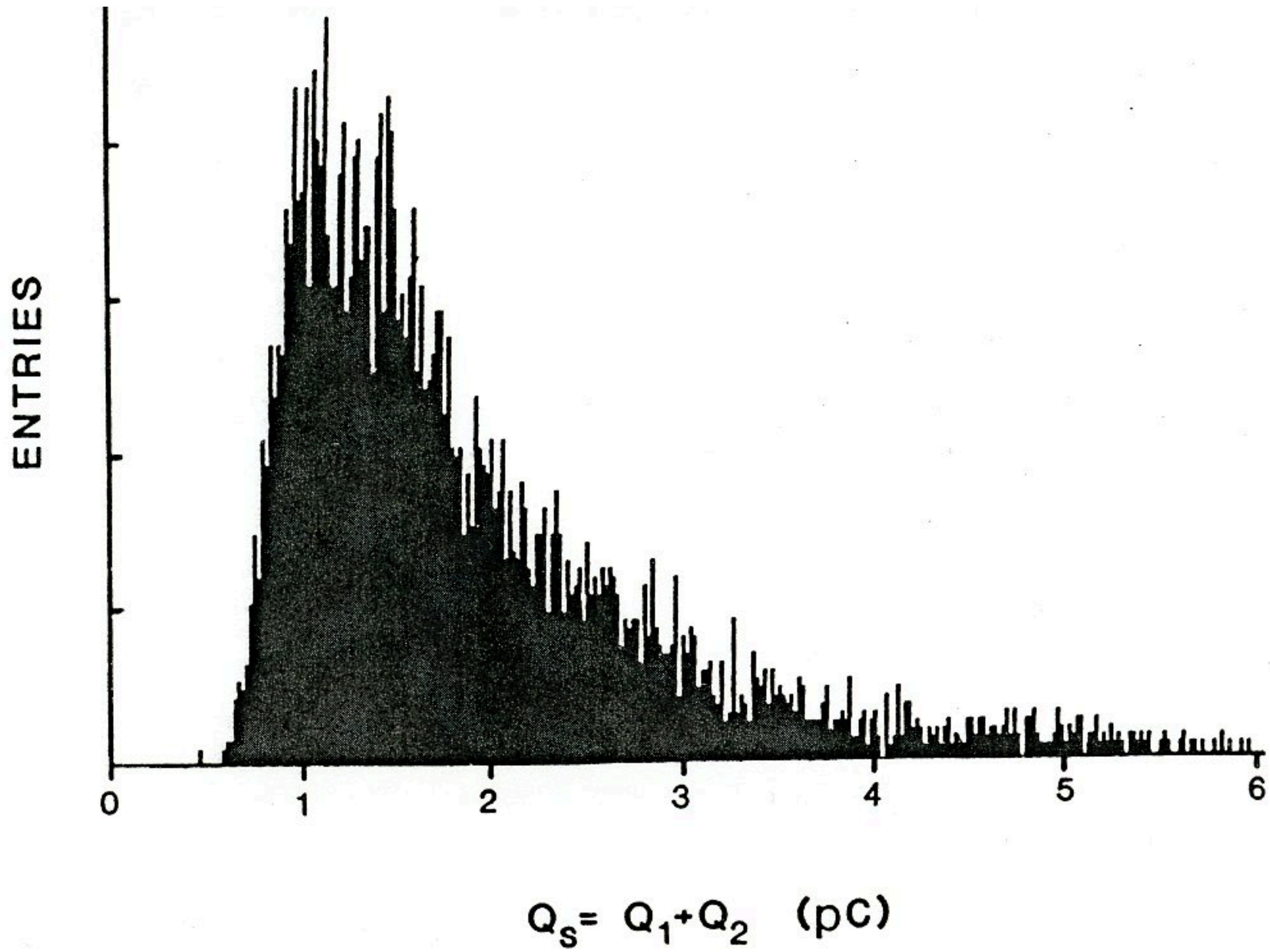
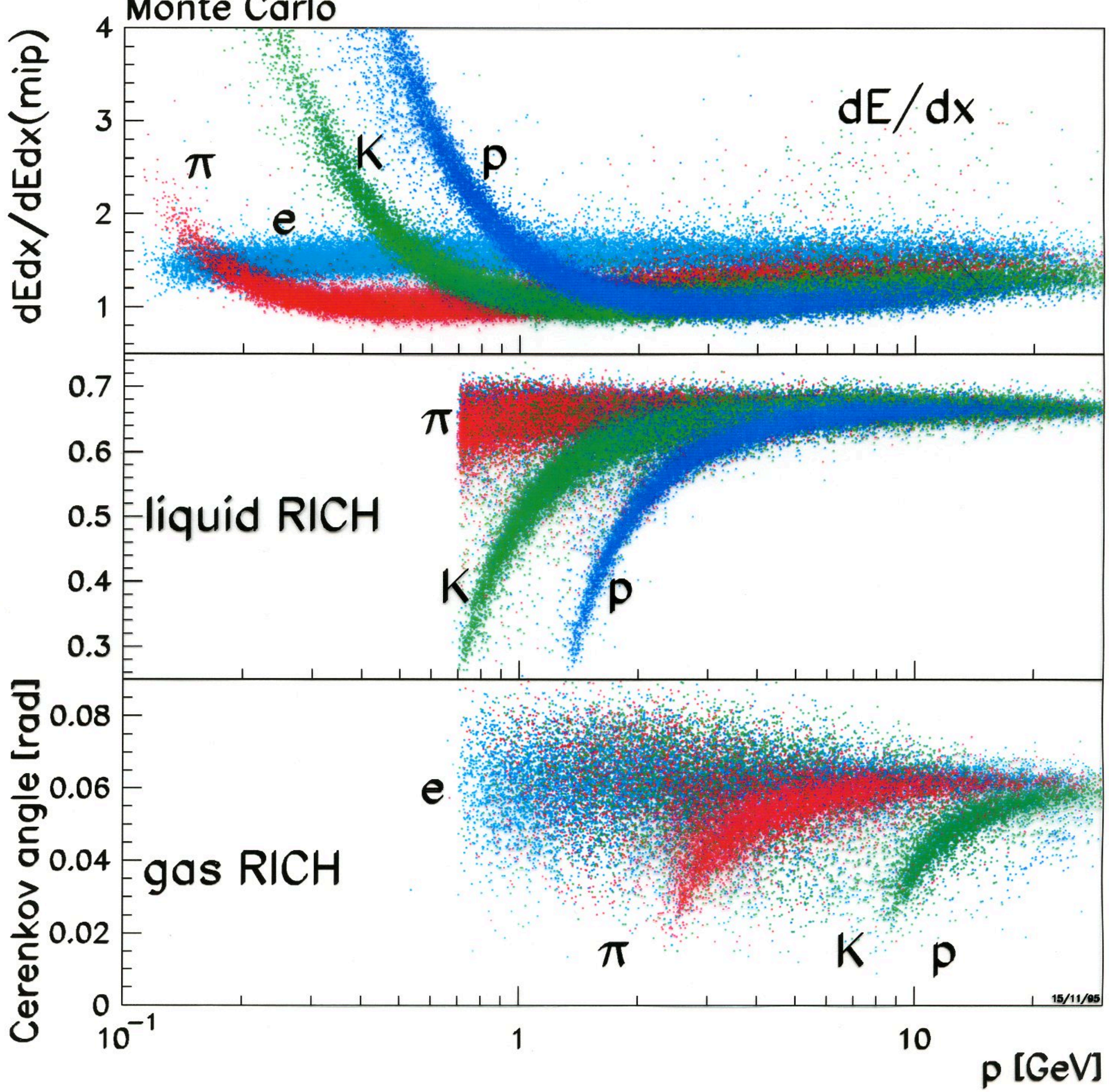


Fig. 6.9. Charge deposited on a wire chamber cathode by the passage of a MIP.

DELPHI particle ID



Cerenkov Radiation

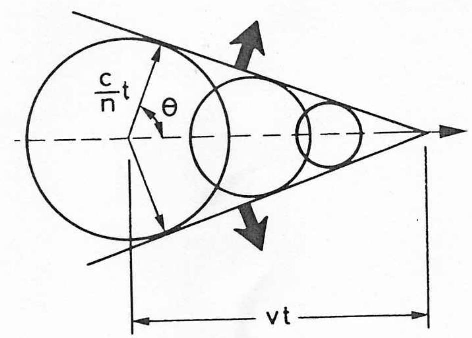
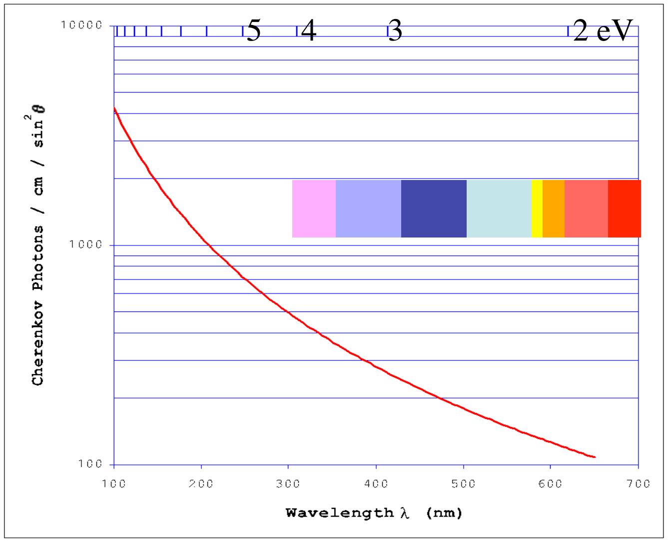


Fig. 2.9. Cherenkov radiation: an electromagnetic shock wave is formed when the particle travels faster than the speed of light in the same medium



$$\cos\theta = \frac{c/nt}{\beta ct} = \frac{1}{\beta n}$$

measure ($\cos\theta$) — *known* (β, n)

$$\frac{dN}{dx} = 2\pi z^2 \alpha \sin^2\theta \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda}$$

$$N[\lambda_1 \rightarrow \lambda_2] = 4.6 \cdot 10^6 \left[\tfrac{1}{\lambda_2(A)} - \tfrac{1}{\lambda_1(A)}\right] L(cm) \sin^2\theta$$

$$475 z^2 \sin^2\theta \text{ photons/cm}$$

350 nm to 550 nm

medium	n	θ_{max} (β=1)	N_{ph} (eV^{-1} cm^{-1})
air	1.000283	1.36	0.208
isobutane	1.00127	2.89	0.941
water	1.33	41.2	160.8
quartz	1.46	46.7	196.4

TRANSITION RADIATION

A CHARGED PARTICLE RADIATES WHEN IT CROSSES THE BOUNDARY BETWEEN MATERIALS WITH DIFFERENT DIELECTRIC CONSTANTS

DEPENDS ON PLASMA FREQUENCY $\omega_p = \sqrt{\frac{4\pi N_e e^2}{m}}$

$$\hbar\omega_p = \frac{r_0}{\alpha}\sqrt{4\pi N_e m c^2} \sim 30\,\mathrm{MeV}, \quad \text{DENSITY} = 1$$

TYPICAL ENERGY OF γ $= \gamma\hbar\omega_p/4$

AVERAGE # PHOTONS $E > \gamma\hbar\omega_p/10$

$$N_\gamma \approx 0.8\,\alpha Z^2 \approx 0.59\times10^{-2} Z^2$$

TOTAL ENERGY EMITTED

$$E = \frac{\alpha Z^2 \gamma\hbar\omega_p}{3}$$

WEAK RADIATION
ONLY SEE FOR STACK OF FOILS / BOUNDARIES

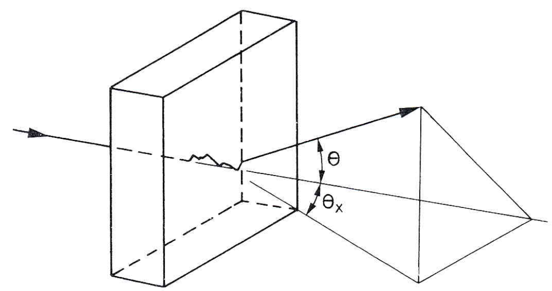
Multiple Coulomb Scattering

- Can be a very important limitation on detector angle/momentum resolution
- For Charged particles traversing a material (ignore radiation)
  - Inelastic collisions with electrons - ionization
  - elastic scattering from atomic nuclei

Rutherford scattering

$$\frac{d\sigma}{d\Omega} = z_1^2 z_2^2 r_e^2 \frac{(m_e c / p\beta)^2}{4\sin^4 \frac{\theta}{2}}$$

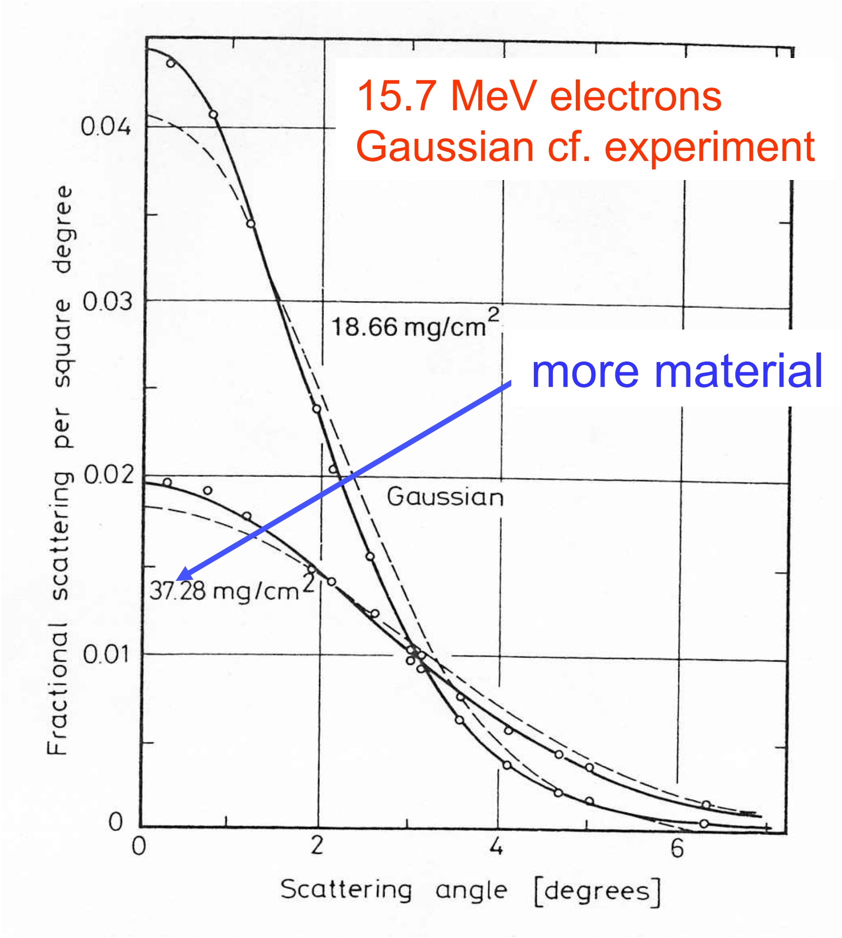
vast majority of scatters – small angle



- *θ is polar angle*
- *number of scatters > 20*
- *negligible energy loss*
- *Gaussian statistical treatment is usually ok*

MOLIÉRE
MORE SOPHISTICATED

Gaussian Multiple Scattering



probability of scattering through θ

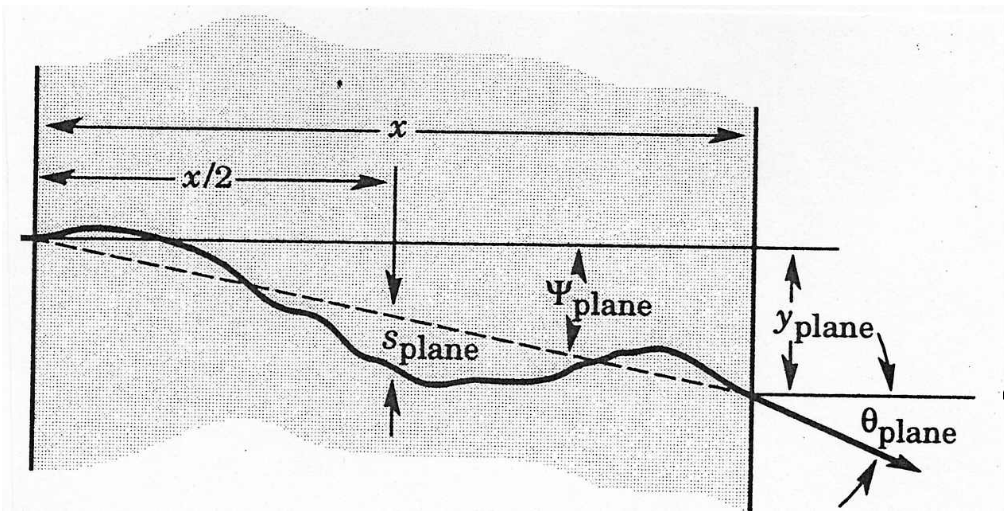
$$P(\theta) \approx \frac{2\theta}{\langle \theta^2 \rangle} \exp\left(-\frac{\theta^2}{\langle \theta^2 \rangle} \right) d\theta$$

$\sqrt{\langle \theta^2 \rangle}$ RMS scattering angle

Gaussian Multiple Scattering

For detectors usually interested in RMS scattering angle – projected on a plane

most detectors measure in a plane



$$\theta_0 = \theta_{PLANE}^{RMS} = \frac{1}{\sqrt{2}}\theta_{SPACE}^{RMS}$$

$$\theta_0 = \frac{13.6MeV}{\beta cp} z \sqrt{\frac{x}{\chi_0}}\left[1 + 0.038\ln\left(\frac{x}{\chi_0}\right)\right]$$

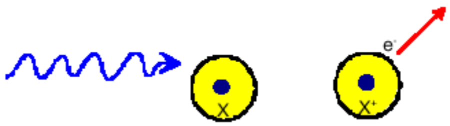
$$\Psi_{PLANE}^{RMS} = \frac{1}{\sqrt{3}}\theta_0$$

$$y_{PLANE}^{RMS} = \frac{x}{\sqrt{3}}\theta_0$$

$$S_{PLANE}^{RMS} = \frac{x}{4\sqrt{3}}\theta_0$$

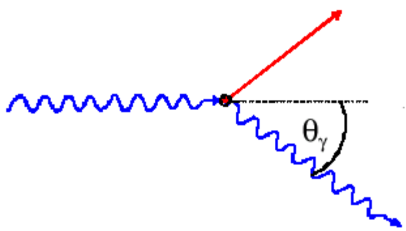
Energy Loss of Photons in Matter

Important for Electromagnetic Showers

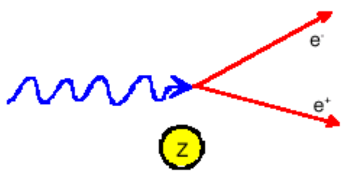


$$\gamma + \text{atom} \rightarrow \text{atom}^{+} + e^{-}$$

- Photoelectric Effect



- Compton Scattering



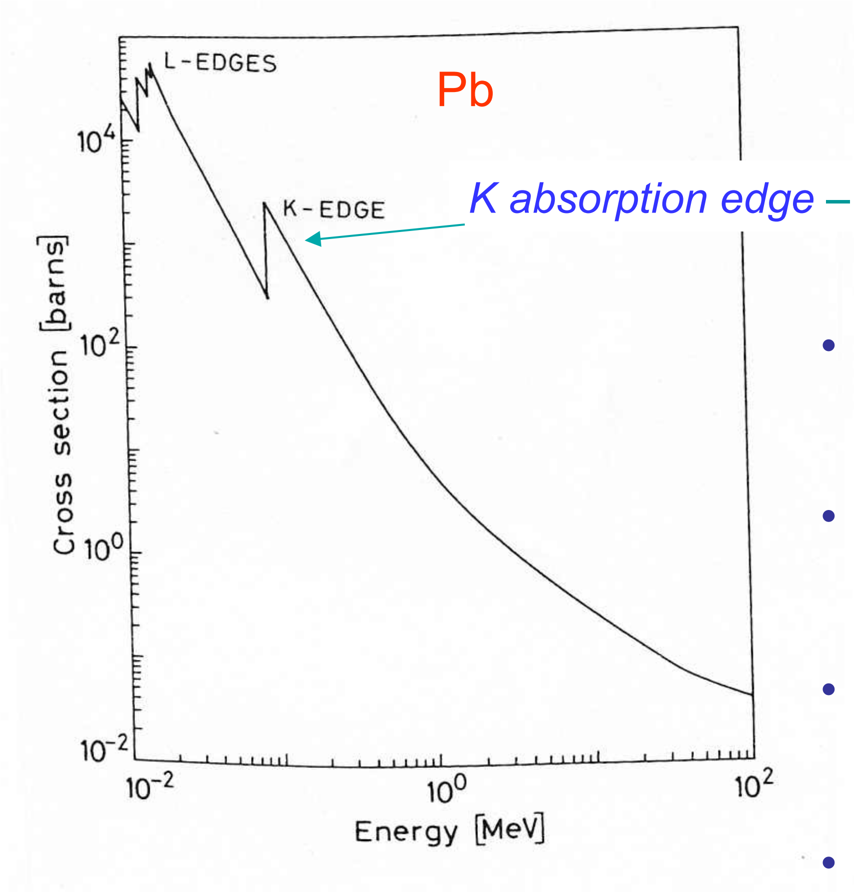
- Pair Production – completely dominant above a few MeV

- For a beam of γ or survival probability of a single γ

$$I(x) = I_0 e^{-\mu x}$$

absorption coefficient

Photoelectric Effect



- Atomic electron absorbs photon and is ejected
- Cross section for absorption increases with decreasing energy until $E_e = h\nu - Binding\ Energy$
- Then drops because not enough energy to eject K-shell electrons
- Dependence on material : $Z^{4 \to 5}$

PHOTO ELECTRIC EFFECT IS HARD TO TREAT ACCURATELY

→ DEPENDS ON KNOWLEDGE OF ATOMIC ELECTRON WAVE FUNCTIONS

CLOSE TO K-EDGE

$$\text{CROSS SECTION} = \frac{6.3 \times 10^{-18}}{Z^2} \left(\frac{\nu_K}{\nu} \right)^{8/3}$$

(ν_K ← E_K; ν ← E)

DEPENDENCE ON Z

$$\sim Z^{4-5} \text{ AT HIGH } Z$$

Compton Scattering

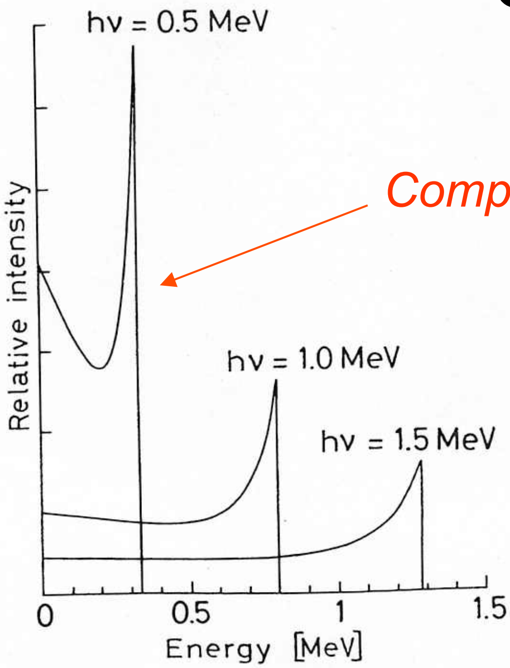
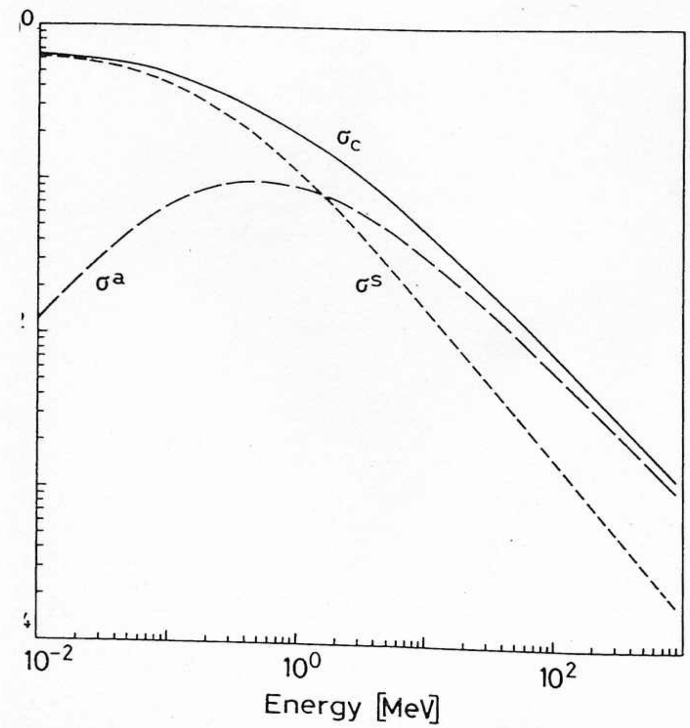


Fig. 2.24. Energy distribution of Compton recoil electrons. The sharp drop at the maximum recoil energy is known as the *Compton edge*

Compton Edge – Detector Calibration

$$T_{MAX} = h\nu\left(\frac{2\gamma}{1+2\gamma}\right)$$

$$\gamma = \frac{h\nu}{m_e c^2}$$



Pair Production

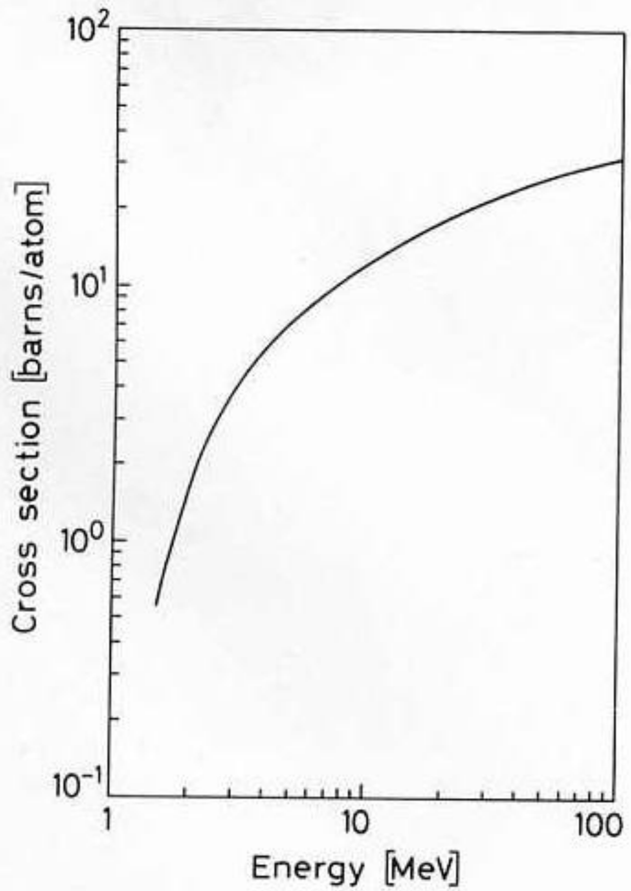


Fig. 2.25. Pair production cross section in lead

- Central to electromagnetic showers
- Can only occur in field of nucleus
- Rises with energy cf. Compton and PE
- Same Feynman diagram as Brems

Mean Free Path

$$\frac{1}{\lambda_{PAIR}} = \frac{7}{9} 4Z(Z+1) N r_e^2 \alpha \left[\ln\left(\frac{183}{Z^{1/3}}\right) - f(z) \right]$$

$$\sim Z^2$$

$$\lambda_{PAIR} \approx \frac{9}{7} \chi_0$$ *Closely related to Radiation Length*

Photon Absorption as Function of Energy