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Optimizing the Top Mass Measurement

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Abstract

We present a method for optimizing the top mass measurement in the lepton + jets mode. This method makes use of the additional information contained in the pretagged sample as well as the different signal-to-background ratio for SVX and SLT tagged events. When we apply this method to the data we obtain a top mass of $176.8 \pm 4.4(\mathit{stat.}) \pm 4.8(\mathit{sys.})\ GeV/c^2$.

1 Introduction

The standard top mass measurement for the lepton + jets channel, in 110 pb^{-1} of data, is $175 \pm 5.7(stat) \pm 5.9(syst)\ GeV/c^2$. This method uses a sample of events that have 4 or more jets with at least one of these jets b-tagged [1, 2]. This method neglects information that is in the untagged sample and it also does not use the differences in the signal-to-background ratio for SVX and SLT tagged samples. We would like to make optimal use of our present data sample. In this note, we investigate changes to the top mass measurement when we incorporate information from the untagged sample and distinguish between the types of b-tags. We have done this *a priori*, using Monte Carlo before looking at the data. We attempt to optimize the mass measurement by first looking for the method which gives the greatest reduction in the statistical uncertainty. Finally, we have applied this method to the data and evaluated the systematic uncertainties associated with it.

2 Finding the Optimal Method in Monte Carlo

Table 1 gives a list of possible sub-samples of the pretagged mass data sample. It also shows the number of observed events in the data and the number of expected background and top events for these subsamples [3]. Most of the background numbers in Table 1 come from Brian Harral's background note[4]. The background numbers for the SVX single tagged and the SLT tagged (no SVX tags) subsamples were calculated by scaling the background numbers from the SVX tagged and SLT tagged subsamples, respectively. The background number for the SVX double tagged subsample comes from a calculation done by Rob Roser. To reduce the amount of background in the top sample we also tried changing the kinematic cuts. In particular, we changed the E_T and η cuts on the fourth jet to match the cuts of the three highest E_T jets; $E_T > 15\ GeV$ and $\eta < 2.0$. The efficiencies of the tighter kinematic requirements on the fourth jet are 31% on background and 68% on top. Table 2 shows the number of expected events for background and top when the four mass jets all have $E_T > 15\ GeV$ and $\eta < 2.0$. The

Subsample	$N_{observed}$	N_{back}	N_{top}
Pretag	153	$98\ ^{+11}_{-11}$	55
No tag	119	$91.6\ ^{+10}_{-10}$	27.4
SVX tag	20	$1.6\ ^{+0.65}_{-0.43}$	18.4
SVX single tag	15	$1.5\ ^{+0.6}_{-0.6}$	13.5
SVX double tag	5	$0.14\ ^{+0.04}_{-0.04}$	4.9
SLT tag	21	$5.0\ ^{+1.56}_{-1.07}$	16
SLT tag (No SVX)	14	$4.8\ ^{+1.5}_{-1.5}$	9.2
SVX and SLT tag	7	$0.15\ ^{+0.09}_{-0.06}$	6.9
SVX or SLT tag (Standard Method)	34	$6.4\ ^{+2.1}_{-1.4}$	27.6

Table 1: The number of observed top events in 110 pb^{-1} of data and the expected number of background and top events for the various subsamples.

expectation numbers in Table 2 are obtained by scaling the number of background and top in Table 1 by their measured efficiencies.

We run 500 pseudo-experiments for each of the subsamples listed in Table 1. For each pseudo-experiment of a particular subsample, we input the number of observed events and the expected background. The pseudo-experiment is compared to templates of various top masses which were generated using Herwig. The Minuit mass fitter[5, 6] returns a log likelihood value for each of the template masses. The log likelihood vs. top mass is fit to a parabola [7] to obtain a top mass value and its statistical uncertainty [8]. Table 3 gives the median mass and median statistical uncertainty from the 500 pseudo-experiments generated for various subsamples.

The last line of Table 3 shows the mass and statistical uncertainty for the untagged subsample where the four highest E_T jets must have $E_T > 15\ GeV$ and $\eta < 2$. We found that raising the E_T cut on the fourth jet only improved the statistical uncertainty for the untagged and pretagged cases. In the tagged subsamples the tighter kinematic cuts reduce the number of events in the sample appreciably and the statistical uncertainty for these samples increases. The more restrictive kinematic cuts help reduce the amount of background in the untagged sample without severely reducing the statistics. For the remainder of this analysis we will apply the more restrictive kinematic cuts only to the untagged sample.

Table 1 shows that the amount of background is significantly less in the SVX tagged sample than the SLT tagged sample and that the tagging efficiency is higher for SVX than SLT. Based on the better signal-to-background ratio of the SVX tagged events, we considered using only the SVX tagged sample and discarding the SLT tagged sample. From Table 3 we notice that we cannot improve the mass measurement by using the SVX tagged sample or the SVX double tagged sample alone. We reduce the amount of background by using only SVX tagged samples but we also lose a sufficient number of events to cause the statistical uncertainty to increase instead of decrease. In fact the subsample with the smallest expected statistical uncertainty is our current method of using SVX or SLT tags. However, by using this sample we are not using all of our available information. We are not making use of the information stored in the larger pretagged sample and we are not taking into account the differences between the signal-to-background ratio for the SVX and SLT tagged samples.

Subsample	N_{back} Expected	N_{top} Expected
Pretag	$31\ ^{+3.5}_{-3.5}$	38
No tag	$29.3\ ^{+3.2}_{-3.2}$	18.9
SVX tag	$0.5\ ^{+0.21}_{-0.14}$	12.7
SVX single tag	$0.5\ ^{+0.2}_{-0.2}$	9.2
SVX double tag	$0.04\ ^{+0.01}_{-0.01}$	3.4
SLT tag	$1.6\ ^{+0.5}_{-0.3}$	11
SLT tag (No SVX)	$1.5\ ^{+0.5}_{-0.5}$	6.3
SVX and SLT tag	$0.05\ ^{+0.03}_{-0.02}$	4.7
SVX or SLT tag (Standard Method)	$2.1\ ^{+0.7}_{-0.5}$	19

Table 2: The expected number of background and top events for the various categories when all 4 jets used in the mass fitter have $E_T > 15\ GeV$ and $\eta < 2$.

In order to use all of the available information, we can divide the pretagged sample up into non-overlapping (exclusive) subsamples. The subsamples are therefore uncorrelated and their likelihoods can then be multiplied together. As an example, we could combine the SVX tagged sample, the SLT tagged sample[9] and the not tagged sample. We take the 500 pseudo-experiments generated for each of these subsamples and multiply their likelihood values together for each pseudo-experiment. We fit each of the 500 combined likelihoods to obtain a mass and statistical uncertainty. We compare the median mass and median statistical uncertainty for different combinations of uncorrelated subsamples to determine which combination is the best.

Table 4 shows the masses and statistical uncertainties for several different combinations of uncorrelated subsamples. The combination which gives the smallest expected statistical uncertainty uses the following four subsamples:

- SVX single tagged
- SVX double tagged
- SLT tagged (No SVX)
- Not tagged with the tighter kinematic requirement that all 4 jets have $E_T > 15\ GeV$ and $\eta < 2$.

Using this combination of subsamples and running 500 pseudo-experiments of Herwig top 175, we obtain a top mass of $175.3 \pm 5.4(stat)\ GeV/c^2$. From Monte Carlo, the standard top mass analysis, using an SVX or SLT tag, gives a top mass of $175.1 \pm 6.4(stat)\ GeV/c^2$. Comparing the standard method to the new optimized method we notice that the statistical uncertainty on the top mass has been significantly reduced. This reduction in the statistical uncertainty is equivalent to increasing the size of our current SVX or SLT tagged data sample by approximately 40%. Figure 1 shows the mass distribution and statistical error for the pseudo-experiments using the optimized method.

Subsample	Median Mass GeV/c^2	Stat. Uncertainty GeV/c^2
Pretag	175.3	7.4
No tag	173.8	12.9
SVX tag	176.2	7.2
SVX single tag	175.7	8.7
SVX double tag	175.4	9.7
SLT tag	177.4	9.6
SLT tag (No SVX)	175.3	12.1
SVX and SLT tag	176.2	11.1
SVX or SLT tag (Standard Method)	175.1	6.4
No tag ($E_t > 15$)	174.3	11.6

Table 3: The median mass and median statistical uncertainty from 500 pseudo-experiments for various subsamples.

3 Applying the Optimal Method to the Data

After finding the optimal method in the Monte Carlo we can now apply it to our full 110 pb^{-1} data sample. Figure 2 shows the top mass distribution for the not-tagged, SVX single tagged, SVX double tagged, and SLT tagged samples, along with the best fit background and top mass distributions. Also shown on each plot is the parabolic likelihood fit. Table 5 shows the mass and its statistical uncertainty in the data [7] for the subsamples which are used in the optimal method. When we combine the SVX single tagged, SVX double tagged, and SLT tagged sample with the not tagged sample, where the fourth jet must have $\mathrm{E_T} > 15\ GeV$ and $\eta < 2$, the top mass is $176.8 \pm 4.4(stat) GeV/c^2$. Figure 3 shows the mass distribution from the data overlayed with the background and background plus top Monte Carlo ($M_{top} = 175$) distributions. Figure 4 shows the mass distribution for all samples combined and the combined likelihood fit for the data.

3.1 Comments on the Statistical Uncertainty

In the previous version of the note the top mass was quoted to be $176.8 \pm 3.7(stat)\ GeV/c^2$. To be consistent with the standard lepton + jets analysis we were asked to use a smoothing technique on the Monte Carlo templates. Using the smoothed templates our top mass measurement is $176.8 \pm 4.04(stat)\ GeV/c^2$. There is a large uncertainty on the statistical uncertainty, $\pm 0.36\ GeV/c^2$, so we quote a final statistical uncertainty of $\pm 4.4\ GeV/c^2$. From figure 1 we find that 8.2 ± 0.9 % of pseudo-experiments using smoothed templates have statistical uncertainties less than 4.4 GeV/c^2.

We have done a number of checks to make sure that the statistical uncertainty returned from the fitter is accurate. Figure 5 shows the pull distribution from pseudo-experiments. If the mass distribution was gaussian we would expect the pull distribution to have a mean of zero and a width of one. Figure 5 has a mean of zero and a width of 0.87 which indicates that we are overestimating the statistical uncertainty. We have also studied the Δ mass vs. the returned

Combined Subsamples	Median Mass GeV/c^2	Stat. Uncertainty GeV/c^2
(SVX or SLT) + No Tag	175.9	5.9
(SVX or SLT) + No Tag ($E_t > 15$)	176.1	5.9
SVX + SLT (No SVX) + No Tag	175.8	5.9
SVX + SLT (No SVX) + No Tag ($E_t > 15$)	176.1	5.8
SVX single + SVX double + SLT (No SVX) + No Tag	175.8	5.5
SVX single + SVX double + SLT (No SVX) + No Tag ($E_t > 15$)	175.3	5.4

Table 4: The median mass and median statistical uncertainty from 500 pseudo-experiments for various subsamples.

Subsample	Fit Mass GeV/c^2	Stat. Uncertainty GeV/c^2
SVX or SLT tag (Standard Method)	175.6	5.7
No tag ($E_t > 15$)	180.9	6.4
SVX single tag	176.3	8.2
SVX double tag	174.3	7.9
SLT tag (No SVX)	140.0	24.1
Optimized Method	176.8	4.4

Table 5: The fit mass and statistical uncertainty for the various subsamples in $110pb^{-1}$ of data.

statistical uncertainty. Figure 6 shows that the statistical uncertainty which is returned by the mass fitter is indicative of the width of the mass distribution. The x-axis is the statistical uncertainty returned from the mass fitter. The y-axis is the width of the mass distribution for events which have statistical uncertainties in the range of the x-error bar. We ran 1000 Monte Carlo pseudo-experiments and then divided the statistical uncertainty distribution up into roughly equal bins. We plotted the difference between the output fitted mass and the input mass (fitted mass - 175) for all events which fell in a particular statistical uncertainty bin. We fit this to a gaussian to get the width (σ) of the Δ mass distribution. If the statistical uncertainty returned by the mass fitter is correct it should be equal to the width of the mass distribution and should lie along the line which has a slope equal to one. Points which low below the line of slope one indicate that the statistical uncertainty from the fitter is an overestimate. We also combined the 4 subsamples in an average error weighted manner (as described by the Particle Data Book) and obtained a mass measurement of 176.6 $\pm$ 4.2 GeV/c^2.

Note: Some of the systematic uncertainties were calculated using templates which were not smoothed so some of the following tables and plots show a statistical uncertainty of $\pm 3.7 GeV/c^2$, instead of the blessed value of $\pm 4.4 GeV/c^2$.

4 Systematic Uncertainties

Table 6 contains the types of systematic uncertainties and their values for the standard lepton + jets mass analysis. We have evaluated each of these sources of systematic uncertainty for our analysis, using similar or identical methods. In addition we have re-evaluated the systematics for the standard analysis. They are found in Table 14. The following sections explain the calculation of each systematic uncertainty.

Systematic	Standard $\mathrm{GeV/c^2}$
Jet E_T Scale	3.1
Soft Gluon Effects	1.9
Different Generators	0.9
Hard Gluon Effects	3.6
Fit Configuration	2.5
b-tagging Bias	2.3
Background Spectrum	1.6
Likelihood Method	2.0
Monte Carlo Statistics	2.3
Total	7.1

Table 6: The blessed systematics for the standard lepton + jets mass analysis.

4.1 Jet E_T Shift Systematics

The systematic uncertainties, jet E_T scale and soft gluon radiation, are obtained by shifting the jet energies and measuring the shift in the top mass. For our analysis we have combined these two systematics into an overall jet E_T shift systematic.

The jet E_T shift systematic contains many different jet shifts. We apply a jet shift for each of the following effects:

- Absolute Corrections - which includes the following:
  - Behrends curve
  - Calorimeter Stability
  - Underlying Event
- Relative Correction
- QFL Shift
- Soft Gluon Radiation Annulus Study
- Energy Outside Cone of 1.0

Previous notes [10, 11] document the procedures used to evaluate the jet E_T scale and soft gluon systematics listed in the blessed systematics table for the standard lepton + jets analysis, see table 6. We give a brief description of the how we calculated the E_T shift for each of the above systematics.

4.1.1 Absolute Corrections

The absolute corrections actually consist of three parts: the Behrends curve, calorimeter stability and underlying event. For the Behrends curve correction we use the same parameterization that was used for the standard analysis in note 3501. The Behrends curve was parameterized as the following:

$$f_b = 0.0245 + 0.0550/P_T$$

The correction for calorimeter stability is taken to be 2% and the correction for underlying event is 100 MeV.

4.1.2 Relative Correction

For the relative correction we obtain the value for the relative correction, f_{rel}, from JTC96S and then shift the jet by 2% of $(f_{rel} - 1)$.

4.1.3 QFL Shift

We apply a jet shift to take into account the energy scale shift in QFL between Run 0 and Run 1B. We apply a 2% shift. Originally this shift was only applied in the upward direction. At the preblessing we were requested to apply the QFL shift in both directions.

4.1.4 Soft Gluon Annulus Study

The soft gluon radiation systematic was studied in detail by L.Galtieri et al. and S.B. Kim et al. This systematic is used to explain the difference in the way that data and Monte Carlo model the energy between cones of 0.4 and 1.0. We use the parameterization used by the LBL group of

$$f_{soft} = e^{(2.4666 - 0.0736 \times P_T)} + 1.4379$$

4.1.5 Energy Outside a Cone of 1.0

We apply an additional shift of 1 GeV to take into account any energy which may be beyond a cone of 1.0.

In the standard lepton + jets analysis a separate jet shift was applied for each of the above effects. The mass shifts for each of these where then added in quadrature. We have written an analysis control module which can implement these shifts separately (as was done previously) or can implement all the shifts at once. To do this we calculate the size of jet shift needed for each of the above effects and add them in quadrature. We then apply this total jet shift in both the positive and negative directions. The shift is applied to the raw 4-vector of the jet taken from the JETS bank. Once the jet energies have been shifted the $\not\!E_T$ of the event is recalculated.

We checked that adding all the jet E_T shifts in quadrature and obtaining a single mass shift was equivalent to applying separate jet E_T shifts and adding the mass shifts in quadrature. Table 7 shows the downward jet E_T shift systematic uncertainties for the standard mass analysis from CDF note 3501 by Galtieri and Lys. Also shown are the values obtained using our analysis control module. The last line is the systematic uncertainty we obtain when we do all the jet shifts at once. There is good agreement between this and the line above it which is all of the jet shifts done separately and the mass shifts added in quadrature. We see that the two values are equivalent within their uncertainties. Table 8 shows the downward jet E_T shift systematic uncertainties for the optimization method. Once again the last line of the table is the systematic uncertainty when all jet shifts are applied together. Tables 7 and 8 do not include the downward shift due to the QFL shift.

Systematic	CDF3501 Values GeV/c^2	Our Values GeV/c^2
Annulus Study	1.1 ± 0.6	1.3 ± 0.4
Outside Cone 1.0	1.3 ± 0.6	1.4 ± 0.4
Total - Soft Gluon	1.7 ± 0.8	1.9 ± 0.6
Absolute	3.0 ± 0.3	2.3 ± 0.4
Relative	0.1 ± 0.3	0.5 ± 0.4
Total - Jet E_T Scale	3.0 ± 0.4	2.4 ± 0.6
Soft Gluon + Jet Scale	3.4 ± 0.9	3.1 ± 0.8
All Shifts Done Together	–	2.7 ± 0.4

Table 7: The values for the jet E_T shift systematics for the standard analysis from CDFNOTE 3501 compared to the values we get using our module for the standard analysis. The bottom line of the table is the systematic uncertainty we get when we added all the jet shifts in quadrature and apply a single overall jet shift. The numbers shown are for a downward shift only and do not include the QFL shift.

When we combine all the jet E_T shifts together and a apply a single, overall upward and downward shift to the jet E_T's we get the following systematic uncertainty for the optimization method:

$$^{+4.1}_{-3.0}\ GeV/c^2$$

When we use this same method on the standard SVX or SLT tagged mass sample we get a systematic uncertainty due to jet shifts of:

$$^{+4.0}_{-3.5}\ GeV/c^2$$

The systematic uncertainties shown above do include the QFL shift in both the upward and downward directions. The jet E_T shift systematic uncertainties are symmetrized in Table 14.

4.2 Different Generators

To study the effects of different Monte Carlo generators we generate our pseudo-experiments from Isajet Monte Carlo instead of Herwig. We compare 500 Isajet pseudo-experiments to the

Systematic	Shift Down GeV/c^2
Annulus Study	1.0 ± 0.3
Outside Cone 1.0	1.1 ± 0.3
Total - Soft Gluon	1.5 ± 0.4
Absolute	1.2 ± 0.3
Relative	0.4 ± 0.3
Total - Jet E_T Scale	1.3 ± 0.4
Soft Gluon + Jet Scale	2.0 ± 0.6
All Shifts Done Together	2.5 ± 0.3

Table 8: The values of the jet E_T shift systematics for the optimization method. The bottom line is the systematic uncertainty when all jet shifts were done together. The values shown are for the shifted downward case only and do not include the QFL shift.

standard Herwig templates. We obtain the systematic uncertainty by taking the difference in the median masses of the two distributions, the mass distribution for Isajet pseudo-experiments with Herwig templates and the mass distribution for Herwig pseudo-experiments with Herwig templates. The shift in the median masses is 0.3 ± 1.1 GeV/c^2. In the standard mass analysis the systematic uncertainty due to different generators is 0.6 ± 1.0 GeV/c^2. Due to the large uncertainties on these values we quote the different generator systematic as 1.4 GeV/c^2 and 1.6 GeV/c^2 for the optimized and standard methods, respectively.

4.3 Hard Gluon Radiation

We repeated the procedure which was used to measure the hard gluon systematic for the standard lepton + jets analysis [12]. We measured the fraction of gluon events in each of the subsamples and then generated new subsamples which had $\pm$ 30% of the measured gluon fraction. We combined the likelihoods for all of the subsamples which had the 30% higher gluon percentages and got a combined mass and statistical uncertainty. We repeated this for the subsamples which had the 30% lower gluon percentages. Table 9 shows the median mass and median statistical uncertainty for the optimized method with varying amount of gluon radiation. The statistical uncertainty has two components which are added in quadrature. The size of the mass shift from the standard case to the higher gluon percentage case and the value needed to be added in quadrature with the statistical uncertainty from the standard case to get the larger statistical uncertainty for the higher gluon percentage case. The hard gluon systematic for the optimized case is 2.2 GeV/c^2. In the standard mass analysis the hard gluon systematic is 3.6 GeV/c^2. The large decrease in the size of the hard gluon systematic was not expected but can be explained by the method in which this systematic is measured. When this systematic was measured for the standard mass analysis we did not notice a sizeable mass shift but we did see a large change in the width of the top mass distribution which is reflected in the size of the statistical uncertainty. We decided to quote a systematic which took into account the change in the statistical uncertainty on the top mass. When we combine the subsamples together we decrease the statistical uncertainty so the hard gluon systematic, which is based on the statistical uncertainty, must also decrease.

Gluon %	Median Mass GeV/c^2	Statistical Uncertainty GeV/c^2	Systematic Uncertainty GeV/c^2
Lower	176.3	5.0	2.0
Standard	175.3	5.4	–
Upper	175.0	5.8	2.1

Table 9: The median mass, median statistical uncertainty and systematic uncertainty for the optimized method when the gluon percentage is varied.

4.4 Kinematic and Likelihood Fitting Methods

In the past the table of systematics had two categories labeled:

- Fit Configuration
- Likelihood Method

The fit configuration systematic was originally assigned to take into account differences between the Minuit and Squaw mass fitters. Since then the mass fitters have been tuned to each other so that this definition for fit configuration is no longer appropriate. There has been much discussion on what these two systematics should entail. We have redefined the fit configuration systematic and combined it in quadrature with the likelihood method systematic. It is now called Kinematic and Likelihood Fitting Methods.

4.4.1 Fit Configuration

Traditionally the systematic uncertainty due to fit configuration was taken as the difference between the SQUAW and MINUIT fit results. We now have several different mass fitting techniques which use approximately the same data sample. These include:

- **Velev et al.** - Uses a different likelihood function which includes jet probability and jet charge information
- **Galtieri et al.** - Uses the SQUAW mass fitter and smooths the templates
- **Eddy et al.** - Uses the MINUIT mass fitter and smooths the templates in a slightly different way than Lina
- **Kirsten et al.** - Uses the MINUIT mass fitter and a different set of templates which are not smoothed (smoothing was used for the blessed optimized mass value)

These results are given in Table 10.

We estimate the systematic due to different fitting methods by using the largest difference between the above results. Using this method we obtain a systematic of: $\pm 1.4\ GeV/c^2$. It has

Analysis	Mass (GeV/c^2)
George Velev L**	174.2 $\pm$ 5.5
Squaw	175.6 $\pm$ 5.7
Minuit Nathan	174.4 $\pm$ 5.6
Minuit Kirsten	174.3 $\pm$ 5.8

Table 10: Mass and statistical uncertainty for various analyses of the lepton + jets data.

been noted that the current difference between SQUAW and MINUIT is most likely *not* due to differences in the kinematic fitting of each event. Rather the differences are most likely due to differences in the smoothing algorithms in the top mass templates. Thus we re-group this systematic with the likelihood systematic uncertainty.

4.4.2 Likelihood Method

In determining a top mass from the data, we separated the data into the four subsamples listed in Section 2 and constrained the background to be within errors of the calculated value. By fitting the mass distribution from data to the appropriate Herwig Monte Carlo templates, we determined, for each subsample, a log likelihood value for each of 12 top masses ranging from 140-210 GeV/c^2. Because the four subsamples are statistically independent, the *combined* log likelihood for each mass was determined by simply adding the log likelihoods for each of the individual subsamples. Using the point with the minimum combined log likelihood value and the three points to either side, a seven-bin fit to a parabola was performed on the plot of log likelihood values versus mass. The minimum of this parabola is the top mass and the change in mass corresponding to a change in 0.5 units of log likelihood is the statistical uncertainty.

In order to determine the likelihood method systematic, we considered other possible fitting methods. We tried the following alternative fitting procedures:

- leaving the background unconstrained
- fitting a parabola over a fixed range of top masses

For the first alternative, we use a fitting procedure identical to the one described above but we allow the fitter to choose the amount of background it prefers. Table 11 gives the mass and number of background events used in the fit when we did and did not constrain the background. The number of background events in the constrained case should be close to the calculated value of the expected number of background events found in the last column. The mass shift due to the change in the background constraint is 0.3 GeV/c^2. For the second choice, we again performed a fitting procedure identical to the one previously described but this time we required the parabolic fit to include nine likelihood points from 150-200 GeV/c^2. The effect of this fixed range fit was to shift the mass value by 0.4 GeV/c^2. To obtain the overall systematic due to the likelihood method we add these two effects in quadrature. The total systematic uncertainty is 0.5 GeV/c^2.

Subsample	Constrained		Unconstrained		Calculated
	Mass	N_{back}	Mass	N_{back}	N_{back}
Not Tagged	180.9 ± 6.4	27.9 ± 3.1	178.8 ± 5.5	10.0 ± 9.4	29.3 ± 3.2
SLT (No SVX)	140.0 ± 24.1	5.1 ± 1.4	140.0 ± 41.8	9.7 ± 3.1	4.8 ± 1.5
SVX Single	176.3 ± 8.2	1.3 ± 0.6	176.8 ± 7.3	0.01 ± 0.8	1.46 ± 0.6
SVX Double	174.3 ± 7.9	0.14 ± 0.04	174.5 ± 8.2	0.01 ± 0.8	0.14 ± 0.04

Table 11: Results of the constrained and unconstrained background likelihood fits for the subsamples used in the optimized method.

4.5 *b*-tagging Bias

The *b*-tagging bias systematic is meant to estimate the top mass uncertainty due to the uncertainty in:

- the efficiency vs. Et for SVX and SLT tagging
- the rate of tagging non-B jets in real top events
- the effect of the b-tagging E_T bias on the W+jets background

In the PRD, it is stated that the last effect causes a negligible change in the top mass. The first two sources led to a top mass systematic of 2.4 GeV/c^2.

In the past, the uncertainty in the SVX and SLT tagging turn-on curve was modeled in the following way. In GENP, real b's matched to found jets passing the kinematics criteria ($E_t > 15\ GeV, |\eta| < 2.0$) are called tagged and treated as such in the mass fitting routines. For SLT tags, real semileptonic b decays were selected. This changes the gradual turn-on above 15 GeV to a step function at 15 GeV. It has been suggested that we examine in greater detail our true uncertainty in the tagging efficiency instead of using this rather *ad hoc* approach. We have attempted to do this, first for SVX tagging, then for SLT tagging.

The SVX tagging efficiency is determined by Monte Carlo, then corrected by a scale factor, determined from data. Recently Unal and others (CDF3563) have determined the scale factor from CTC tracking studies. This scale factor is determined as a function of jet E_T, as shown in Figure 7. The uncertainties are statistical and systematic, where the latter are determined from reasonable variations in the tracking dependance on Q, the track quality variable. The first variation assumed no Q dependence and the second variation assumed a one standard deviation stronger Q dependence than the standard for the CTC tracking efficiency.

In all top analyses, the scale factor is assumed to be flat with E_T. We estimated the systematic uncertainty due to SVX b tagging by using the maximum deviation from flat allowed by Figure 7. This deviation was used to sculpt the standard Monte Carlo for SVX tagging. The resultant Monte Carlo $M_{top} = 175\ GeV/c^2$ pseudo experiments were then processed with the standard Minuit fitting routines. The deviation in the top mass from the nominal is given in Table 12.

The SLT efficiency is determined from data. The SLT muon selection is studied in detail with the very large $\psi \rightarrow \mu\mu$ sample. Systematic effects like how isolated the muon is have been studied and are flat with tagged jet E_T. SLT electron tags are studied with the high statistics $\gamma \rightarrow ee$ sample. Again, tagging rate vs. track isolation variables have been studied and residual

Sample Type	SVX Mass Shift	SLT Mass Shift	Combined
SLT or SVX	-0.2 $\pm$ 0.4	0.25	0.3
SVX Single	-0.5	0	0.5
SVX Double	+0.2	0	0.2
SLT	0	$\pm$2.2	2.2
Not Tagged	0	0	0
Combined	0.1	0.4	0.4

Table 12: Top Mass Shift for SVX and SLT b Tagging Bias. Shifts are given for the standard mass analysis (svx or slt), the 4 sub-samples, and the combined optimized analysis. All uncertainties are approximately 0.4 GeV/c^2.

uncertainties are negligible. Also small are the uncertainties in the had/Em cut efficiency vs. E_T.

One uncertainty in the SLT tagging which is relevant is the exact ratio of real-to-fake SLT tags in top events. This ratio is uncertain around the 10-20% level. In order to estimate the effect of this on the top mass, we generate top pseudo experiments which have 100% fake SLT tags in top events. We also generate pseudo experiments with 100% real SLT tags in top events. The distribution is more broad for fake SLT tags. Half the difference between these two extremes is clearly a conservative estimate of this systematic error, and it is given in Table 12.

We add the mass shift due to SVX tagging bias in quadrature with the SLT tagging bias to obtain the total b-tagging bias systematic uncertainty of 0.4 GeV/c^2 for the optimized analysis. We have also calculated the b-tagging bias systematic for the standard top mass analysis in much the same way. We changed the SVX tagging, leaving the SLT tagging alone, then changed the SLT tagging, leaving the SVX tagging alone. These two deviations from the standard result are then added in quadrature. The change in top mass due to the SVX scale factor change is 0.2 ± 0.4 GeV/c^2. The change in top mass due to changing the type of SLT tag from 100% real to 100% fake in the top Monte Carlo is 0.25 GeV/c^2. Thus the systematic uncertainty in the standard top mass analysis due to b-tagging bias is 0.3 GeV/c^2. These results are also found in Table 12 .

4.6 Background Spectrum

To measure the effect of different background shapes on the top mass we vary the Q^2 scale in Vecbos. The mass group decided previously that the standard Q^2 value to use in Vecbos would be P_T^2. We changed the Q^2 value in Vecbos to M_W^2 and generated new background templates. We then refit the data using the background templates with $Q^2 = M_W^2$. We take the shift in the mass value as the systematic uncertainty due to the background spectrum. Table 13 shows the mass values for the data using the two different Q^2 scales. We measure a systematic uncertainty due to the background spectrum of 0.7 GeV/c^2 in both the standard mass analysis and for the optimization method.

We use Vecbos to model the QCD $W + jets$ backgrounds. A small amount of non-W background is also expected. Previous studies done by Lina et al. [13] have shown that the non-W background shapes are similar to Vecbos. So we make the assumption that all backgrounds are adequately modeled by Vecbos background templates. We intend to revisit this study, with larger data samples, and look at the following distributions:

- Non-isolated lepton + 4 jets
- Lepton + 4 low E_T jets
- Z + 4 jets

Q^2 Value in Vecbos	Mass (GeV/c^2)
P_T^2 (Standard Value)	176.8 ± 4.4
M_W^2	176.1 ± 4.2

Table 13: The mass values for the data using different Q^2 scales to generate our Vecbos background templates.

4.7 Monte Carlo Statistics

The top mass is determined by comparing masses observed in the data to templates for top (at different masses) and background as determined from Monte Carlo. These templates have finite statistics, and thus there is some dependence on the fitted top mass.

To determine the effect of such finite statistics, we fit the data to *fluctuated* templates. To construct *fluctuated* templates, we vary the number of entries in each bin, using the standard template entry as the mean. This is done for all top mass templates, and the background template. The data is then fit using these templates, and a mass value is returned. This procedure is repeated 500 times for the standard analysis (using SVX or SLT tags). The systematic uncertainty we quote is then the sigma of the returned distribution, or 0.9 GeV/c^2.

The same method is used for the optimized analysis, in which all of the templates in each of the 4 sub-samples are *fluctuated.* The systematic uncertainty we quote is then the sigma of the returned distribution, or 0.8 GeV/c^2.

5 Total Systematics

For completeness we list the previous and re-evaluated systematic uncertainties for the standard method, and also the systematic uncertainties for optimized top mass analysis in Table 14.

6 Conclusion

We found a method to optimize the top mass measurement which makes use of more information than the standard analysis. We divided the pretagged sample up into non-overlapping subsamples and did a Monte Carlo study to find which combination gave the smallest statistical uncertainty. The optimized combination of subsamples used the SVX single tagged, SVX double tagged, SLT tagged and not tagged, with tighter kinematic cuts, samples. When we applied the optimal method to the data we obtained a top mass of

Systematic	Standard GeV/c^2	New Standard GeV/c^2	Optimized GeV/c^2
Soft Gluon + Jet E_T Scale	3.6	3.8	3.6
Different Generators	0.9	1.6	1.4
Hard Gluon Effects	3.6	3.6	2.2
Kinematic and Likelihood Fitting Methods	–	2.0	1.5
Fit Configuration (now included above)	2.5	–	–
Likelihood Method (now included above)	2.0	–	–
b-tagging Bias	2.3	0.3	0.4
Background Spectrum	1.6	0.7	0.7
Monte Carlo Statistics	2.3	0.9	0.8
Total	7.1	5.9	4.8

Table 14: The systematic uncertainties for the blessed and re-evaluated standard analysis and also the systematic uncertainties for optimized top mass analysis.

$$176.8 \pm 4.4(stat.) \pm 4.8(sys.)\ GeV/c^2.$$

We have also updated the systematics for the standard lepton + jets analysis so the new standard lepton + jets mass value is

$$175.6 \pm 5.7(stat.) \pm 5.9(sys.)\ GeV/c^2.$$

References

[1] F.Abe *et al.*, Phys. Rev. D 50, (1994) 2966

[2] F.Abe *et al.*, Phys. Rev. Lett., **74**, 14 (1995) 2626

[3] Our table does not include the category of double tags that is used by Shin Aota (see CDFNOTE 3604) and Rick Wilkinson (see CDFNOTE 3543). My double tagged sample consists of only events which have two SVX tags. Shin and Rick's double tag samples allow double b-tagged events which have SVX, SLT and jet probability tags. In the future I can include a subsample which uses their definition of double b-tagged events.

[4] B.Harral, CDF Note 3526 : Background Calculations for Mass-fit Event Selection (110 pb^{-1}), Version 1.0. I need to update my numbers to correspond with Brian's new background calculations in Version 2.0 of this note.

[5] B.Harral, CDF Note 2503 : Comments on Likelihood Method for Extracting M_{top}

[6] B.Harral, S.Kim, and S.Vejcik, CDF Note 3000 : Top Mass Determination Using the MINUIT Fitter

[7] We use a slightly different method for minimizing the log likelihood function than the Michigan and Penn groups. (For a description of their method see CDF note 2503.) This method was first suggested and used by the LBL group. We find the minimum likelihood point and then fit a parabola to 7 points, three points on each side of the minimum and the minimum. We found this method to be more robust when analyzing many Monte Carlo pseudo-experiments. This method will be also be applied to the data.

[8] The statistical uncertainty returned by the fitter is the change in the mass associated with a change in log likelihood of 0.5.

[9] If an event is SVX and SLT tagged it is included in the SVX tagged subsample and not in the SLT tagged sample. So the SLT tagged subsample has events with only SLT tags and no SVX tags.

[10] L.Galtieri, J.Lys, CDF Note 3501 : Systematics on Top Mass from Jet P_T Uncertainties

[11] S.B.Kim, S.Vejcik, CDF Notes 3503 and 3504, Soft Gluon Radiation Effects on the Jet $\mathrm{E_T}$ Scale and Jet $\mathrm{E_T}$ Scale from Momentum Balancing in $Z + 1$ jet and Photon+1 jet

[12] K.Tollefson, R.Hughes, P.Tipton CDF Note 3511 : Effects of Hard Gluon Radiation on the Top Mass

[13] L.Galtieri, B.Harral, S.B.Kim, J.Lys, S. Vejcik, and W.M.Yao, CDF Note 3026 : Summary of mass fits with the SQUAW and MINUIT Fitters

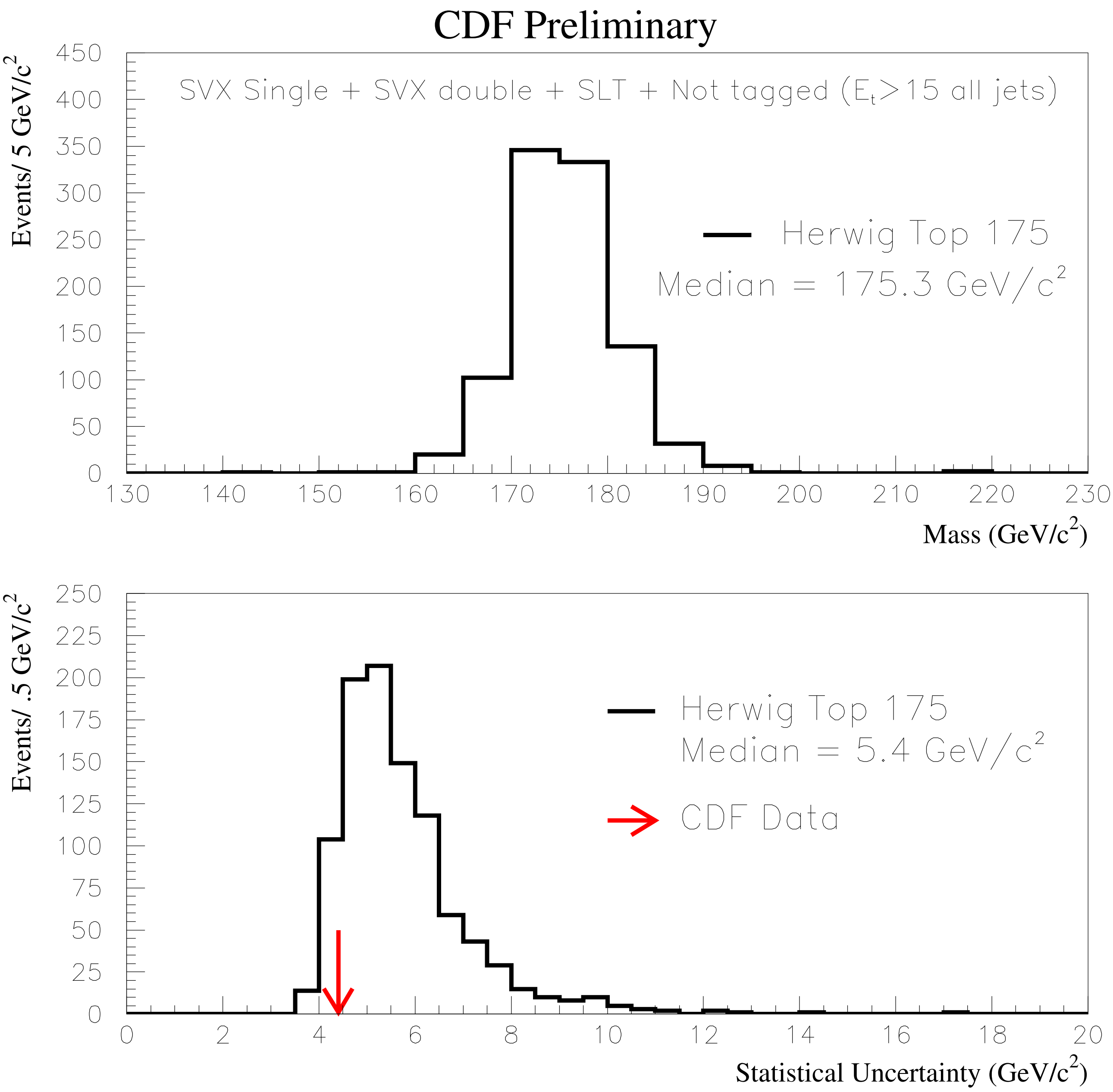

Figure 1: The top mass distribution and statistical error for the pseudo-experiments using the optimized method. Also shown is the result from the data.

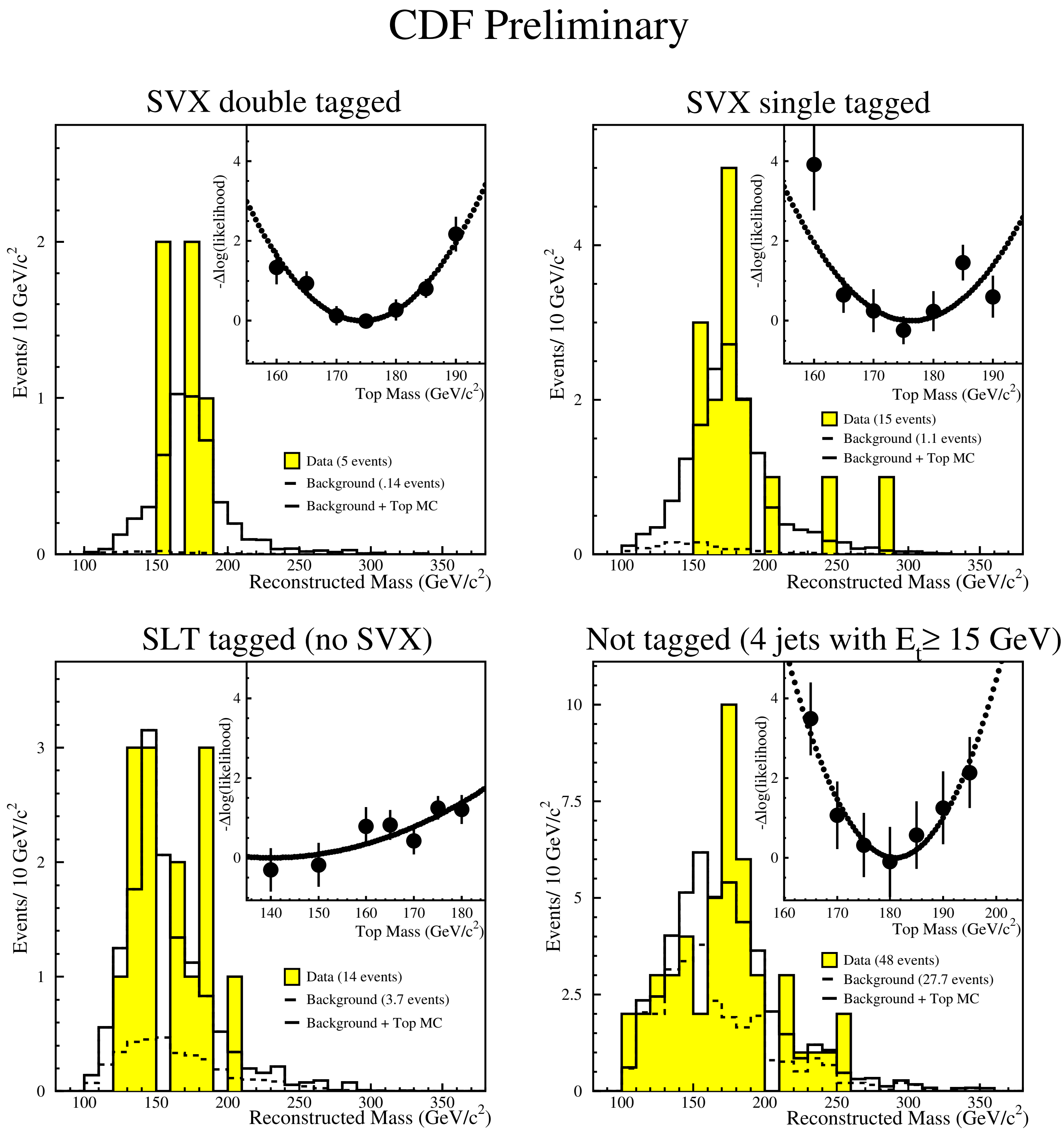

Figure 2: The top mass distribution for the svx double tagged, SVX single tagged, SLT tagged, and not tagged samples, along with the background (dashed) and top Monte Carlo plus background (solid) distributions. The top Monte Carlo mass used was 175, 180, 140, and 180 GeV/c^2, respectively. Also shown is the parabolic likelihood fit to each sample.

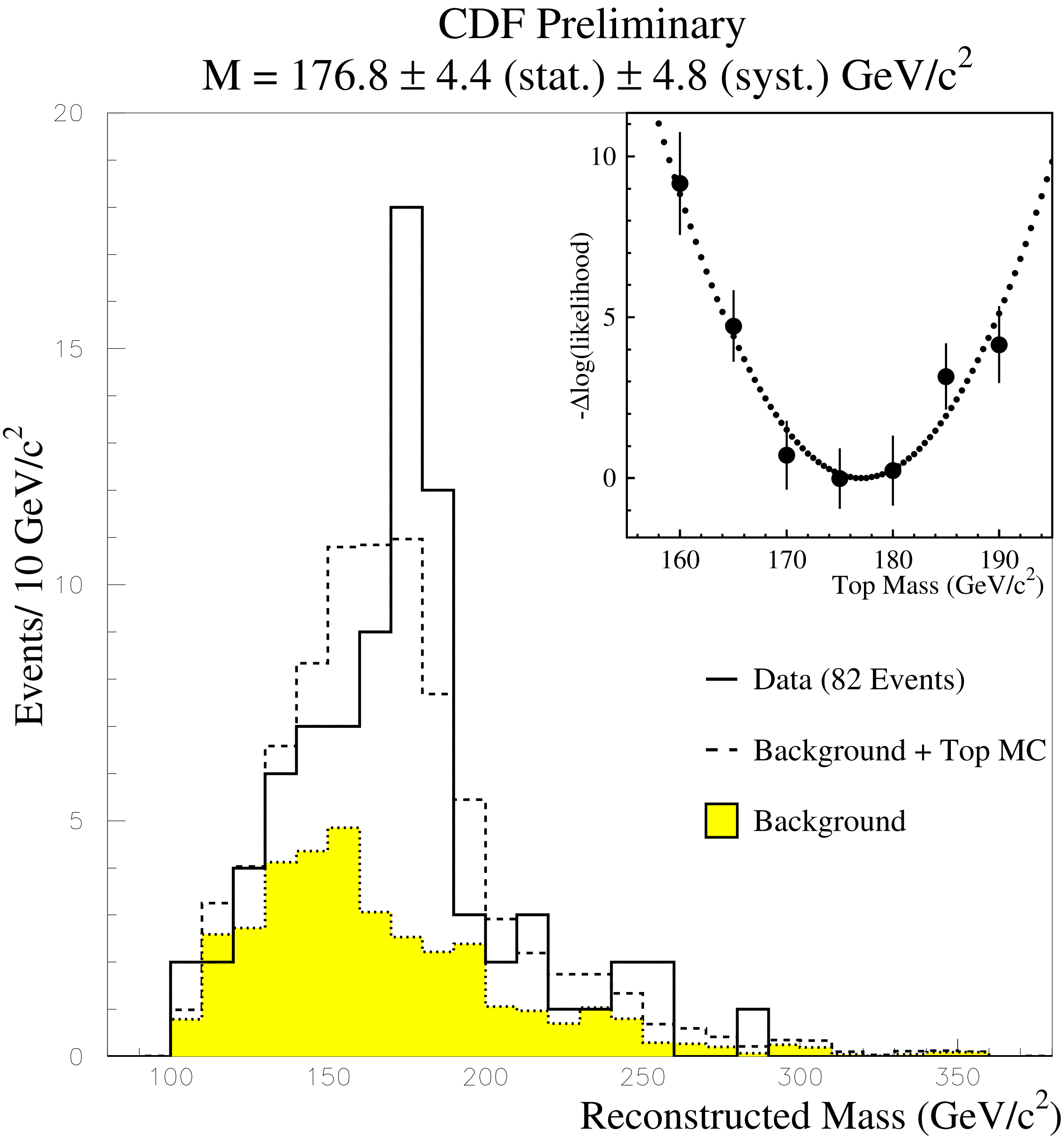

Figure 3: The top mass distribution for the optimized mass with background and background plus top Monte Carlo ($M_{top} = 175$) overlayed.

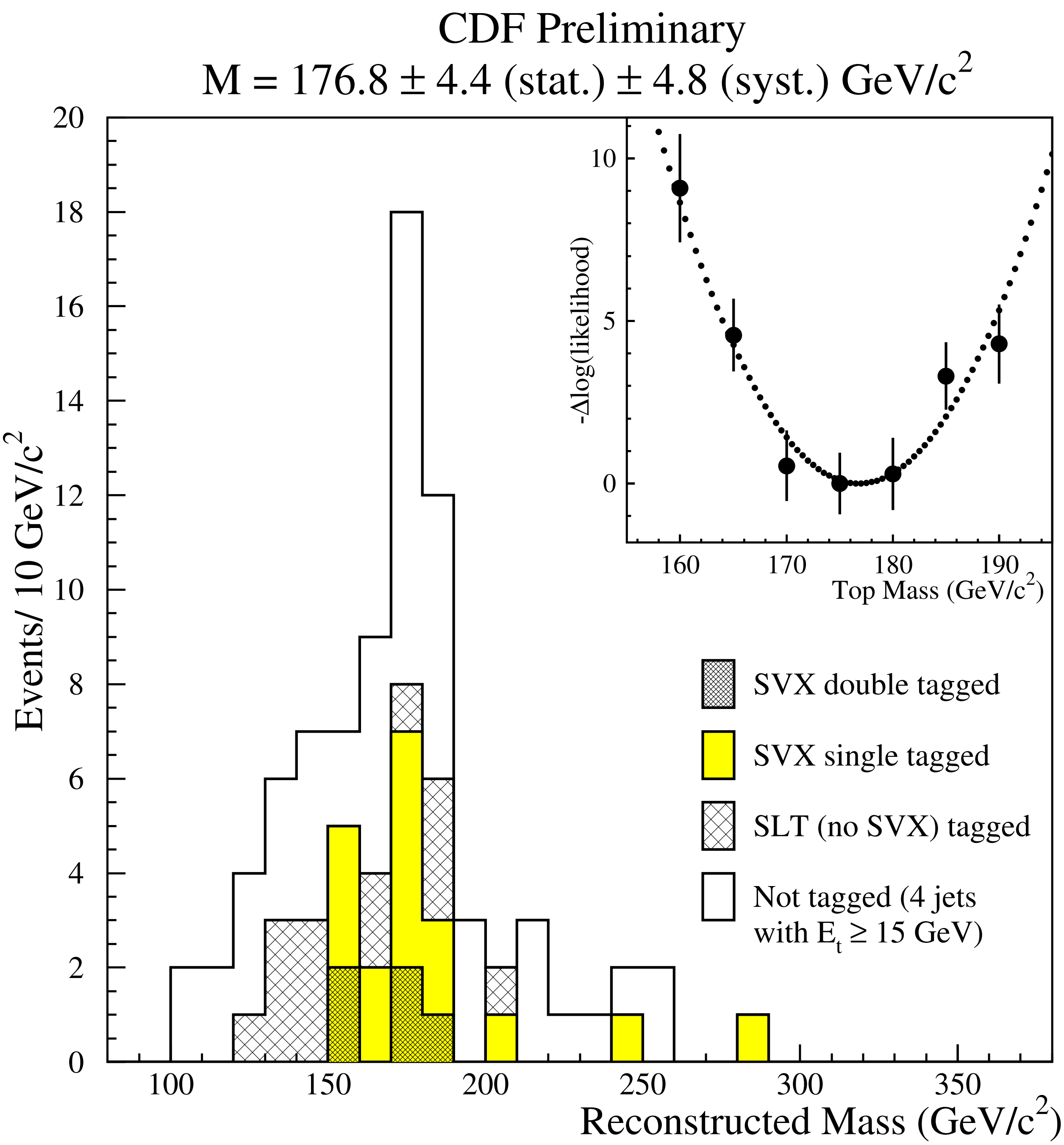

Figure 4: The top mass distribution for the all four samples combined.

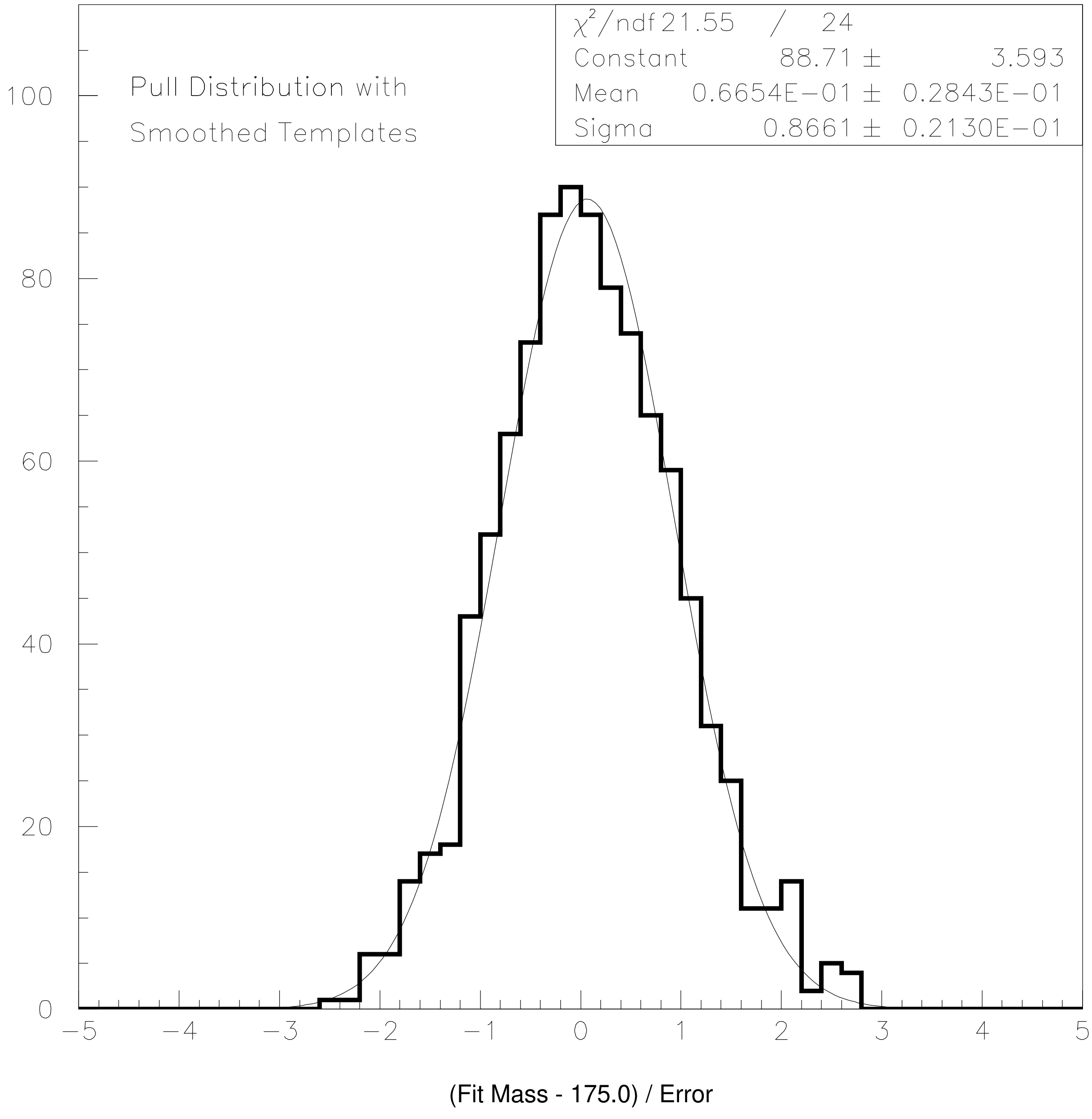

Figure 5: The pull distribution from pseudo-experiments.

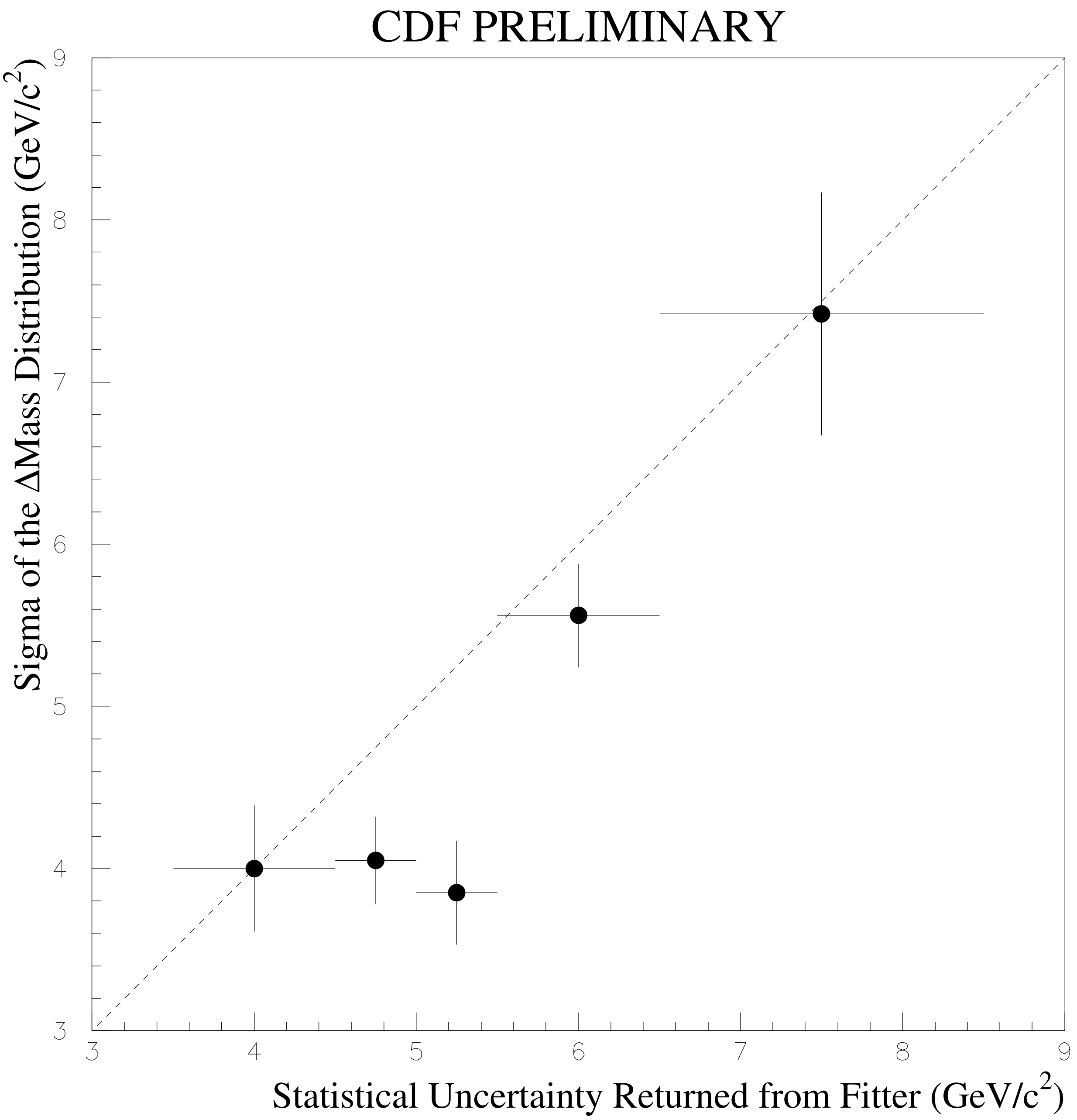

Figure 6: The width of the mass distribution as a function of statistical uncertainty versus the returned statistical uncertainty for pseudo-experiments.

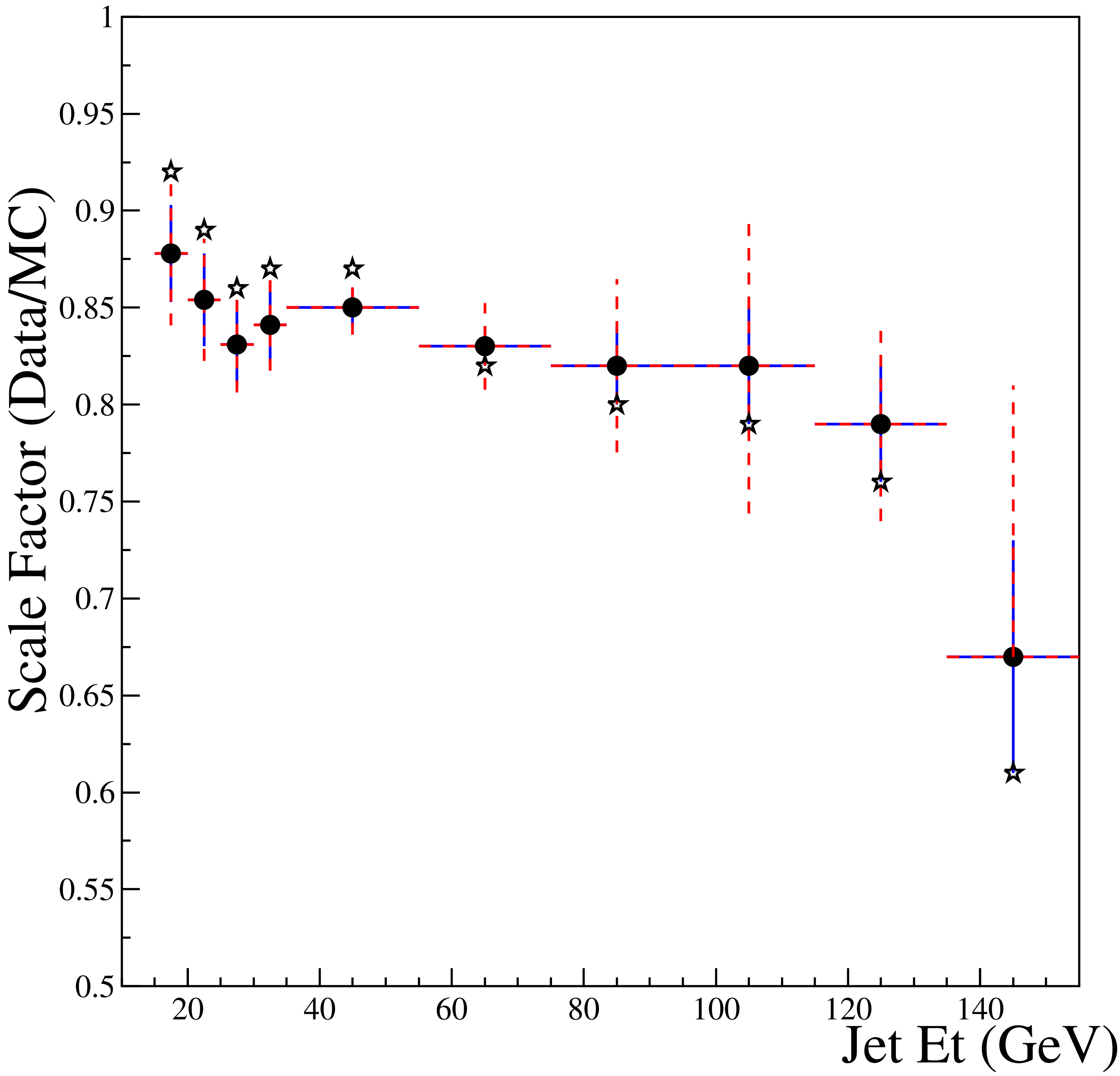

Figure 7: SVX B-tagging scale factor (data-over-Monte Carlo) vs. E_T as given in the analysis presented in CDF3563. The systematic uncertainty comes from assuming no Q dependence and a one standard deviation stronger Q dependence in the CTC tracking efficiency. The largest deviation from a flat distribution (stars) was used to sculpt the standard top SVX Monte Carlo.